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Neutrino Processes in Neutron Stars

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Abstract. The aim of these lectures is to introduce basic processes responsible for cooling of neutron stars and to show how to calculate the neutrino production rate in dense strongly interacting nuclear medium. The formalism is presented that treats on equal footing one-nucleon and multiple-nucleon processes and reactions with virtual bosonic modes and condensates. We demonstrate that neutrino emission from dense hadronic component in neutron stars is subject of strong modifications due to collective effects in the nuclear matter. With the most important in-medium processes incorporated in the cooling code an overall agreement with available soft X ray data can be easily achieved. With these findings the so-called *"standard"* and *"non-standard"* cooling scenarios are replaced by one general *"nuclear medium cooling scenario"* which relates slow and rapid neutron star coolings to the star masses (interior densities).
The lectures are split in four parts.

Part I: After short introduction to the neutron star cooling problem we show how to calculate neutrino reaction rates of the most efficient one-nucleon and two-nucleon processes. No medium effects are taken into account in this instance. The effects of a possible nucleon pairing are discussed. We demonstrate that the data on neutron star cooling cannot be described without inclusion of medium effects. It motivates an assumption that masses of the neutron stars are different and that neutrino reaction rates should be strongly density dependent.

Part II: We introduce the Green's function diagram technique for systems in and out of equilibrium and the optical theorem formalism. The latter allows to perform calculations of production rates with full Green's functions including all off-mass-shell effects. We demonstrate how this formalism works within the quasiparticle approximation.

Part III: The basic concepts of the nuclear Fermi liquid approach are introduced. We show how strong interaction effects can be included within the Green's function formalism. Softening of the pion mode with an baryon density increase is explicitly incorporated. We show examples of inconsistencies in calculations without inclusion of medium effects. Then we demonstrate calculations of different reaction rates in non-superfluid nuclear matter with taking into account medium effects. Many new reaction channels are open up in the medium and should be analyzed.

Part IV: We discuss the neutrino production reactions in superfluid nuclear systems. The reaction rates of processes associated with the pair breaking and formation are calculated. Special attention is focused on the gauge invariance and the exact fulfillment of the Ward identities for the vector current. Finally we present comparison of calculations of neutron star cooling performed within nuclear medium cooling scenario with the available data.

References

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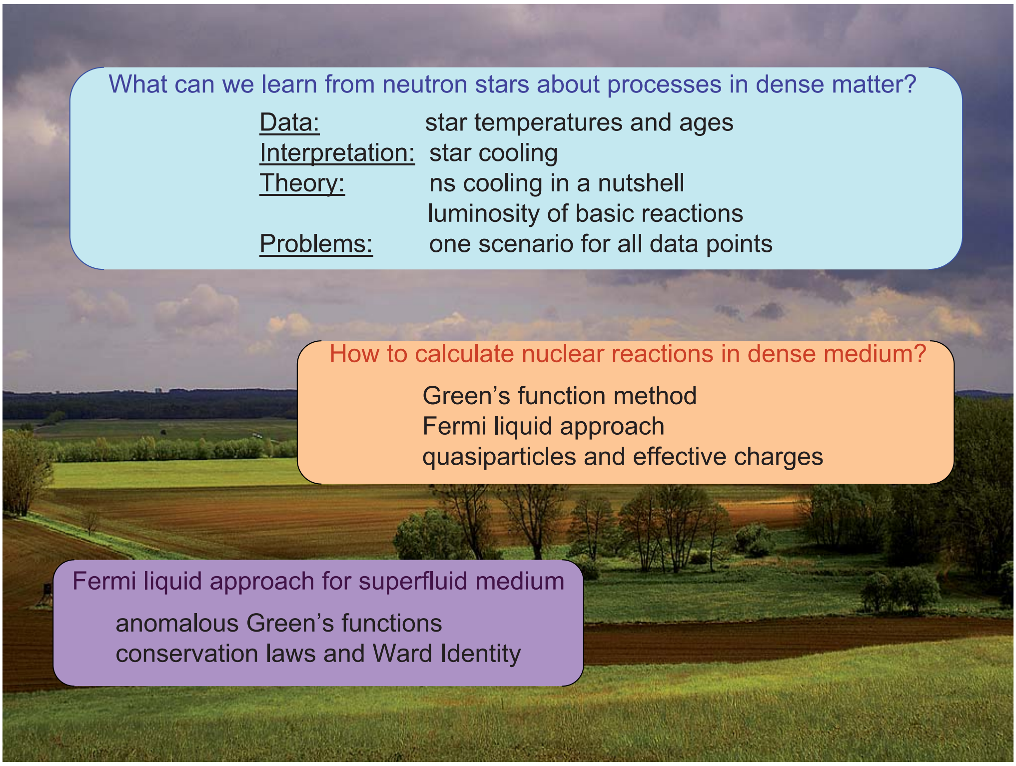
Neutrino Processes in Neutron Stars


What can we learn from neutron stars about processes in dense matter?
Data: star temperatures and ages
Interpretation: star cooling
Theory: ns cooling in a nutshell
luminosity of basic reactions
Problems: one scenario for all data points
How to calculate nuclear reactions in dense medium?
Green's function method
Fermi liquid approach
quasiparticles and effective charges
Fermi liquid approach for superfluid medium
anomalous Green's functions
conservation laws and Ward Identity

Structure of NS

- **atmosphere** mm — 10 cm, $\rho \lesssim 10^6$ g/cm^3,
  plasma: determines photon radiation; $T_{\rm surf}$, $B_{\rm surf}$ affect EOS;
- **outer crust** $\sim 10^2$ m, $\rho < \rho_d = 4 \cdot 10^{11}$ g/cm^3,
  solid of heavy nuclei and relativistic electrons;
- **inner crust** $\sim$ km, $\rho \lesssim 0.5 \div 0.7\rho_0$,
  neutronized nuclei + neutron gas + rel. electrons;
  $0.3\rho_0 \lesssim \rho \lesssim 0.5 \div 0.7\rho_0$,
  nuclear pasta = mixed phase: nuclear drops, rods, slabs, etc.;
- **outer core** $\gtrsim$ from several km, maybe, up to the center, $\rho \lesssim 2 \div 4\rho_0$,
  superfluid (at $T \lesssim$ MeV) of nn, pp + normal electrons;
- **inner core** up to center (larger for massive NS),
  Kingdom of Exotics:
  possible mixed phases between npe, π_c, K_c, H, q-CSC(?),
  pure phases π_c, K_c, H, q-CSC?

Neutrino probe

At temperatures **smaller** than the opacity temperature (T^{opac}~1-few MeV)
mean free path of neutrinos and antineutrinos is **larger** than the neutron star radius

$$\lambda_\nu \gg R \simeq 10\text{km}$$

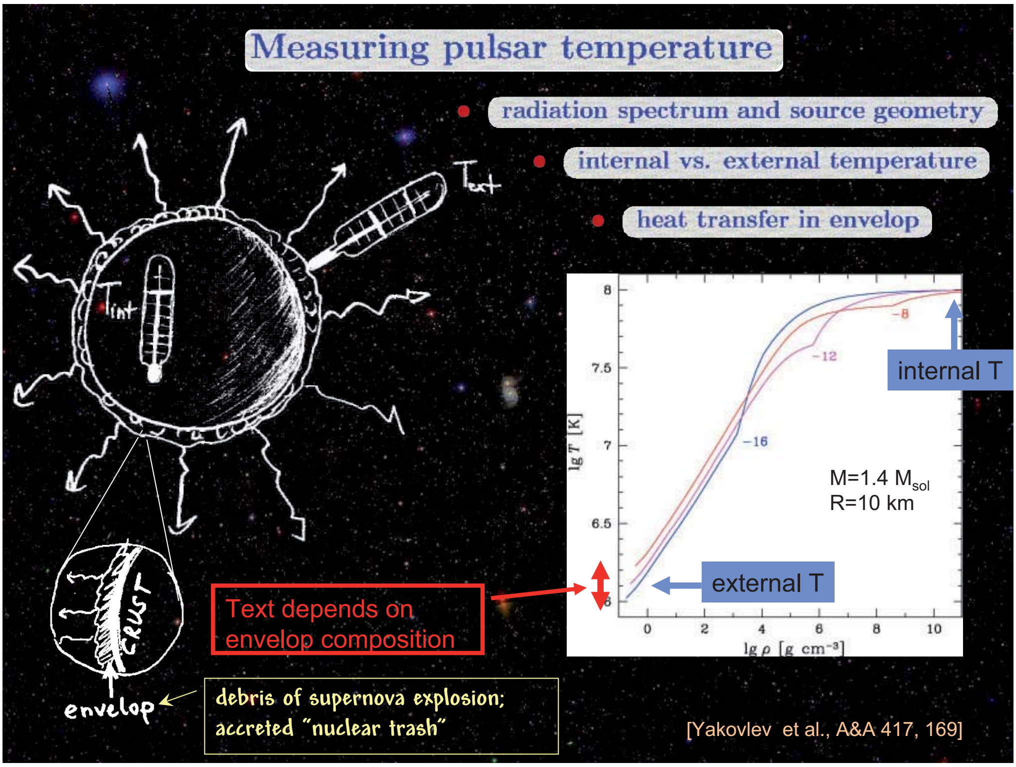
➡ **white body radiation problem**

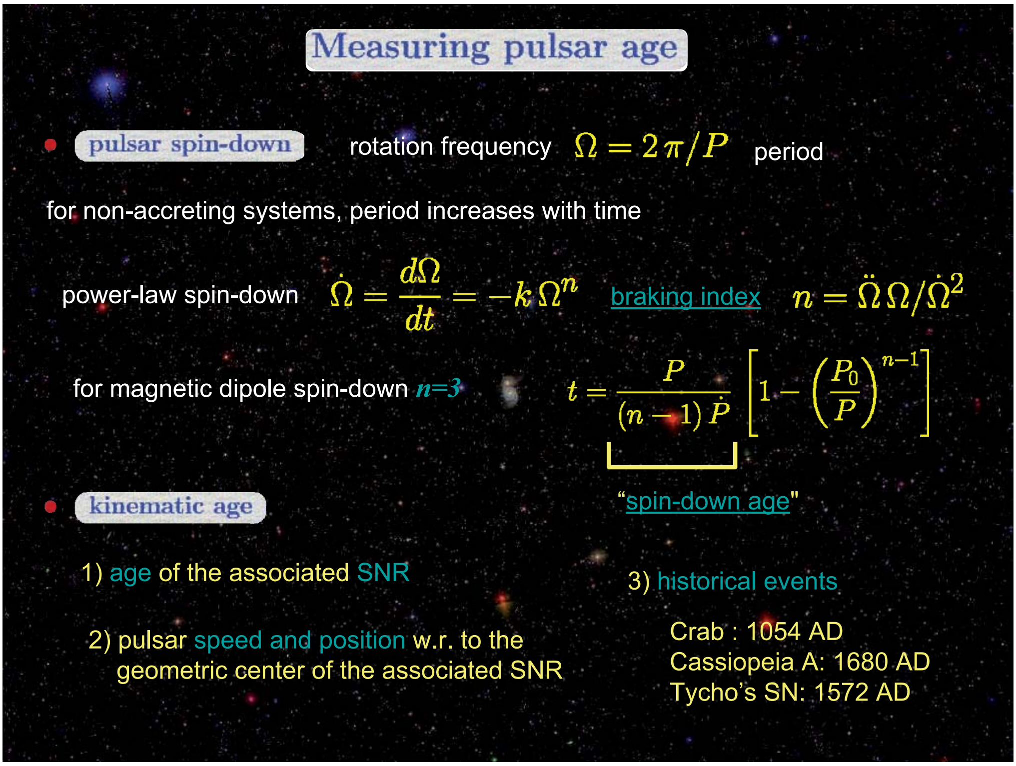
After >10^5 yr –black body radiation of photons

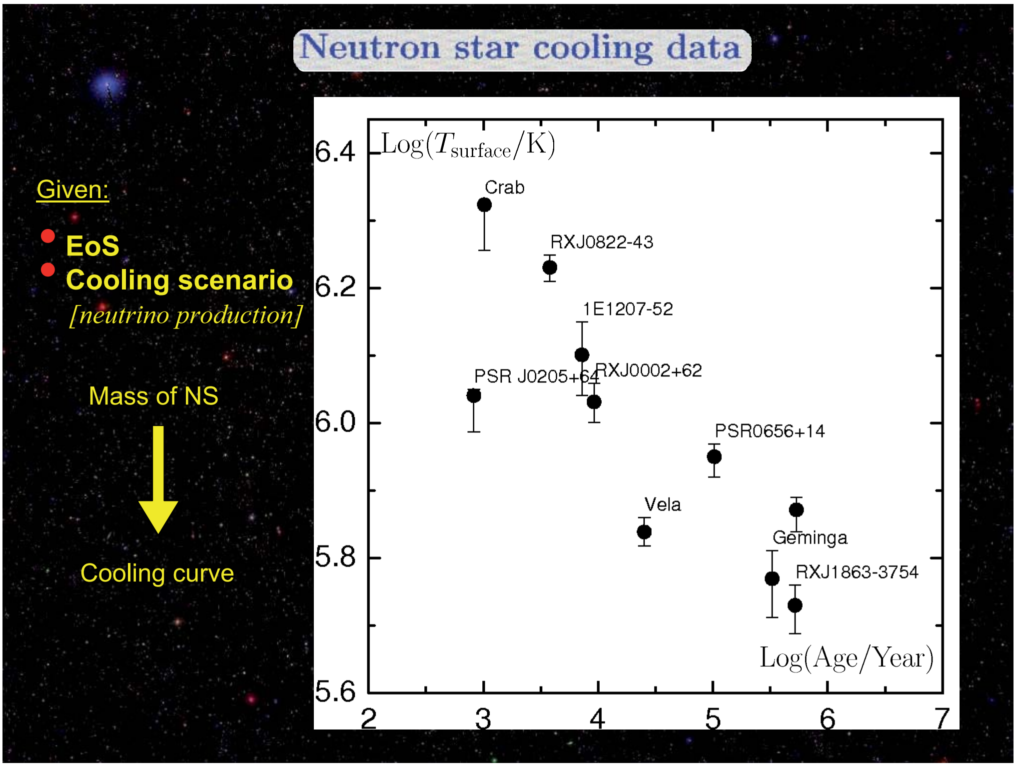
At temperatures $T > T^{opac}$ $\lambda_\nu < R$

➡ **neutrino transport problem**

important for supernova


Measuring pulsar temperature
radiation spectrum and source geometry
internal vs. external temperature
heat transfer in envelop
T_{ext}
T_{int}
CRUST
envelop
Text depends on envelop composition
debris of supernova explosion; accreted "nuclear trash"
internal T
external T
-8
-12
-16
M=1.4 M_{sol}
R=10 km
lg T [K]
lg ρ [g cm^{-3}]
[Yakovlev et al., A&A 417, 169]


Measuring pulsar age
pulsar spin-down
rotation frequency $\Omega = 2\pi/P$ period
for non-accreting systems, period increases with time
power-law spin-down $\dot{\Omega} = \frac{d\Omega}{dt} = -k\,\Omega^n$
braking index $n = \ddot{\Omega}\,\Omega/\dot{\Omega}^2$
for magnetic dipole spin-down $n=3$
$t = \underbrace{\frac{P}{(n-1)\,\dot{P}}}\left[1 - \left(\frac{P_0}{P}\right)^{n-1}\right]$
"spin-down age"
kinematic age
1) age of the associated SNR
2) pulsar speed and position w.r. to the geometric center of the associated SNR
3) historical events
Crab : 1054 AD
Cassiopeia A: 1680 AD
Tycho's SN: 1572 AD



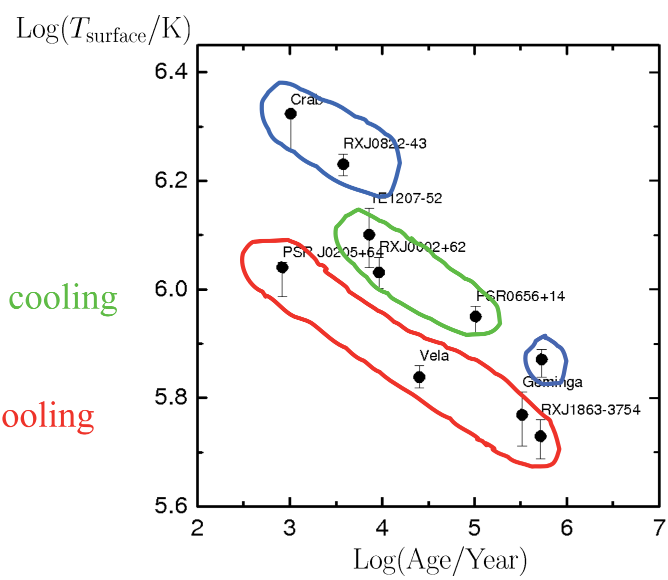
Neutron star cooling data

3 groups:

slow cooling

intermediate cooling

rapid cooling



How to describe all groups within one cooling scenario?

Neutrino emission reactions

$T < T_{\rm opac} \sim 10^{-1} \div 10^{0}$ MeV   neutron star is transparent for neutrino

$$C_V \frac{\mathrm{d}T}{\mathrm{d}t} = -L$$

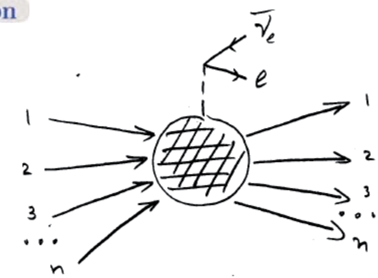
C_V - *specific heat,* L - *luminosity*

$$L = \int \mathrm{d}V \sum_{\text{reaction r}} \epsilon_\nu^{(r)}$$

emissivity

✔ n-nucleon reaction

$T \ll \epsilon_{\rm F}$



$\Rightarrow \epsilon_\nu \sim T^{2n+4}$

each leg on a Fermi surface $\propto T$

neutrino phase space × neutrino energy $\omega_{\bar\nu} \times \delta(\omega_{\bar\nu} - \ldots)\, \omega_{\bar\nu}^2\, d\omega_{\bar\nu} \sim T^3$

Direct reactions

Cooling: role of crust and interior?

most important are reactions in the interior

(The baryon density is $n \gtrsim n_0$ where n_0 is the nuclear saturation density)

one-nucleon reactions:	$n \to p + e + \bar\nu$	direct URCA (DU)	$\sim T^6$
two-nucleon reactions:	$n + n \to n + p + e + \bar\nu$	modified URCA (MU)	$\sim T^8$
	$n + n \to n + n + \nu + \bar\nu$	nucleon bremsstrahlung (NB)	

URCA=Gamow's acronym for "Un-Recordable Coolant Agent"

NS cooling in a nutshell

black body radiation $$L_\gamma = 4\,\pi\,R^2\,\sigma_{\rm SB}\,T_{\rm ext}^4 = 7.8\cdot 10^{43}\,T_{\rm ext,9}^{4}\,\frac{\rm erg}{s}$$

external temperature $$T_{\rm ext}/K \simeq \begin{cases} \alpha\,T_{\rm int}/K & T > T_{\rm crust} \\ 45(T_{\rm int}/K)^{0.65} & T < T_{\rm crust} \end{cases}$$

star is too hot; crust is not formed

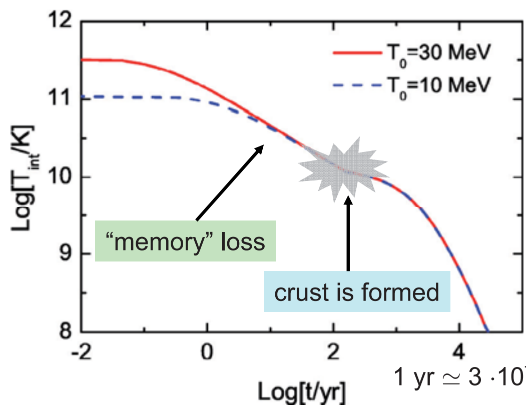
heat transport thru envelop

$$C_V = C\cdot T$$

$$C = 10^{30}\frac{\rm erg}{\rm K^2} = 10^{48}\,\frac{\rm erg}{(10^9\,{\rm K})^2}$$

$$C\int\limits_{T_0}^{T(t)}\frac{T\,dT}{L_\gamma(T)} = -t$$

$$T_9 = T/(10^9{\rm K})$$



1 yr $\simeq 3\cdot 10^7$s

NS cooling in a nutshell

volume neutrino radiation $\quad T < T_{\rm opac} = 1$ MeV

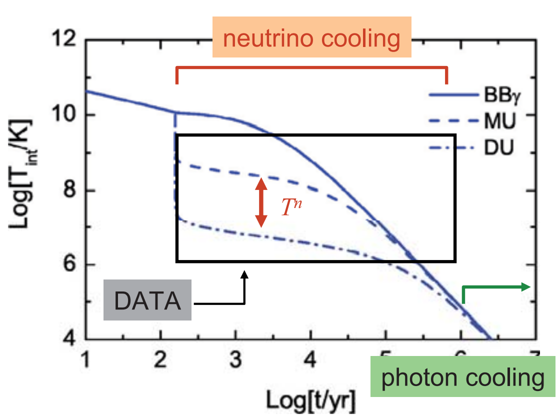
DU: $\epsilon_{\rm DU} = 10^{27}\,T_9^{6}\,\frac{\rm erg}{\rm s\ cm^3}$

$L_{\rm DU} = V\,\epsilon_{\rm DU} = 4\cdot 10^{45}\,T_9^{6}\,\frac{\rm erg}{\rm s}$

MU: $\epsilon_{\rm MU} = 2\cdot 10^{21}\,T_9^{8}\,\frac{\rm erg}{\rm s\ cm^3}$

$L_{\rm MU} = V\,\epsilon_{\rm MU} = 8\cdot 10^{39}\,T_9^{8}\,\frac{\rm erg}{\rm s}$

$$\int\limits_{T(t)}^{T_0}\frac{C\,T\,dT}{L_\gamma(T) + L_\nu(T)\,\theta(T_{\rm opac}-T)} = -t$$



$V = \frac{4\pi}{3}\,R^3 = \frac{4\pi}{3}\,(10\,{\rm km})^3 = 4\cdot 10^{18}\,{\rm cm}^3$

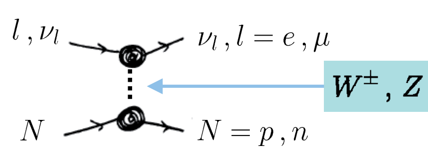
We consider, first, how to calculate the basic neutrino production processes.

No in-medium effects are taken into account

In-medium effects will be studied in lectures 2 and 3
in terms of Green's function formalism

Weak interactions

Low-energy weak interaction of nucleons



effective weak coupling constant $\quad G = 1.16\cdot 10^{-5}\,\mathrm{GeV}^{-2}$

$$L_{\mathrm{weak}} = \frac{G}{\sqrt{2}}\, j_\mu \times l^\mu$$

lepton current $\quad l_\mu = \bar{u}(q_1)\,\gamma_\mu(1-\gamma_5)\,u(q_2)$

nucleon current

$$j_\mu \;=\; V_\mu - A_\mu = \#\,(\bar{N}\,\gamma_\mu N) - \#\,(\bar{N}\,\gamma_\mu\,\gamma_5 N)$$

Weak interactions

Weinberg's Lagrangian:

$$\mathcal{L} = \frac{g}{\sqrt{2}}\left(J_\mu^- W_\mu^+ + J_\mu^+ W_\mu^-\right) + \frac{g}{\cos\theta_W}\left(J_\mu^{(3)} - \sin^2\theta_W J_\mu^{\text{e.m.}}\right) Z_\mu + g\sin\theta_W J_\mu^{\text{e.m.}} A_\mu$$

<u>lepton current</u> $\quad l_\mu = \bar{u}(q_1)\,\gamma_\mu(1-\gamma_5)\,u(q_2)$

<u>nucleon current</u> $\quad < N|j_\mu|N > \; = \; V_\mu^{NN} - A_\mu^{NN} = \bar{g}_V\,(\bar{N}\,\gamma_\mu N) - \bar{g}_A(\bar{N}\,\gamma_\mu\,\gamma_5 N)$

$$V_\mu^{np} \approx g_V\,\chi_p^\dagger(p')\big(1,\boldsymbol{v}\big)\chi_n(p)$$

$$V_\mu^{nn} \approx -\frac{g_V}{2}\,\chi_n^\dagger(p')\big(1,\boldsymbol{v}\big)\chi_n(p)$$

$$V_\mu^{pp} \approx +\frac{g_V}{2}\,\mathrm{c_V}\chi_p^\dagger(p')\big(1,\boldsymbol{v}\big)\chi_p(p)$$

$$g_V = 1 \qquad \boldsymbol{v} = \frac{\boldsymbol{p}+\boldsymbol{p}'}{2\,m_N}$$

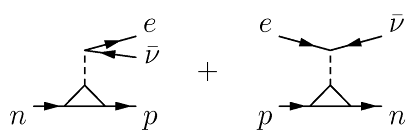
$$\mathrm{c_V} = 1 - 4\sin^2\theta_W \simeq 0.08$$

$$A_\mu^{np} \; = \; -2\,A_\mu^{pp} = -2\,A_\mu^{nn}$$
$$\approx \; g_A\,\chi_p^\dagger(p')\big(\boldsymbol{\sigma}\cdot\boldsymbol{v},\boldsymbol{\sigma}\big)\chi_n(p)$$

$$g_A = 1.26$$

Note 1/2 in neutral channel,
since *Z* boson is neutral and *W* is charged!

One-nucleon processes (DU)



emissivity:

$$\epsilon_\nu^{\rm DU} = 2\int\frac{d^3p_n}{(2\pi)^3}f_n\int\frac{d^3p_p}{(2\pi)^3}(1-f_p)\int\frac{d^3q_e}{2\omega_e(2\pi)^3}(1-f_e)\int\frac{d^3q_{\bar\nu}\,\omega_{\bar\nu}}{2\omega_{\bar\nu}(2\pi)^3}(2\pi)^4\delta^{(4)}(P_f-P_i)\sum_{\rm spins}|M|^2$$

<u>matrix element</u>

$$\sum_{\rm spins}|M|^2 = \frac{G^2}{2}\left[g_V^2\,{\rm Tr}\{l^0\,l^0\}\;{\rm Tr}\{\mathbf{1}\} + g_A^2{\rm Tr}{\rm Tr}(\boldsymbol{l}\cdot\boldsymbol{\sigma})\,(\boldsymbol{l}\cdot\boldsymbol{\sigma})\right] \simeq 8\,G^2\omega_e\omega_\nu(1+3g_A^2)$$

$${\rm Tr}\,l^\mu\,l^{\dagger\nu} = 8\,[q_1^\mu\,q_2^\nu + q_1^\nu\,q_2^\mu - (q_1\cdot q_2)\,g^{\mu\nu} - i\epsilon^{\mu\nu\eta\rho}(q_1)_\eta\,(q_2)_\rho]$$

$${\rm Tr}\,l^0\,l^{\dagger 0} = 8\,[\omega_1\,\omega_2 + \boldsymbol{q}_1\,\boldsymbol{q}_2] \qquad \sum_i^3{\rm Tr}l^i l^{\dagger i} = 8\,[3\,\omega_1\,\omega_2 - \boldsymbol{q}_1\boldsymbol{q}_2]$$

phase space integration

simplifications for $T \ll \epsilon_{Fn}, \epsilon_{Fp}$ $\qquad \omega_{\bar{\nu}} = q_{\bar{\nu}} \sim T$

$$d^3 p_i \simeq d^3 p_i \int dE_i \, \delta(E_i - E_{pi}) \simeq d^3 p_i \int dE_i \, \delta(E_i - E_{\mathrm{F}i})$$

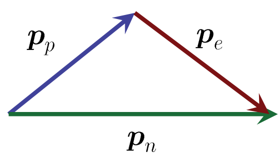
$$\delta(E_i - E_{\mathrm{F}i}) \simeq \frac{m_i}{p_{\mathrm{F}i}} \delta(p_i - p_{\mathrm{F}i})$$

angle integration

on Fermi surfaces

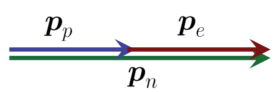
$$A_{\mathrm{DU}} = \int d\Omega_n \, d\Omega_p \, d\Omega_e \, \delta(\boldsymbol{p}_n - \boldsymbol{p}_p - \boldsymbol{p}_e) = \frac{8\,\pi^2}{p_{\mathrm{F}e}\, p_{\mathrm{F}p}\, p_{\mathrm{F}n}} \Theta(p_{\mathrm{F}p} + p_{\mathrm{F}e} - p_{\mathrm{F}n})$$

triangle inequality



$$|\boldsymbol{p}_p| = |\boldsymbol{p}_e|$$

critical condition



energy integration

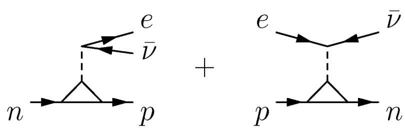
since the integration over energy goes from -∞ to +∞ and $\quad 1 - f(-E) = f(E)$

under integral we can replace $\quad 1 - f_i \Longrightarrow f_i$

$$I = \int_0^{+\infty} dx_{\bar{\nu}} \, x_{\bar{\nu}}^3 \prod_{j=1}^{3} \int_{-\infty}^{+\infty} dx_j \, f_j(x_j) \, \delta(x_1 - x_2 - x_3 - x_{\bar{\nu}}) = \frac{457\,\pi^6}{5040}$$

$$\epsilon_\nu^{\mathrm{DU}} = \frac{457\,\pi}{10080} G^2 \, (1 + 3\, g_A^2) \, m_n \, m_p \, p_{\mathrm{F},e} \, T^6 \, \Theta(2\, p_{\mathrm{F}p} - p_{\mathrm{F}n})$$

One-nucleon processes (DU)



allowed if $|p_{\mathrm{F},n} - p_{\mathrm{F},p}| < p_{\mathrm{F},e}$ ⟶ proton concentration > 11-14%

$$\epsilon_\nu^{\mathrm{DU}} \simeq 4 \cdot 10^{27} \left(\frac{n_e}{n_0}\right)^{1/3} T_9^6 \, \Theta(2p_{\mathrm{F},p} - p_{\mathrm{F},n}) \, \frac{\mathrm{erg}}{\mathrm{cm}^3 \cdot \mathrm{s}}$$

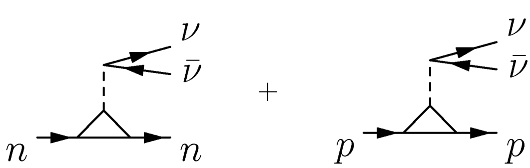
- large prefactor $\sim 10^{27} - 10^{28}$
- weak temperature dependence T^6

✔ very efficient reaction

BUT does not always occur

$$n > n_c^{\mathrm{DU}} \; (M > M_c^{\mathrm{DU}})$$

One-nucleon processes on neutral currents



energy-momentum conservation

$$\delta(\boldsymbol{p}_1 - \boldsymbol{p}_2 - \boldsymbol{q}_1 - \boldsymbol{q}_2)\, \delta(E_1 - E_2 - \omega_1 - \omega_2)$$

⇒
$$E_1 = E(p_1) = \frac{p_1^2}{2m_N} = \frac{(\boldsymbol{p}_1 - \boldsymbol{q}_1 - \boldsymbol{q}_2)^2}{2m_N} + \omega_1 + \omega_2$$
$$\approx \frac{p_1^2}{2m_N} - v_F\, |\boldsymbol{q}_1 + \boldsymbol{q}_2| \cos\theta + |\boldsymbol{q}_1| + |\boldsymbol{q}_2|$$

⇒ requires $v_{\mathrm{F}} \geq 1$

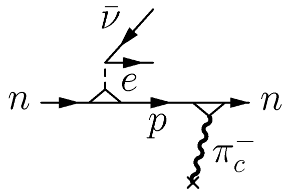
processes on neutral currents are forbidden!

Process on π^- condensate (PU)

assume μ_e reaches m_π $\quad e^- \to \pi^-$ $\quad n \to p + \pi^-$

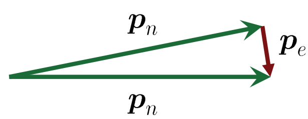
Bose condensate of pions

$k_\mu = (m_\pi, 0)$



neutrons in both initial and final states

$$A_{\rm PU} = \int d\Omega_{n_1}\, d\Omega_{n_2}\, d\Omega_\pi\, \delta(\boldsymbol{p}_{n_1} - \boldsymbol{p}_{n_2} - \boldsymbol{p}_e) \simeq \frac{8\,\pi^2}{p_{{\rm F},n}^2\, p_{{\rm F},e}} \cdot 1$$



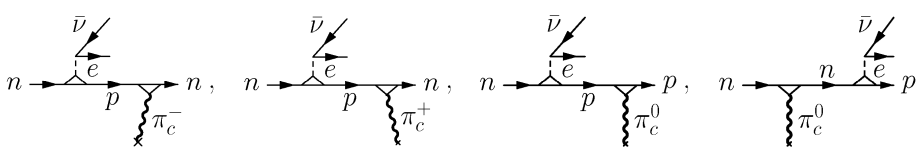
energy-momentum conservation is easily fulfilled

[Bahcall Wolf, Phys. Rev. (1965)]

Process on pion condensate (PU)

Migdal's pion condensate $k=(\mu_\pi, k_c)$: $\mu_\pi < m_\pi$, $k_c \sim p_{F,e}$ *p-wave condensate*

several processes are possible



$$\epsilon_\nu \sim 10^{26}\, T_9^6 (n/n_0)^{1/3} \;\frac{\rm erg}{\rm cm^3\ sec}$$

Kaon condensate processes yield a smaller contribution $[\propto \sin^2\theta_{\rm C} \simeq (0.23)^2]$

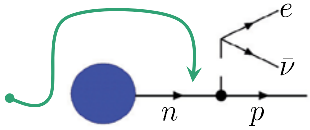
All "exotic" processes start only when the density exceeds some critical density

In order to calculate these reactions we have to introduce new diagrammatic elements:

Intermediate fermion propagator

outgoing proton:

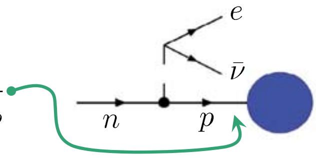
$$G = \frac{1}{E + \omega + \mu_n - E_n(p_1)} \approx \frac{1}{\omega + \mu_n - \mu_p}$$



since $\quad E \approx E_p(p) - \mu_p, \quad \text{for} \quad v_{\rm F}^N \ll 1$

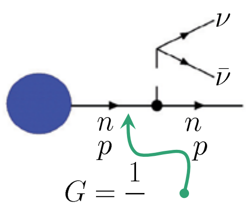
incoming neutron

$$G = \frac{1}{E - \omega + \mu_p - E_p(\boldsymbol{p} - \boldsymbol{q})} = -\frac{1}{\omega + \mu_n - \mu_p}$$

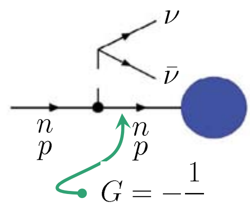


since $\quad E = E_n(p) - \mu_n, \quad \text{for} \quad v_{\rm F}^N \ll 1$

neutral processes



$$G = \frac{1}{\omega}$$



$$G = -\frac{1}{\omega}$$

πNN Vertices

$$\boxed{H_{\rm int} = f_{\pi NN}\, \psi_N^\dagger\, \sigma_i \tau_a \psi_N\, (\nabla_i \varphi_a)}$$

$(\varphi_1, \varphi_2, \varphi_3)$ three neutral pseudoscalar fields

$\psi_N = \begin{pmatrix} \psi_p \\ \psi_n \end{pmatrix}$ nucleon isospin dublet

$\sigma_1,\ \sigma_2,\ \sigma_3$ Pauli matrices acting in <u>spin</u> space

$\tau_1,\ \tau_2,\ \tau_3$ Pauli matrices acting in <u>isospin</u> space

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$$\tau_a\,\varphi_a = \tau_1\,\varphi_1 + \tau_2\,\varphi_2 + \tau_3\,\varphi_3 = \sqrt{2}\,\tau_+\,\pi^+ + \sqrt{2}\,\tau_-\,\pi^- + \tau_3\,\pi^0$$

$\pi^\pm = \dfrac{\varphi_1 \mp i\,\varphi_2}{\sqrt{2}}, \quad \pi^0 = \varphi_3$ pion annihilation operators

$\tau_\pm = \dfrac{\tau_1 \pm i\,\tau_2}{2}$ rising and lowering isospin matrices

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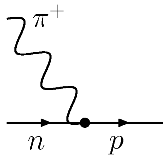
$$H_{\rm int} = f_{\pi NN}\left\{\sqrt{2}\,\psi_p^\dagger \boldsymbol{\sigma}\,\psi_n\, \boldsymbol{\nabla}\pi^+ + \sqrt{2}\,\psi_n^\dagger \boldsymbol{\sigma}\,\psi_p\, \boldsymbol{\nabla}\pi^- + \psi_p^\dagger \boldsymbol{\sigma}\psi_p\, \boldsymbol{\nabla}\pi^0 - \psi_n^\dagger \boldsymbol{\sigma}\psi_n\, \boldsymbol{\nabla}\pi^0\right\}$$

πNN Vertices

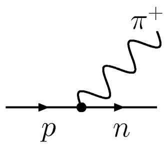
$< p|\,(-i)\,H_{\rm int}\,|n\,\pi^+ >=$



$= +\sqrt{2}\,f_{\pi NN}\,(\boldsymbol{\sigma}\cdot\boldsymbol{k})$

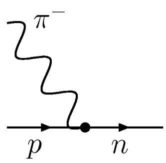
incoming vs. outgoing meson

$< n\,\pi^+|\,(-i)\,H_{\rm int}\,|p >=$



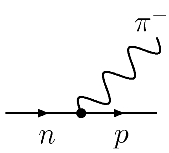
$= -\sqrt{2}\,f_{\pi NN}\,(\boldsymbol{\sigma}\cdot\boldsymbol{k})$

$< n|\,(-i)\,H_{\rm int}\,|p\,\pi^- >=$



$= +\sqrt{2}\,f_{\pi NN}\,(\boldsymbol{\sigma}\cdot\boldsymbol{k})$

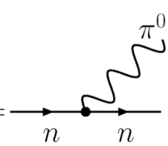
$< p\,\pi^-|\,(-i)\,H_{\rm int}\,|n >=$



$= -\sqrt{2}\,f_{\pi NN}\,(\boldsymbol{\sigma}\cdot\boldsymbol{k})$

πNN Vertices

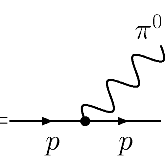
$< n\,\pi^0|\,(-i)\,H_{\rm int}\,|n >=$



$= +f_{\pi NN}\,(\boldsymbol{\sigma}\cdot\boldsymbol{k})$

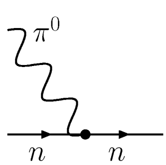
isospin τ_3 matrix

$< p\,\pi^0|\,(-i)\,H_{\rm int}\,|p >=$



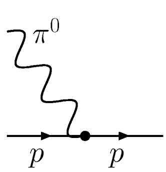
$= -f_{\pi NN}\,(\boldsymbol{\sigma}\cdot\boldsymbol{k})$

$< n|\,(-i)\,H_{\rm int}\,|n\,\pi^0 >=$



$= -f_{\pi NN}\,(\boldsymbol{\sigma}\cdot\boldsymbol{k})$

$< p|\,(-i)\,H_{\rm int}\,|p\,\pi^0 >=$

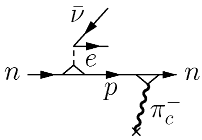


$= +f_{\pi NN}\,(\boldsymbol{\sigma}\cdot\boldsymbol{k})$

Emissivity of PU reaction on π^- condensate

Maxwell et al., AJ (1977)

PU process operates for $n>n_c^{\pi^-}$



$$iM = \chi_3^\dagger \sqrt{2}\, f_{\pi NN}\, (\boldsymbol{\sigma} \boldsymbol{k}_c \, < \phi_c >) \frac{-i}{\omega + \Delta\mu} i \frac{G}{\sqrt{2}} \big(l_0 - g_A\, (\boldsymbol{\sigma}\, \boldsymbol{l})\big) \chi_1$$

$$\omega + \Delta\mu \simeq \mu_e$$

Condensate field $\phi_c = a\, e^{-i\omega_c t + i\boldsymbol{k}_c \boldsymbol{r}}$ with fixed momentum and frequency and amplitude *a*

Using: $\mathrm{Tr}\{\sigma_i\, \sigma_j\} = 2\, \delta_{ij}$

$$\sum_{\text{spins}} |M|^2 = \frac{16\, f_{\pi NN}^2\, (1 + 3g_A^2)\, G^2\, k_c^2}{(\omega + \Delta\mu)^2} \omega_1\, \omega_2\, a^2$$

Inversed reaction gives extra factor 2

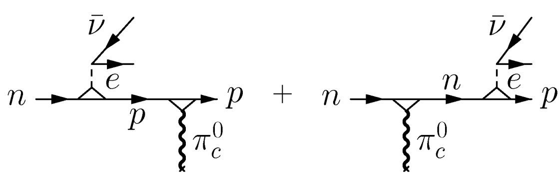
$$\epsilon_\nu^{\pi_c^-} = \frac{457\,\pi}{5040} G^2\, (1 + 3\, g_A^2)\, m_n^2\, p_{\mathrm{F},e}\, a_\pi^2\, T^6\, \Theta(n - n_c) \qquad a_\pi^2 = \frac{< |\pi|^2 >}{4\, f_\pi^2}$$

condensate amplitude

$$a^2 \sim (0.01 - 0.1) \quad \Rightarrow \quad \epsilon_\nu^{\pi_c^-} \sim (0.1 \div 1)\epsilon_\nu^{\mathrm{DU}}$$

Emissivity of PU reaction on π^0 condensate

PU process operates for $n>n_c^{\pi^0}$



D.V.,Senatorov JETP Lett.(1984)

$$\begin{aligned} iM = \quad & \chi_3^\dagger\, f_{\pi NN}\, (\boldsymbol{\sigma} \boldsymbol{k}_c) < \phi > \; \frac{i}{-\omega}\, \frac{i\,G}{\sqrt{2}}\, \big(l_0 - g_A\, (\boldsymbol{\sigma}\, \boldsymbol{l})\big)\chi_1 \\ - \; & \chi_3^\dagger\, \frac{i\,G}{\sqrt{2}}\, \big(l_0 - g_A\, (\boldsymbol{\sigma}\, \boldsymbol{l})\big) \; \frac{i}{\omega} \; f_{\pi NN}\, (\boldsymbol{\sigma} \boldsymbol{k}_c) < \phi > \chi_1 \\ = + \; & 2\, \frac{f_{\pi NN}\, G}{\sqrt{2}\, \omega}\, \chi_3^\dagger \Big(\boldsymbol{\sigma} \boldsymbol{k}_c < \phi > \, l_0 - g_A\, \boldsymbol{l} \boldsymbol{k}_c \; < \phi > \Big) \chi_1 \end{aligned}$$

For

$$\phi = a\, \cos(\boldsymbol{k}_c \boldsymbol{r}) \qquad \sum_{\text{spins}} |M|^2 = 32\, f_{\pi NN}^2\, (1 + g_A^2)\, G^2\, k_c^2\, \frac{\omega_1 \omega_2}{\omega^2}\, a^2\, \underbrace{< \sin^2(\boldsymbol{k}_c\, \boldsymbol{r}) >}_{=\frac{1}{2}}$$

Inversed reaction gives extra factor 2

$$\epsilon_\nu^{\pi_c^0} = \frac{457\,\pi}{5040}\, G^2\, (1 + g_A^2)\, f_{\pi NN}^2\, m_n\, m_p\, k_c\, a^2\, T^6 \sim \epsilon_\nu^{\pi_c^-}$$

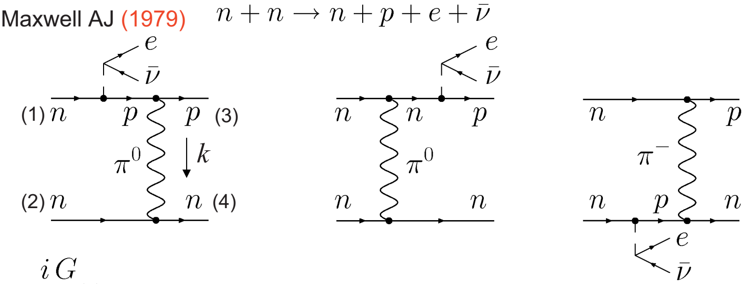
Two-nucleon process (MU)

$$n + p \to p + p + e + \bar{\nu} \qquad\qquad n + n \to n + p + e + \bar{\nu}$$

- ✔ no critical density
- ✔ 5 fermions ➡ suppressed phase-space volume (compared to one-nucleon processes)
- ✔ T^8 dependence of the emissivity
  (5 fermions ➡ $\sim T^5$, $\quad \omega_{\bar{\nu}}\,\delta(\omega_{\bar{\nu}} + \ldots)\,\omega_{\bar{\nu}}^2\, d\omega_{\bar{\nu}}$ ➡ T^3)

Two-nucleon process (Modified Urca)

Friman & Maxwell AJ (1979) $\quad n + n \to n + p + e + \bar{\nu}$



$$i\,M = f_{\pi NN}^2 \frac{i\,G}{\sqrt{2}} \times$$

$$\Big\{ \chi_3^\dagger \,[(-\boldsymbol{\sigma}\boldsymbol{k})\,(l_0 - g_A\,\boldsymbol{l}\,\boldsymbol{\sigma})]\,\chi_1 \frac{i^2\,D_{\pi^0}(\omega_\pi, k)}{-(\omega + \Delta\mu)}\,\chi_4^\dagger(-\boldsymbol{\sigma}\,\boldsymbol{k})\,\chi_2$$

$$+\chi_3^\dagger \,[(l_0 - g_A\,\boldsymbol{l}\,\boldsymbol{\sigma})\,(+\boldsymbol{\sigma}\boldsymbol{k})]\,\chi_1 \frac{i^2\,D_{\pi^0}(\omega_\pi, k)}{+(\omega + \Delta\mu)}\,\chi_4^\dagger(-\boldsymbol{\sigma}\,\boldsymbol{k})\,\chi_2$$

$$+\chi_3^\dagger \sqrt{2}(-\boldsymbol{\sigma}\boldsymbol{k})\chi_1 \frac{i^2\,D_{\pi^-}(\omega_\pi, k)}{-(\omega + \Delta\mu)}\,\chi_4^\dagger[\sqrt{2}(+\boldsymbol{\sigma}\boldsymbol{k})(l_0 - g_A\boldsymbol{l}\boldsymbol{\sigma})]\chi_2 \Big\}$$

Additionally one should take into account exchange reactions (identical nucleons)

Two-nucleon process (Modified Urca)

Emissivity:

$$\epsilon_\nu^{\rm MU} = \prod_{i=1}^{4}\int\left[\frac{d^3p_i}{(2\pi)^3}\right] f_1\, f_2\,(1-f_3)\,(1-f_4)\,\frac{d^3q_e\,(1-f_e)}{2\,\omega_e\,(2\pi)^3}$$
$$\times\ \frac{d^3q_{\bar\nu}}{2\,\omega_{\bar\nu}\,(2\pi)^3}\,\omega_{\bar\nu}\,(2\,\pi)^4\,\delta^{(4)}(P_f-P_i)\,\frac{1}{s}\sum_{spins}|M|^2,$$

$s=2$ is symmetry factor. Reactions with the electron in an initial state yield extra factor 2.

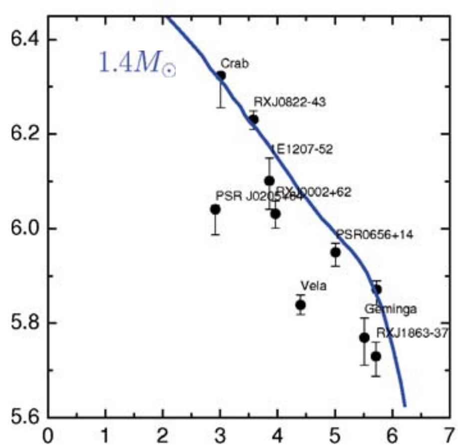
Finally

$$\epsilon_\nu^{\rm MU} = \frac{11513}{60480\,\pi}\,G^2\,g_A^2\,f_{\pi NN}^4\,m_n^3\,m_p\,p_{{\rm F},e}\,T^8\,\underbrace{1.3}_{\text{due to exchange reactions}} \simeq 8\cdot 10^{21}\,(n_p/n_0)^{1/3}\,T_9^8\,\times\,\frac{\rm erg}{{\rm cm}^3\cdot {\rm s}}$$

Coherence: only axial-vector term contributes (!)
whereas for PU processes both vector and axial-vector terms contribute

Standard scenario. DU scenario

standard scenario (MU)
only part of the data can be described

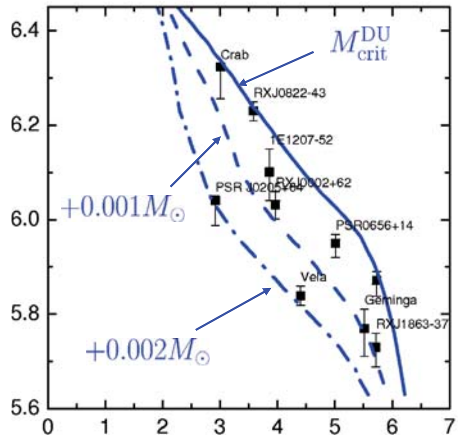


[Blaschke, Grigorian, Voskresensky A&A (2004)]

Standard scenario. DU scenario

standard scenario (MU)
only part of the data can be described

Direct-Urca scenario
NS masses close to $M_{\rm crit}^{\rm DU}$



[Blaschke, Grigorian, Voskresensky A&A (2004)]

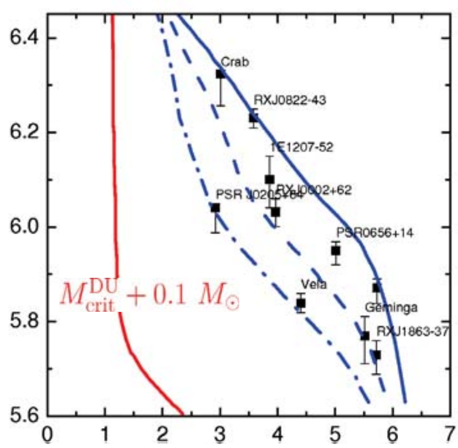
Standard scenario. DU scenario

standard scenario (MU)
only part of the data can be described

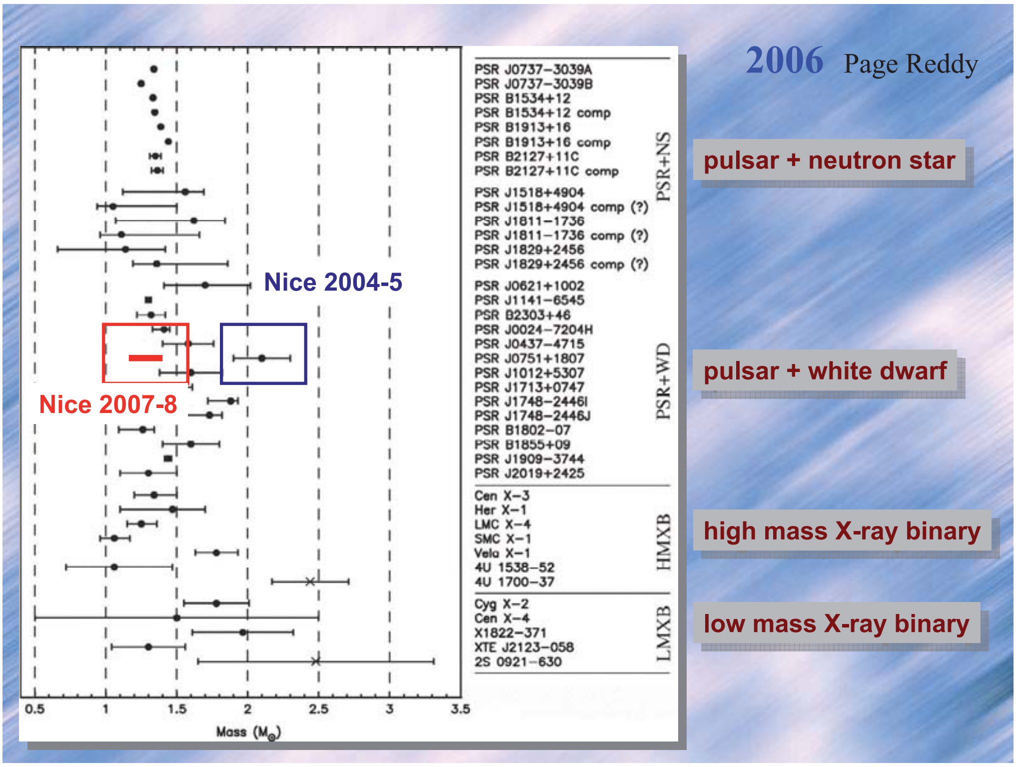
Direct-Urca scenario
NS masses close to $M_{\rm crit}^{\rm DU}$

Neutron stars with $M > M_{\rm crit}^{\rm DU}$
will be too cold

But masses of NS
are not close to each others



[Blaschke, Grigorian, Voskresensky A&A (2004)]



Direct-Urca Cooling Scenario ?

DU process schould be „exotics“

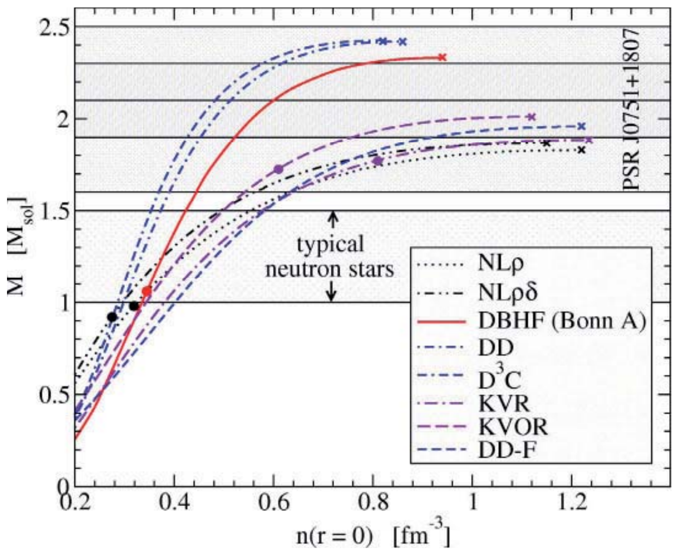
(if DU starts it is dificult to stop it)

- $M_{\rm crit}^{\rm DU} \gtrsim 1.3\ M_{\odot}$
- $n_{\rm crit}^{\rm DU} \gtrsim 4\, n_0$

EoS should produce a large DU threshold in NS matter !

[Kolomeitsev, Voskresensky NPA759 (2005)]

DU thresholds



Klähn et al. PRC (2006)

Pairing in nuclear matter

PHYSICAL REVIEW VOLUME 110, NUMBER 4 MAY 15, 1958

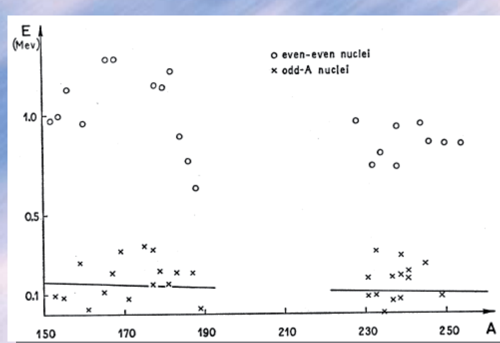
Possible Analogy between the Excitation Spectra of Nuclei and Those of the Superconducting Metallic State

A. BOHR, B. R. MOTTELSON, AND D. PINES*

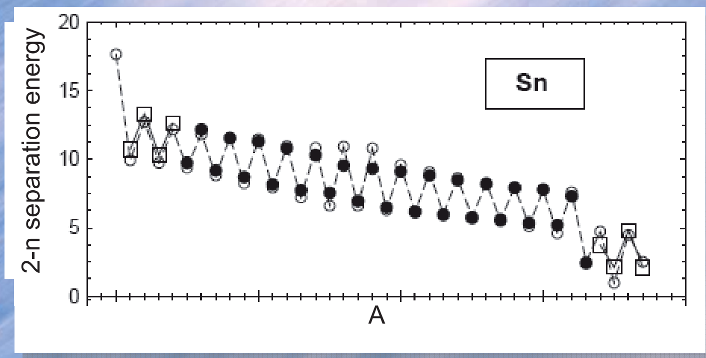
Institute for Theoretical Physics, University of Copenhagen, Copenhagen, Denmark, and Nordisk Institut for Teoretisk Atomfysik, Copenhagen, Denmark

(Received January 7, 1958)

The evidence for an energy gap in the intrinsic excitation spectrum of nuclei is reviewed. A possible analogy between this effect and the energy gap observed in the electronic excitation of a superconducting metal is suggested.



Pairing in nuclear matter



Nuclear Physics **13** (1959) 655—674; © *North-Holland Publishing Co., Amsterdam*

SUPERFLUIDITY AND THE MOMENTS OF INERTIA OF NUCLEI

A. B. MIGDAL

Atomic Energy Institute of USSR, Academy of Sciences, Moscow

It should be noted that superfluidity of nuclear matter may lead to some interesting cosmological phenomena if stars exist which have neutron cores. A star of this type would be in a superfluid state with a transition temperature corresponding to 1 MeV.

Pairing in nuclear matter

excitation spectrum

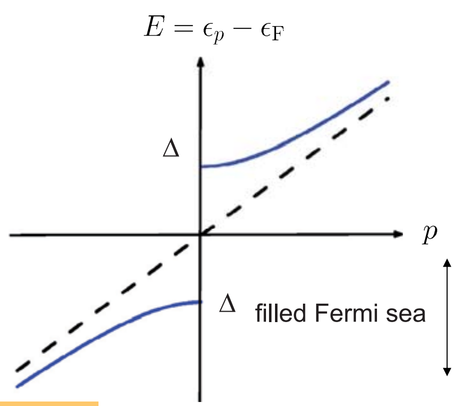
- without pairing

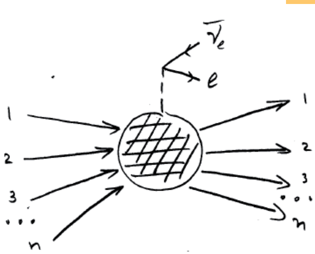
$$E = \epsilon_p - \epsilon_F \simeq v_F\,(p - p_F)$$

- with pairing

$$E = \pm\sqrt{v_F^2\,(p - p_F)^2 + \Delta^2}$$

pairing gap

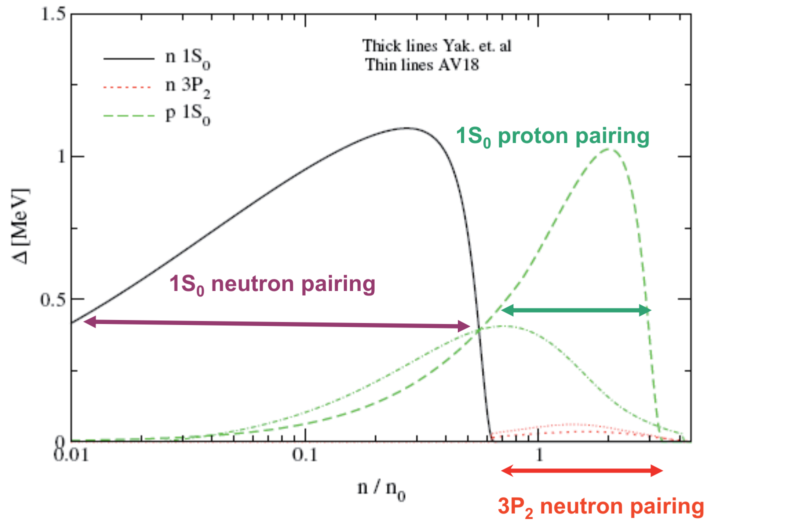




suppressed by factor $\sim e^{-n\Delta/T}$

Pairing gaps in nuclear matter

✔ pairing gaps

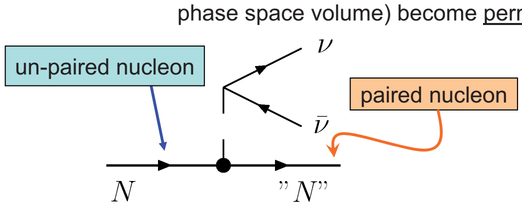


[Schwenk Friman, PRL (2004)]

Breaking and Formation of Cooper pairs (PBF)

In superfluid ($T<T_c<0.1$-1 MeV) all two-nucleon processes are suppressed by factor $exp(-2\Delta/T)$

new "quasi"-one-nucleon-like processes (one-nucleon phase space volume) become permitted



[Flowers, Ruderman, Sutherland, AJ 205 (1976), Voskresensky& Senatorov, Sov. J. Nucl. Phys. 45 (1987)]

$$\epsilon_\nu \sim 10^{29} \left[\frac{\Delta_{nn}}{\mathrm{MeV}}\right]^7 \left[\frac{T}{\Delta_{nn}}\right]^{1/2} (n/n_0)^{1/3}\, \xi_{nn}^2, \quad \frac{\mathrm{erg}}{\mathrm{cm}^3\,\mathrm{sec}}$$

Δ_{nn} is neutron gap and $\xi_{nn}=\exp(-\Delta_{nn}/T)$

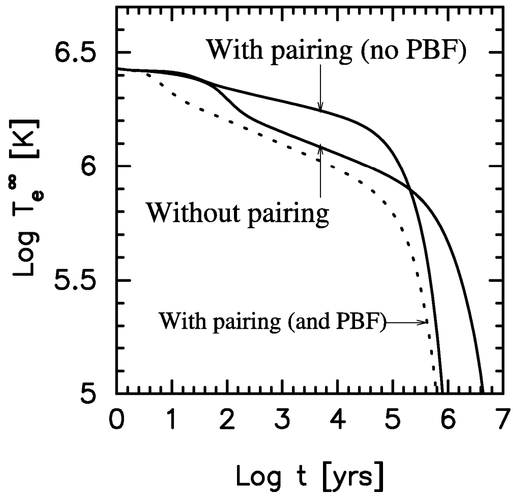
[Voskresensky, Senatorov, Sov. J. Nucl. Phys. 45 (1987); Senatorov, Voskresensky, Phys. Lett. B184 (1987); Voskresensky astro-ph/0101514]

not $\epsilon_\nu \sim 10^{20} T_9^7\, \xi_{nn}^2$ as in Flowers et al. (1976)

Naively one expect the emissivity of $p \to p \nu\bar{\nu}$ to be suppressed by extra $c_V^2 \sim 0.006$ factor.

Effects of pairing on the neutron star cooling

- ✔ enhanced cooling
- ✔ mass dependence

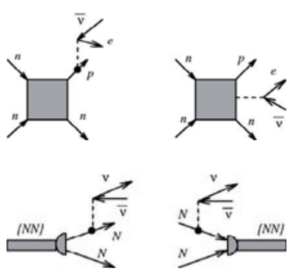


pair breaking and formation (PBF) processes are important!

[Page, Geppert, Weber , NPA (2006)]

Neutrino emission reactions

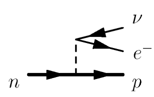
standard $\quad T < T_{\rm opac} \sim 10^{-1} \div 10^{0}\ \mathrm{MeV}$



$$10^{22} \times \left(\frac{m_N^*}{m_N}\right)^4 T_9^8 \left(\frac{n_e}{n_0}\right)^{\frac{1}{3}} \frac{\mathrm{erg}}{\mathrm{cm^3\,s}} \times e^{-2\Delta/T}$$

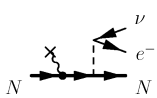
$$10^{29} \times \left(\frac{m_N^*}{m_N}\right) \left(\frac{\Delta}{\mathrm{MeV}}\right)^7 \left(\frac{T}{\Delta}\right)^{\frac{1}{2}} e^{-2\Delta/T} \frac{\mathrm{erg}}{\mathrm{cm^3\,s}}$$

exotic



$$10^{27} \times \left(\frac{m_N^*}{m_N}\right)^2 T_9^6 \left(\frac{n_e}{n_0}\right)^{\frac{1}{3}} \frac{\mathrm{erg}}{\mathrm{cm^3\,s}} \times e^{-\Delta/T}$$

allowed if $|p_{{\rm F},n} - p_{{\rm F},p}| < p_{{\rm F},e}$



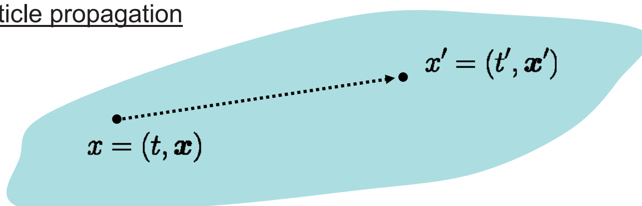
$$7 \cdot 10^{26} \times \left(\frac{m_N^*}{m_N}\right)^2 T_9^6 \frac{|\varphi_c|^2}{m_\pi^2} \frac{\mathrm{erg}}{\mathrm{cm^3\,s}} \times e^{-\Delta/T}$$

Green's functions

N-body system: wave function of the whole system $\Psi(x_1,x_2,\ldots,x_N)$
encodes the dynamics of all particles and is very complicated

Introduce the object which describes the dynamics of the reduced number of particles of interest

<u>one-particle propagation</u>



Amplitude of particle transition from a point (x,t) to a point (x',t')

$$\Psi(\boldsymbol{x}', t') = \int d\boldsymbol{x}\, G^{(+)}(\boldsymbol{x}', t'; \boldsymbol{x}, t)\Psi(\boldsymbol{x}, t) \qquad t' > t$$

for $t' = t + 0$ $\qquad \Psi(\boldsymbol{x}', t+0) = \int d\boldsymbol{x}\, G^{(+)}(\boldsymbol{x}', t+0; \boldsymbol{x}, t)\Psi(\boldsymbol{x}, t)$

$$G^{(+)}(\boldsymbol{x}', t+0; \boldsymbol{x}, t) = \delta(\boldsymbol{x}' - \boldsymbol{x})$$

If $\Psi(\boldsymbol{x},t)$ obeys the Schrödinger equation $\qquad [i\partial_t - H(\boldsymbol{x})]\,\Psi(\boldsymbol{x}, t) = 0$

$$[i\partial_t - H(\boldsymbol{x})]\, G^{(+)}(\boldsymbol{x}, t; \boldsymbol{x}', t') = i\,\delta(t - t')\,\delta(\boldsymbol{x} - \boldsymbol{x}')$$

for homogeneous system : $\quad G^{(+)}(\boldsymbol{x}', t'; \boldsymbol{x}, t) = G^{(+)}((\boldsymbol{x}' - \boldsymbol{x})^2, t' - t > 0)$

eigenfunctions: $\quad H\,\varphi_\lambda(\boldsymbol{x}) = \epsilon_\lambda(\boldsymbol{x})\,\varphi(\boldsymbol{x})$

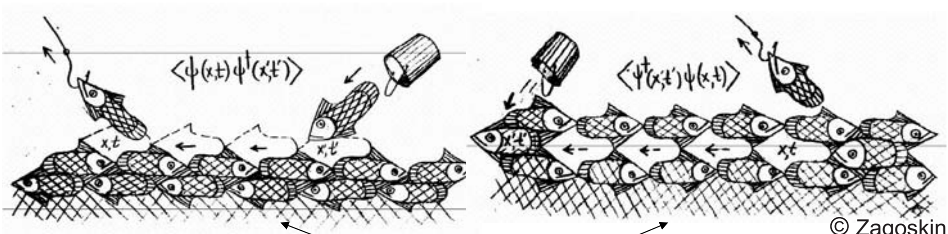
$$G^{(+)}(\boldsymbol{x}', \boldsymbol{x}, \tau = t' - t) = -\sum_\lambda \int_{-\infty}^{\infty} \frac{d\epsilon}{2\pi i}\, e^{-i\epsilon\tau} \frac{\varphi_\lambda(\boldsymbol{x}')\,\varphi^*_\lambda(\boldsymbol{x})}{\epsilon - \epsilon_\lambda + i\,0}$$

$$G^{(+)}(\boldsymbol{x}', t'; \boldsymbol{x}, t) =< N|\hat\Psi(\boldsymbol{x}', t')\,\hat\Psi^\dagger(\boldsymbol{x}, t)|N >$$

$$\hat\Psi(\boldsymbol{x}, t) = \sum_\lambda \varphi_\lambda(x)\,\hat a_\lambda\, e^{-i\,\epsilon_\lambda t} \qquad |N >= a^\dagger_a\, a^\dagger_2\, a^\dagger_3 \ldots a^\dagger_N\, |0 >$$

Green's function of non-interacting fermions

$$
\begin{aligned}
i\,G(\boldsymbol{x},t;\boldsymbol{x}',t') &= <N|T\{\hat{\Psi}(\boldsymbol{x},t)\,\hat{\Psi}^{\dagger}(\boldsymbol{x}',t')\}|N> \\
&= <N|\hat{\Psi}(\boldsymbol{x},t)\,\hat{\Psi}^{\dagger}(\boldsymbol{x}',t')|N> - <N|\hat{\Psi}^{\dagger}(\boldsymbol{x}',t')\,\hat{\Psi}(\boldsymbol{x},t)|N>
\end{aligned}
$$



© Zagoskin

$$
G(\epsilon,p) = \frac{1-n_p}{\epsilon-\epsilon_p+i\,0} + \frac{n_p}{\epsilon+\epsilon_p^h-i\,0}
$$

$T=0$

$$
n_p = \theta(p_F - p) \qquad \epsilon_p^h = -\epsilon_p
$$

$$
G(\epsilon,\boldsymbol{p}) = \frac{1}{\epsilon-\epsilon_p+i\,\mathrm{sign}(\epsilon-\epsilon_F)}
$$

Diagram technique

Ground state:

$$
iG(x,y) = <N|\widehat{T}\{\widehat{\Psi}(x)\widehat{\Psi}^{\dagger}(y)\}|N> = <N|\widehat{S}^{-1}\widehat{T}\{\widehat{\Psi}_I(x)\widehat{\Psi}_I^{\dagger}(y)\}\widehat{S}|N>
$$

in interaction picture: $iG = <N|\widehat{T}\{\widehat{\Psi}_I(x)\widehat{\Psi}_I^{\dagger}(y)\}\widehat{S}|N><\widehat{S}^{-1}>$

transition from the ground state to the ground state under action of evolution operator

$$
\widehat{S} = \underset{\substack{\uparrow \\ \text{time ordering}}}{\widehat{T}}\exp\Big\{-i\int_{-\infty}^{\infty}\widehat{V}_I(t)dt\Big\}
\qquad
\widehat{V}_I(t) = e^{i\widehat{H}_0(\mu)\,t}\,\widehat{V}\,e^{-i\widehat{H}_0(\mu)t}
$$

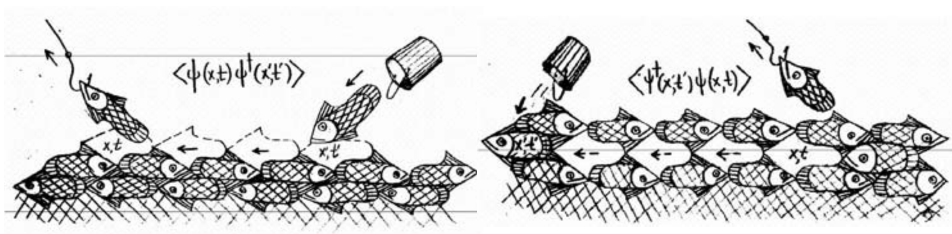
$$
\widehat{H}_0(\mu) = H_0 - \sum_a \mu_a \widehat{N}_a
$$

Only one type of Green's functions

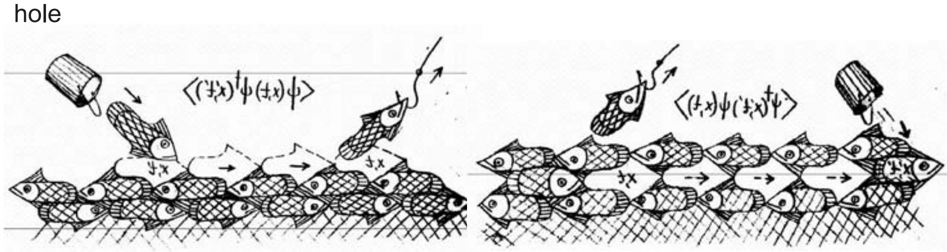
Freeman Dyson

particle

G(ε,p)



hole



G^h(ε,p)

Diagram technique "out of non-equilibrium"

For a non-equilibrium state $|N>$

$$iG^{--}(x,y) =< N|\widehat{T}\{\widehat{\Psi}(x)\,\widehat{\Psi}^{\dagger}(y)\}\,|N> \;=\; < N|\,\widehat{S}^{-1}\,\widehat{T}\{\widehat{\Psi}_I(x)\,\widehat{\Psi}_I^{\dagger}(y)\}\,\widehat{S}\,|n>$$

$$\neq\; < N|\widehat{T}\,\{\widehat{\Psi}_I(x)\,\widehat{\Psi}_I^{\dagger}(y)\}\,\widehat{S}\,|N> < \widehat{S}^{-1} >$$

non-equilibrium ground state at -∞ **does not** transit to the same ground state at +∞

due to possible decays **4 Green's functions**

$$iG^{--}(x,y) =< N|\,\widehat{T}\{\widehat{\Psi}(x)\,\widehat{\Psi}^{\dagger}(y)\}\,|N> \quad iG^{++}(x,y) =< N|\,\widehat{T^{\dagger}}\{\widehat{\Psi}(x)\,\widehat{\Psi}^{\dagger}(y)\}\,|N>$$

inverse time ordering

Wigner densities (no time ordering operations)

$$iG^{-+}(x,y) = \mp < N|\,\widehat{\Psi}^{\dagger}(y)\,\widehat{\Psi}(x)\,|N> \qquad iG^{+-}(x,y) =< N|\,\widehat{\Psi}(x)\,\widehat{\Psi}^{\dagger}(y)\,|N>$$

Green functions are not independent !

$$G^{--} + G^{++} = G^{-+} + G^{+-}.$$

Diagram technique "out of non-equilibrium"

Assume suppression of initial correlations ($t \gg t_{cor}$) ➡ Wick theorem

➡ averaging of equations of motion for operators we obtain coupled Schwinger-Dyson equations for G^{--}, G^{++}, G^{+-}, G^{-+}

$$G(x,y) = G^0(x,y) + \int G^0(x,z)\Pi(z,z')G(z',y)$$

matrix of full G.F. — matrix of bare G.F. — matrix of self-energies

general structure of the matrix

$$F(x,y) = \begin{pmatrix} F^{--}(x,y) & F^{-+}(x,y) \\ F^{+-}(x,y) & F^{++}(x,y) \end{pmatrix} \qquad F = \{G, \Pi\}$$

"covariant metric"

$$F_i^j = \sigma_{ik}F^{kj}, \quad \sigma_{ik} = (\sigma_3)_{ik}, \quad \sigma_i^k = \delta_{ik}, \quad i,k = \{-,+\}$$

[Ivanov, Knoll, Voskresensky. NPA 657 (1999); NPA 672 (2000)]

different notations compared with LP (different signs in G^{ij} and Π^{ij}) and KB text books ($>, <$)

Factor $(-i)$ for " $-$ " vertex and $(+i)$ for " $+$ " vertex is used.

- **retarded Green's function** $G^R = G^{--} - G^{-+}$

decouples and defines excitation spectrum

$$G^R_{12} = G^{0,R}_{12} + G^{0,R}_{13} \cdot \Pi^R_{34} \cdot G^R_{42}$$

- No Wick rotation
- Same diagrams as for ground-state system

Thermal equilibrium

In equilibrium only one Green's function (G^R) is required :

$$F(p) = \begin{pmatrix} F^R \pm i f(E)\mathcal{A} & \pm i\, f(E)\mathcal{A} \\ -i(1 \mp f(E))\mathcal{A} & -F^A \mp i\, f(E)\mathcal{A} \end{pmatrix}$$

for Green's functions $\mathcal{A} = A$

for self-energies $\mathcal{A} = \Gamma$

particle occupation factors: $f(E) = \dfrac{1}{e^{(E/T)} \pm 1}$ Wigner densities: $G^{-+} \propto f(E)$

spectral function: $A(p) = -2\,\mathrm{Im}\,G^R = \dfrac{\Gamma}{M^2 + \Gamma^2/4}$ **width** $\Gamma = -2\,\mathrm{Im}\,\Pi^R$

mass operator $M = -Q(p) + \mathrm{Re}\,\Pi$ with $Q = \epsilon - \dfrac{\boldsymbol{p}^2}{2m}$ non-relativistic particles

$Q = p^2 - m^2$ relativistic bosons

Thermal equilibrium

fermion Green's function
$$\begin{pmatrix} G^R + i\, f(E)\, A & +i f(E)\, A \\ -i(1 - f(E))\, A & -G^A - +i\, f(E)\, A \end{pmatrix}$$

➡ $G^{-+} \propto f(E)$! ➡ specific role of G^{-+}

- **Quasiparticle limit** $\Gamma \to 0$ $A \to 2\,\pi\,\delta(M)$

  that fixes in-medium mass-shell $M = -Q(p) + \mathrm{Re}\,\Pi = 0$

  in quasiparticle limit $T \ll E_{\mathrm{F}}$ each extra G^{-+} line brings a factor $\propto \dfrac{T^2}{E_{\mathrm{F}}^2}$

  "perturbation series" in number of (- +) Green functions (T-counting)

- **gas limit** particles are on mass-shell and $\mathrm{Re}\,\Pi^R \to 0$

$$G_0^{-+} = +2\,\pi\, i\, f(E)\, \delta(E + \mu - E_p) \quad G_0^{+-} = -2\,\pi\, i\,(1 - f(E))\, \delta(E + \mu - E_p)$$

momentum distribution of quantum kinetic equation $$f(\boldsymbol{p}) = \pm \int i\, G^{-+}(E,\boldsymbol{p})\, \frac{dE}{2\,\pi}$$

$\Rightarrow$ $G^{-+}(E,\boldsymbol{p})$ has the meaning of a generalized distributions of virtual particles

In general case - no relation between E and $\boldsymbol{p}$!

Optical theorem in quantum mechanics

In quantum mechanics $$\sigma_{\mathrm{tot}} = \frac{4\pi}{k}\mathrm{Im}F(0) \longleftarrow$$ forward scattering amplitude.

$$\mathrm{Im}F(\boldsymbol{k}',\boldsymbol{k}) = \frac{k}{4\pi}\int F(\boldsymbol{k}',\boldsymbol{k}')\, F^*(\boldsymbol{k}'',\boldsymbol{k})\, d\Omega''$$

Scattering amplitude in the Born approximation

$$f^{\mathrm{B}}(q) = -\frac{m}{2\,\pi}\int e^{-i\boldsymbol{q}\,\boldsymbol{r}}U(\boldsymbol{r})d^3\boldsymbol{r}, \quad q = 2k\sin\frac{\theta}{2}$$

$\Rightarrow$ $\mathrm{Im} f^{\mathrm{B}}(\theta = 0) = 0$ and **the optical theorem is not fulfilled.**

Solution: f^{B} is the first term in the perturbative expansion of f in U.

Since $\sigma \propto |f|^2$ $\Rightarrow$ $\sigma \propto U^2$ for small U.

Therefore in optical theorem $[\sigma \propto \mathrm{Im} f^{\mathrm{B}}(\theta = 0)]$ there is no linear term.

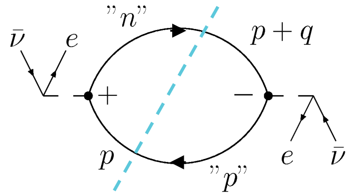
$\Rightarrow$ $\mathrm{Im} f^{\mathrm{B}}(\theta = 0) \propto U^2$ $\Rightarrow$ higher order term is required (!)

Optical theorem in non-equilibrium diagram technique

Let us calculate self-energy

with free Green's functions

$-i\,\Pi_0^{-+} =$



$$-i\Pi_0^{-+} = \frac{G^2}{2}\mathrm{Tr}\{l_1^\mu l_2^\nu\}\int\frac{d^4p}{(2\pi)^4}\mathrm{Tr}\{(-iJ_\mu)\,iG_n^{-+}(p+q)\,(+iJ_\nu)\,iG_p^{+-}(p)(-1)\}$$

$$= -i\mathcal{L}_0^{-+}\sum_{\mathrm{spin}}|M|^2$$

$\boldsymbol{2\pi i\, f(E+\omega)\,\delta(E+\omega+\mu-E^{(n)}_{p+q})}$ $\boldsymbol{-2\pi i\,(1-f(E))\,\delta(E+\mu-E_p)}$

the loop function

$\boldsymbol{\int dE}$

$$-i\mathcal{L}_0^{-+} = \int\frac{d^3p}{(2\pi)^3} f_n(\boldsymbol{p}+\boldsymbol{q})\,[1-f_p(\boldsymbol{p})]\,2\pi\,\delta[E^n(\boldsymbol{p}+\boldsymbol{q})-\omega-E^p(\boldsymbol{p})-\mu_n+\mu_p]$$

Cutting the diagram means removing of *dE* integration due to δ-function

Comparing with standard expression for emissivity

$$\epsilon_\nu^{\mathrm{DU}} = 2\int\frac{d^3p_n}{(2\pi)^3}f_n\frac{d^3p_p}{(2\pi)^3}(1-f_p)\frac{d^3q_e}{2\omega_e(2\pi)^3}(1-f_e)\frac{d^3q_{\bar\nu}\,\omega_{\bar\nu}}{2\omega_{\bar\nu}\,(2\pi)^3}(2\pi)^4\delta^{(4)}(P_f-P_i)\sum_{\mathrm{spins}}|M|^2$$

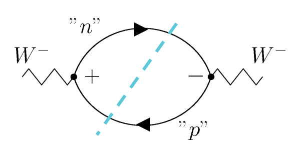
$$\epsilon_\nu^{\mathrm{DU}} = 2\int\frac{d^3q_e}{2\omega_e(2\pi)^3}(1-f_e)\frac{d^3q_{\bar\nu}}{2\,\omega_{\bar\nu}\,(2\pi)^3}\omega_{\bar\nu}\,[-i\Pi_0^{-+}(q_e+q_{\bar\nu})]$$

➡ to calculate Direct Urca emissivity

we need only (no medium effects) simple free W boson "**-** **+**" loop

---

Using relation $i\,\Pi^{-+} = -\dfrac{2\,\mathrm{Im}\Pi^R}{e^{\omega/T}-1}$ we may calculate cross-sections as an integral of $|M|^2$ over the phase space **OR** as an imaginary part of W^- boson self-energy



perturbative expansion: second-order term in weak coupling and **zeroth-order** term in strong coupling

Terms of higher order in strong couplings must be included! $\Pi_0^{-+}\longrightarrow\Pi^{-+}$

[Voskresensky, Senatorov, Sov, J. Nucl. Phys. 45 (1987)]

- **Bose occupation number out of fermion loop**

$$f_{\rm F}(E_1)\,[1-f_{\rm F}(E_2)] = [f_{\rm F}(E_2)-f_{\rm F}(E_1)]\,f_{\rm B}(E_1-E_2)$$

$$\begin{aligned}
-i\mathcal{L}_0^{-+} &= \int \frac{d^3p}{(2\pi)^3} f_{{\rm F}n}(\boldsymbol{p}+\boldsymbol{q})\,[1-f_{{\rm F}p}(\boldsymbol{p})]\,2\,\pi\,\delta[E^n(\boldsymbol{p}+\boldsymbol{q})-\omega-E^p(p)-\mu_n+\mu_p] \\
&= f_B(\omega)\int \frac{d^3p}{(2\pi)^3}[f_{{\rm F}p}(\boldsymbol{p})-f_{{\rm F}n}(\boldsymbol{p}+\boldsymbol{q})]\,2\,\pi\,\delta[E^n(\boldsymbol{p}+\boldsymbol{q})-\omega-E^p(p)-\mu_n+\mu_p] \\
&= -2\,f_B(\omega)\,\mathrm{Im}\int \frac{d^3p}{(2\pi)^3}\frac{f_{{\rm F}p}(\boldsymbol{p})-f_{{\rm F}n}(\boldsymbol{p}+\boldsymbol{q})}{E^n(\boldsymbol{p}+\boldsymbol{q})-\omega-E^p(p)-\mu_n+\mu_p-i0} \\
&= -f_B(\omega)\,2\,\mathrm{Im}\,\mathcal{L}_{np}^R
\end{aligned}$$

$$\mathcal{L}_{ab}^R = \int \frac{d^3p}{(2\pi)^3}\frac{f_{{\rm F}a}(\boldsymbol{p}+\boldsymbol{q})-f_{{\rm F}b}(\boldsymbol{p})}{\omega+E^b(p)-E^a(\boldsymbol{p}+\boldsymbol{q})+\mu_a-\mu_b+i0}$$

Lindhard function

very sharp function of ω and κ

$$\begin{aligned}
\mathcal{L}_{ab}^R &= \int \frac{d^3p}{(2\pi)^3}\frac{f_{{\rm F}a}(\boldsymbol{p}+\boldsymbol{q})-f_{{\rm F}b}(\boldsymbol{p})}{\omega+E^b(p)-E^a(\boldsymbol{p}+\boldsymbol{q})+\mu_a-\mu_b+i0} \\
&= -\frac{m_N}{2\,\pi^2}\left[p_{{\rm F},a}\Phi_{1,a}(\omega,k)+p_{{\rm F},b}\Phi_{1,b}(-\omega,-k)\right]
\end{aligned}$$

Migdal function (T=0):

$$\Phi_{1,a}(\omega,k) = \frac{m_N^2}{2\,p_{{\rm F},a}\,k^3}\left\{\frac{a^2-b^2}{2}\ln\frac{a+b}{a-b}-ab\right\} \qquad a=\omega-\frac{k^2}{2\,m_N} \quad b=\frac{p_{{\rm F},a}\,k}{m_N}$$

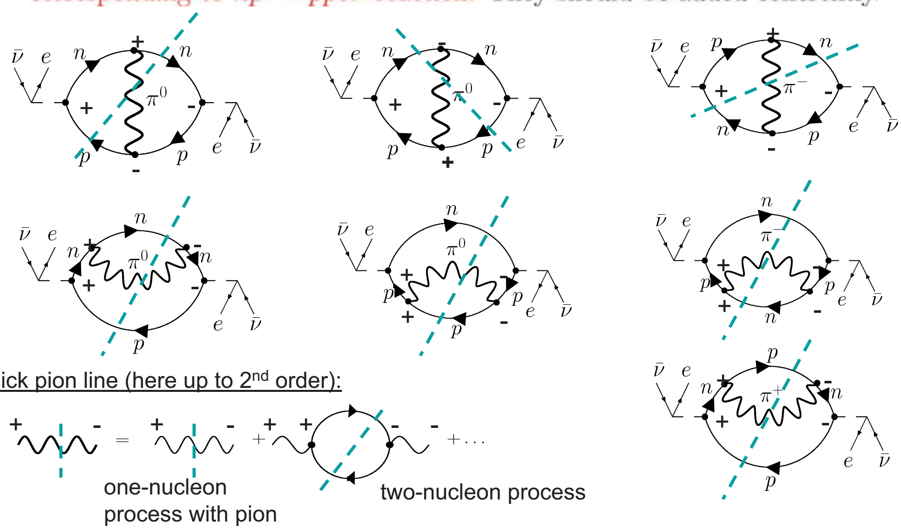
$$|a|\ll b \qquad \Phi_{1,a}\simeq\frac{1}{2}$$

$$|a|\gg b \qquad \Phi_{1,a}\simeq-\frac{\pi^2 n_a}{m_N^2 p_{{\rm F},a}}\frac{1}{\omega-\dfrac{k^2}{2\,m_N}}$$

Optical theorem for modified URCA reactions

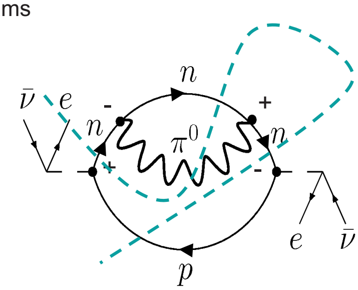
$$\epsilon_\nu^{\rm MU} = \int \frac{d^3 q_e\,(1-f_e)}{2\,\omega_e\,(2\,\pi)^3}\,\frac{d^3 q_{\bar\nu}}{2\,\omega_{\bar\nu}\,(2\,\pi)^3}\,\omega_{\bar\nu}\,[-i\Pi^{-+}_{\rm MU}(q_e+q_{\bar\nu})]$$

To get correct 2-order Π^{-+} one should add diagrams with π^- corresponding to $np \to ppe\bar\nu$ reaction. They should be added coherently.



Optical theorem for modified URCA reactions

Another type of digrams



which can be cut in more than two pieces *vanish* in quasi-particle approximations for fermions

Optical theorem for modified URCA reactions

One and two-nucleon processes are treated on equal footing

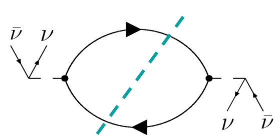
- ✔ Exchange diagrams are automatically included
- ✔ Both $nn \to npe\bar{\nu}$ and $np \to ppe\bar{\nu}$ reactions are treated on equal footing
- ✔ We need second-order diagram in pion propagator (!) ➡
  since $f_{\pi NN} m_\pi \simeq 1$ (not $\ll 1$), in general we should include all diagrams?

➡ We can treat fermions in QPA (for $T \ll E_F$) but (!)
intermediate bosons should be treated beyond QPA, including width effects.

Superfluid matter

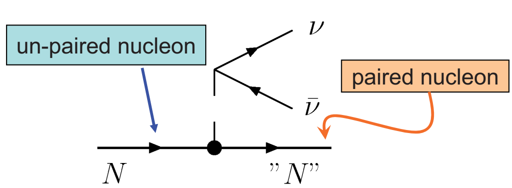
In non-<u>superfluid</u> medium ($T > T_c \sim 0.1\text{-}1$ MeV)

the process $N \to N\bar{\nu}\nu$



is **not allowed**

In <u>superfluid</u> medium ($T<T_c$)



new “quasi”-one-nucleon-like processes become <u>permitted</u>

Green's function formalism for superfluid matter

We will use Green's function formalism in non-equilibrium diagram technique

[Senatorov, Voskresensky, Sov J. Nucl. Phys. 45 (1987)]

Besides <u>normal Green's functions</u> one introduces **anomalous nucleon Green's functions** on the Schwinger-Keldysh contour:

$$i\widehat{F}^{(1)}(x,y) = {}^{a} \rightarrow\leftarrow {}^{b} = < N|\mathcal{T}_{\mathcal{C}}\widehat{\Psi}^{a}(x)\widehat{\Psi}^{b}(y)|N+2>$$
$$i\widehat{F}^{(2)}(x,y) = {}_{a} \leftarrow\rightarrow {}_{b} = < N+2|\mathcal{T}_{\mathcal{C}}\widehat{\Psi}^{\dagger}_{a}(x)\widehat{\Psi}^{\dagger}_{b}(y)|N>$$

$F^{(1,2)}$ are matrices in spin space, a and b are spin indices

$$\widehat{F}^{(1)ab} = F\, g^{ab}, \quad \widehat{F}^{(2)}_{ab} = F\, g_{ab}$$

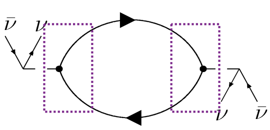
$$g^{ab} = -g^{ba} \qquad g^{12} = 1$$

Green's function formalism for superfluid matter

- vertices

We can have two types of diagram now:

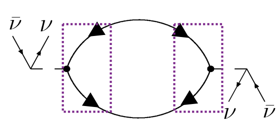
with <u>normal</u> GFs



$\mathcal{T}^{\dagger}$ $\mathcal{T}$

3 types of vertices

with <u>anomalous</u> GFs



$\mathcal{T}^{h}$ $\mathcal{T}$

additional vertex for "hole"

$$\mathcal{T}^{h}(p,q) = \sigma_2\, \mathcal{T}^{T}(-p,-q)\, \sigma_2$$

For vector vertex:

$$\mathcal{T}^{\mu}_{V} = (\mathcal{T}^{0}_{V}\,,\, \boldsymbol{\mathcal{T}}_{V}) \longrightarrow \left[\mathcal{T}^{\mu}_{V}\right]^{h} = (\mathcal{T}^{0}_{V}\,,\, -\boldsymbol{\mathcal{T}}_{V})$$

We use here $\sigma_2\, \boldsymbol{\sigma}^{T}\, \sigma_2 = -\boldsymbol{\sigma}$

For axial-vector vertex:

$$\mathcal{T}^{\mu}_{A} = (\mathcal{T}^{0}_{A}\,,\, \boldsymbol{\mathcal{T}}_{A}) \longrightarrow \left[\mathcal{T}^{\mu}_{A}\right]^{h} = (\mathcal{T}^{0}_{A}\,,\, -\boldsymbol{\mathcal{T}}_{A})$$

Green's function formalism for superfluid matter

$$(G^{-+})^a_b = 2\,\pi\, i\, f(E)\,[u_p^2\,\delta(E-E(p)) + v_p^2\,\delta(E+E(p))]\,\delta^a_b$$

$$(F^{-+})_{ab} = -2\,\pi\, i\, f(E)\, u_p\, v_p\,[\delta(E-E(p)) - \delta(E+E(p))]\, g_{ab}$$

$$u_p^2 = \frac{E_p + \epsilon_p}{2E_p}, \qquad v_p^2 = \frac{E_p - \epsilon_p}{2E_p},$$

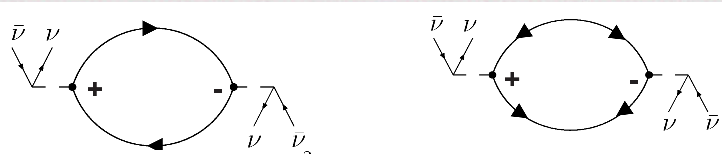
$$E_p^2 = \epsilon_p^2 + \Delta^2(p), \qquad \epsilon_p = v_{\rm F}(p - p_{\rm F})$$

For $v_p \to 0$ ➡ standard "normal matter" case.

Terms $\propto v_p$ are $\propto \Delta$ ➡ G and F are now not free but QP Green's functions (!)

For systems with pairing one cannot proceed without inclusion of medium effects.

Probability of the PBF processes ($1S_0$ pairing)



$$-i\,\Pi_0^{-+} = \frac{G^2}{2}\,{\rm Tr}\,\{l^\nu l^{\dagger\mu}\}\;(-i\Pi^{-+}_{0,\mu\nu}[{\rm nucl}])$$

$$-i\Pi^{-+}_{0,\mu\nu}[{\rm nucl}] \;=\; \int \frac{d^4p}{(2\,\pi)^4}(-1)\,\big[(-i)\,(\eta_v\,J^V_\mu - J^A_\mu)i\,G^{-+}\,i\,(\eta_v J^V_\nu - J^A_\nu)i\,G^{+-}$$
$$-(-i)\,(\eta_v\,J^V_\mu - J^A_\mu)\,i\,F^{-+}\,\hat P\,i\,(\eta_v J^V_\nu - J^A_\nu)\,i\,F^{+-}\big]$$

$$g^{ab}\,g_{bc} = -\delta^a_c$$

Here $\eta_v = 1$ for nn pairing and $\eta_v = c_v$ for pp.

$$\hat P = \begin{cases} +1 & \text{for } \mathcal{T}^0_V \\ -1 & \text{for } \boldsymbol{\mathcal{T}_V} \\ +1 & \text{for } \mathcal{T}^0_A \\ -1 & \text{for } \boldsymbol{\mathcal{T}_A} \end{cases}$$

The probability of the nPBF (pPBF) processes

$$-i\,\Pi_0^{-+} \;=\; -i\,\Pi^{-+}_{0,N} - i\,\Pi^{-+}_{0,A}$$
$$\simeq\; 2\,G^2 \int \frac{d^4p}{(2\,\pi)^4}\,\omega_1\,\omega_2\,\left[\left(\eta_v^2 + 3\,g_A^2\right)\,G^{-+}\,G^{+-} - \left(\eta_v^2 - 3\,g_A^2\right)\,F^{-+}\,F^{+-}\right]$$

Probability of the PBF processes ($1S_0$ pairing)

The first (normal) contribution contains four terms.

Three of them yield zero due to energy conservations. ➡

$$-i\,\Pi^{-+}_{0,N} = \frac{2G^2\omega_1\omega_2}{(2\pi)^2}\left(\eta_v^2+3g_A^2\right)\int d^3p \frac{u^2_{p+q}\,v^2_p\,\delta(\omega-E_p-E_{\boldsymbol{p}+\boldsymbol{q}})}{(\exp[(\omega-E_{\boldsymbol{p}})/T]+1)\,(\exp[(E_{\boldsymbol{p}})/T]+1)}$$

Introducing $\boldsymbol{p}_1=\boldsymbol{p}$, $\boldsymbol{p}_2=\boldsymbol{p}+\boldsymbol{q}$ we rewrite

$$\int d^3p = 2\,\pi\int_0^\infty p_1\,dp_1\,\frac{1}{q}\int_{p_1-q}^{p_1+q} p_2\,dp_2$$

Assuming $v_F^2 \ll 1$, we may put $E_{\boldsymbol{p}_2} \simeq E_{\boldsymbol{p}_1}$. ➡

$$-i\,\Pi^{-+}_{0,N} = \frac{G^2\,\omega_1\,\omega_2\Delta^2}{\omega^2}\,\left(\eta_v^2+3\,g_A^2\right)\frac{\tilde{p}^2}{(dE_p/dp)_{\tilde{p}}\,(\exp[\omega/(2T)]+1)^2}\,\theta(\omega-2\Delta),$$

where $\tilde{p}$ is the root of the relation $2E_p=\omega$, and $(dE_p/dp)_{\tilde{p}} = v_{\rm F}\frac{\sqrt{\omega^2-4\Delta^2}}{\omega}$.

Emissivity of the PBF processes ($1S_0$ pairing)

The anomalous term is $\propto u^2_{p+q}v^2_p$:

$$-i\,\Pi^{-+}_{0,A} = \frac{2\,G^2\,\omega_1\,\omega_2}{(2\pi)^2}\,\left(\eta_v^2-3\,g_A^2\right)\int d^3p\,\frac{u^2_{p+q}\,v^2_p\,\delta(\omega-E_p-E_{\boldsymbol{p}+\boldsymbol{q}})}{(\exp[(\omega-E_{\boldsymbol{p}})/T]+1)\,(\exp[(E_{\boldsymbol{p}})/T]+1)}$$

Thus one obtains

$$-i\,\Pi^{-+}_{0,A} = \frac{2\,m_N\,p_{{\rm F},N}\,G^2\,\omega_1\,\omega_2\,\Delta^2}{\pi\,\omega\,\sqrt{\omega^2-4\Delta^2}}\,\left(\eta_v^2-3g_A^2\right)\,\frac{\theta(\omega-2\Delta)}{(\exp[\omega/(2T)]+1)^2}$$

➡ **Only vector term in $\Pi^{-+}_{0,N}+\Pi^{-+}_{0,A}$ remains.**

For the emissivity of the nPFB reaction we find

$$\epsilon_\nu(1S_0) = \epsilon_\nu^{\rm vect}(1S_0)+\epsilon_\nu^{\rm axial}(1S_0) = N_\nu\,\frac{4}{15}\,\frac{G^2\,m_N\,p_{{\rm F},N}\Delta^7}{\pi^5}I(\Delta/T)\times\eta_v^2,$$

N_ν is the number of neutrino species.

Emissivity of the PBF processes ($1S_0$ pairing)

$$I(x) = \int_0^\infty \mathrm{ch} y dy/(\exp[x \mathrm{ch} y] + 1)^2,$$

$$I(x \gg 1) = \exp(-2x)\sqrt{\pi/(4x)}, \quad I(x \ll 1) \simeq 0.6x^{-5}$$

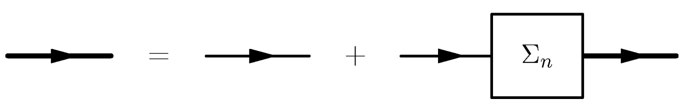
✔ Note that $\epsilon_\nu(x \gg 1) \propto \Delta^7 \exp(-2\Delta/T)$, [Senatorov, Voskresensky, Sov. Nucl. Phys. 45 (1987)], rather than to $T^7 \exp(-2\Delta/T)$, as it is often claimed.
For $T_9 = 1$ then would be $\epsilon_\nu^{\mathrm{PBF}} \sim 0.1\epsilon_\nu^{\mathrm{MU}}$, whereas with correct asymptotic for $\Delta = 0.5$ MeV, $T_9 = 1$ one gets **extra** 10^7.

✔ $c_V^2 \simeq 0.006$ suppression factor for pPBF
but $\Pi_{\mu\nu}^{-+}$ with bare vertices and QP Green's functions does not fulfill vector current conservation (!)

➡ dressing of vertices is required.

Full Green's function

particle-line

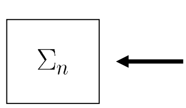


$$\widehat{G}_{\mathrm{n.s.}} = \widehat{G}_0 + \widehat{G}_0\,\widehat{\Sigma}_{\mathrm{n.s.}}\,\widehat{G}_{\mathrm{n.s.}} = \left[[\widehat{G}_0]^{-1} - \widehat{\Sigma}_{\mathrm{n.s.}}\right]^{-1}$$

diagonal in spin-space

$$\hat{G}_0(\epsilon, \boldsymbol{p}) = \frac{\hat{1}}{\epsilon - \boldsymbol{p}^2/2\,m_N + i\,0\,\mathrm{sign}\,(\epsilon - \epsilon_{\mathrm{F}})}$$

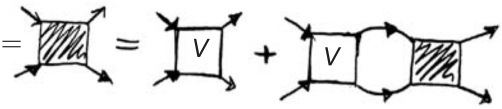
analogously for the hole-line

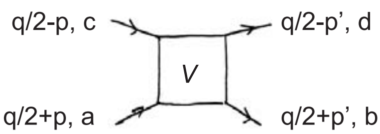


particle-particle, particle-hole hole-hole interactions

Particle-particle interaction

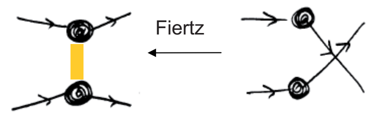
$-i\,T_{pp}(p,p';q) =$

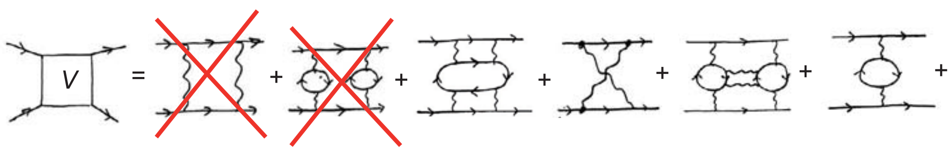




two-particle irreducible interaction

$$\left[\widehat{V}(p,p',q)\right]^{ac}_{bd} = V_0(p,p',q)\,\delta^a_b\,\delta^c_d + V_1(p,p',q)\,(\boldsymbol{\sigma}\,)^a_b\,(\boldsymbol{\sigma}\,)^c_d$$

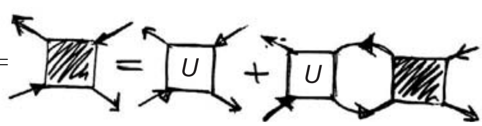


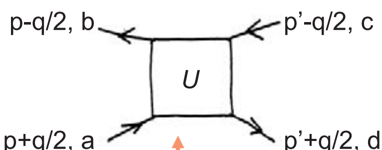


$$\widehat{T}_{\rm pp}(p,p',q) = \widehat{V}(p,p',q) + \int \frac{\mathrm{d}^4 p''}{(2\pi)^4\, i} \widehat{V}(p,p'',q)\,\widehat{G}(q/2+p'')\,\widehat{G}(q/2-p'')\,\widehat{T}_{\rm pp}(p'',p',q)$$

Particle-particle interaction

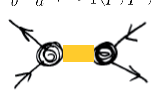
$-i\,T_{\rm ph}(p,p';q) =$

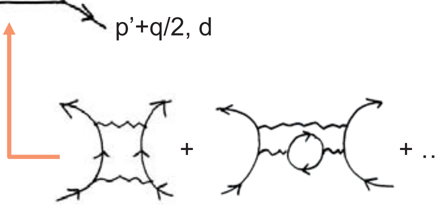




particle-hole irreducible interaction

$$\left[\widehat{U}(p,p',q)\right]^{ac}_{bd} = U_0(p,p',q)\,\delta^a_b\,\delta^c_d + U_1(p,p',q)\,(\boldsymbol{\sigma}\,)^a_b\,(\boldsymbol{\sigma}\,)^c_d$$

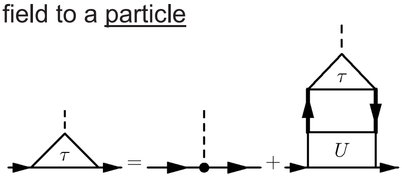




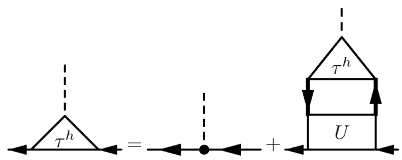
$$\widehat{T}_{\rm ph}(p,p',q) = \widehat{U}(p,p',q) + \int \frac{\mathrm{d}^4 p''}{(2\pi)^4\, i} \widehat{U}(p,p'',q)\,\widehat{G}(q/2+p'')\,\widehat{G}^h(q/2-p'')\,\widehat{T}_{\rm ph}(p'',p',q)$$

"Charge" of particle and hole

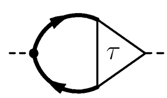
Coupling of the external field to a <u>particle</u>



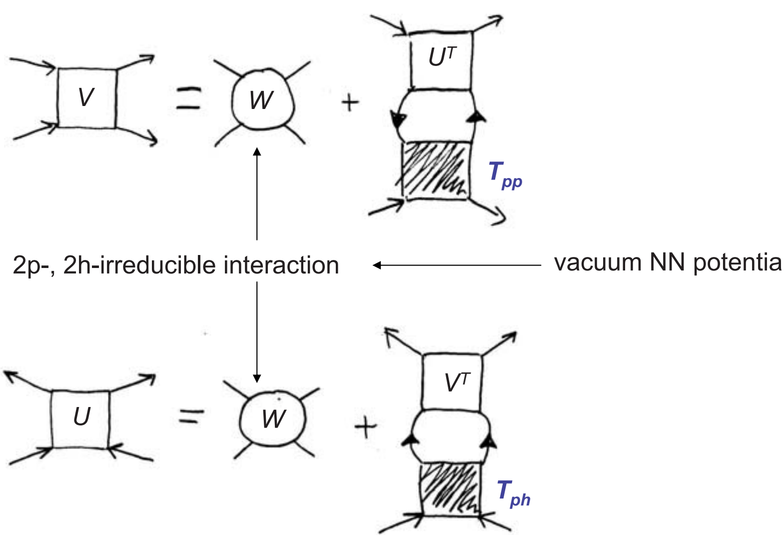
and a <u>hole</u>



- **one-nucleon reaction**

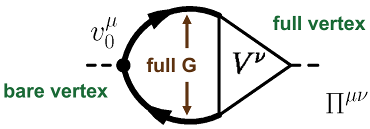
$-i\,\Pi =$ 

$\mathrm{Im}\,\Pi \rightarrow \epsilon^{\mathrm{DU}}$



[Wambach, Ainsworth,Pines NPA (1993)]

Vector current conservation



Current is conserved if $\boxed{\Pi^{\mu\nu}\, q_\nu = 0}$

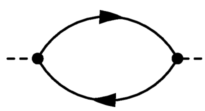
$$\Pi^{\mu\nu} \propto \int d^4p \,\mathrm{Tr}\left\{\gamma^\mu\, G(p+q/2)\, V^\nu(p,-q)\, G(p-q/2)\right\}$$

If the relation $\boxed{q_\mu\, V^\mu(p,q) = G^{-1}(p+q/2) - G^{-1}(p-q/2)}$ Is fulfilled

$$\Pi^{\mu\nu} q^\nu \propto \int d^4p \,\mathrm{Tr}\left\{\gamma^\mu\left[G(p-q/2) - G(p+q/2)\right]\right\} = 0$$

The Ward identities impose non-trivial relations
between vertex functions and Green's functions,
which synchronize any modification of
the Green's function with a corresponding change in the vertex function.

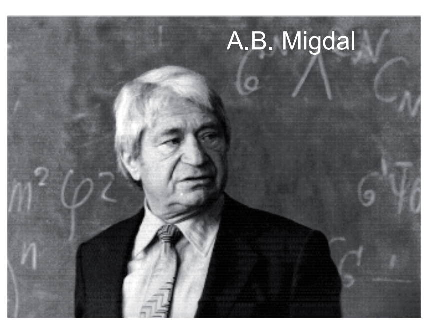
in non-relativistic limit for free G and vertices: $\tau_0^\mu = (1, \boldsymbol{v}) \quad G(p) = (\epsilon - p^2/2m)^{-1}$



$$q \cdot \tau_0 = \omega - \boldsymbol{v}\boldsymbol{q} \equiv G_0^{-1}(p+q/2) - G_0^{-1}(p-q/2)$$

The Ward identity is fulfilled and the current is conserved

NUCLEAR FERMI LIQUID



Landau Fermi liquid approach

system of quasi-particles

interacting fermions

quantized excitations in the system

quasi-particles ≠ original "bare" fermions [constituents of the system]

Landau wrote the Boltzmann eq. for q.p distribution function: $n(\boldsymbol{x}, \boldsymbol{p}, t)$

$$\frac{dn}{dt} = \frac{\partial n}{\partial t} + \dot{\boldsymbol{x}}\frac{\partial n}{\partial \boldsymbol{x}} + \dot{\boldsymbol{p}}\frac{\partial n}{\partial \boldsymbol{p}} = I(n)$$

equations of motion for q.p.

$$\dot{\boldsymbol{x}} = \frac{\partial \bar{e}(\boldsymbol{p}, \boldsymbol{x})}{\partial \boldsymbol{p}}, \qquad \dot{\boldsymbol{p}} = -\frac{\partial \bar{e}(\boldsymbol{p}, \boldsymbol{x})}{\partial \boldsymbol{x}}$$

"generalized" velocity Newton's law

$$\frac{\partial n}{\partial t} + \frac{\partial \bar{e}(\boldsymbol{p}, \boldsymbol{x})}{\partial \boldsymbol{p}}\frac{\partial n}{\partial \boldsymbol{x}} - \frac{\partial \bar{e}(\boldsymbol{p}, \boldsymbol{x})}{\partial \boldsymbol{x}}\frac{\partial n}{\partial \boldsymbol{p}} = I(n)$$

$$\boldsymbol{\mathcal{F}} = \int \boldsymbol{p}\, n \frac{d^3p}{(2\pi)^3}$$

momentum flux density

$$\begin{aligned}\frac{\partial \boldsymbol{\mathcal{F}}}{\partial t} &= \int \boldsymbol{p}\frac{\partial n}{\partial t}\frac{d^3p}{(2\pi)^3} = \int \boldsymbol{p}\, I(n)\frac{d^3p}{(2\pi)^3} - \int \boldsymbol{p}\left[\frac{\partial \bar{e}}{\partial \boldsymbol{p}}\frac{\partial n}{\partial \boldsymbol{x}} - \frac{\partial \bar{e}}{\partial \boldsymbol{x}}\frac{\partial n}{\partial \boldsymbol{p}}\right]\frac{d^3p}{(2\pi)^3} \\ &= -\frac{\partial}{\partial x_j}\int \boldsymbol{p}\, n \frac{\partial \bar{e}}{\partial \boldsymbol{p}_j}\frac{d^3p}{(2\pi)^3} - \int n \frac{\partial \bar{e}}{\partial \boldsymbol{x}}\frac{d^3p}{(2\pi)^3} \\ &= \underline{\frac{\partial}{\partial x_j}\boldsymbol{\Pi}^j + \int \bar{e}\frac{\partial n}{\partial \boldsymbol{x}}\frac{d^3p}{(2\pi)^3}}\end{aligned}$$

<u>momentum conservation</u>

$$0 = \int \boldsymbol{\mathcal{F}} d^3x = \int \cancel{\frac{\partial}{\partial x_j}\boldsymbol{\Pi}^j}\, d^3x + \int d^3x \int \bar{e}\frac{\partial n}{\partial \boldsymbol{x}}\frac{d^3p}{(2\pi)^3}$$

$$\Rightarrow \int d^3x \underbrace{\int \bar{e}\frac{\partial n}{\partial \boldsymbol{x}}\frac{d^3p}{(2\pi)^3}} = 0$$

$$\downarrow$$

$$\int \bar{e}\frac{\partial n}{\partial \boldsymbol{x}}\frac{d^3p}{(2\pi)^3} = \frac{\partial}{\partial \boldsymbol{x}} E$$

$$\downarrow$$

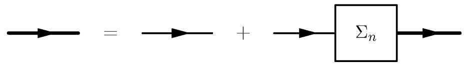
$$\delta E = \int \bar{e}\,\delta n \frac{d^3p}{(2\pi)^3}$$

E → energy of the system

$$\frac{\delta E}{\delta n(p)} = \bar{e}(p)$$

single particle mechanism of excitation

[G.E. Brown, RMP (1971)]



$$\widehat{G} = \widehat{G}_0 + \widehat{G}_0\,\widehat{\Sigma}\,\widehat{G} = \left[[\widehat{G}_0]^{-1} - \widehat{\Sigma}\right]^{-1}$$

$T \ll \varepsilon_{\mathrm{F},n}, \varepsilon_{\mathrm{F,p}}$ and $\epsilon \sim \epsilon_{\mathrm{F}}\,,\ p \sim p_{\mathrm{F}}$

pole residue $\quad a^{-1} = 1 - \left.\dfrac{\partial}{\partial\epsilon}\Sigma(\epsilon, 0, T)\right|_{\epsilon\simeq\epsilon_{\mathrm{F}}}$

$$G(\epsilon, \boldsymbol{p}) = \frac{a}{\epsilon - \epsilon_p + i\,\gamma\,\epsilon^2\,\mathrm{sign}\epsilon} + G_{\mathrm{reg}}(\epsilon, \boldsymbol{p})$$

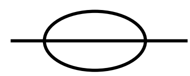
q.p. energy

q.p. width

$$\epsilon_p = \frac{p^2 - 2m_N\mu_N}{2\,m_N^*} \approx \frac{p^2 - p_{\mathrm{F}}^2}{2\,m_N^*} \approx v_{\mathrm{F}}\,(p - p_{\mathrm{F}})$$

$$\gamma = -\lim_{\epsilon\to 0} \mathrm{Im}\,\Sigma^R(\epsilon, p^2 = 2m_N\mu_N, T)/\epsilon^2$$

<u>small for $T<<\varepsilon_F$</u>

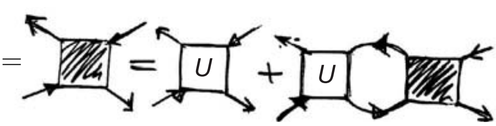


q.p. effective mass

$$\frac{1}{m_N^*} = \frac{a}{m_N} + 2\,a\,\left.\frac{\partial}{\partial p^2}\Sigma(\epsilon, \boldsymbol{p}, T)\right|_{p=0,\epsilon\simeq\epsilon_{\mathrm{F}}}$$

$G_{\mathrm{reg}}(\epsilon, \boldsymbol{p})$ complicated background part

$$-i\,T_{\mathrm{ph}}(p, p'; q) =$$



$$\widehat{T}_{\mathrm{ph}}(p, p', q) = \widehat{U}(p, p', q) + \int \frac{\mathrm{d}^4 p''}{(2\pi)^4\, i}\widehat{U}(p, p'', q)\,\widehat{G}(q/2 + p'')\,\widehat{G}^h(q/2 - p'')\,\widehat{T}_{\mathrm{ph}}(p'', p', q)$$

$$G(q/2 + p)\,G^h(q/2 - p) = G(q/2 + p)\,G(p - q/2)$$

$$= \frac{a}{[\epsilon + \omega/2 - \epsilon_{\boldsymbol{p}+\boldsymbol{q}/2} + i\,0\,\mathrm{sign}\,(\epsilon + \omega/2)]}\,\frac{a}{[\epsilon - \omega/2 - \epsilon_{\boldsymbol{p}-\boldsymbol{q}/2} + i\,0\,\mathrm{sign}\,(\epsilon - \omega/2)]} + \tilde{B}(p, q)$$

$$\simeq a^2\,\delta(\epsilon)\int d\epsilon\,\frac{1}{[\epsilon + \omega/2 - \epsilon_{\boldsymbol{p}+\boldsymbol{q}/2} + i\,0\,\mathrm{sign}\,(\epsilon + \omega/2)]}\,\frac{1}{[\epsilon - \omega/2 - \epsilon_{\boldsymbol{p}-\boldsymbol{q}/2} + i\,0\,\mathrm{sign}\,(\epsilon - \omega/2)]} + B(p, q)$$

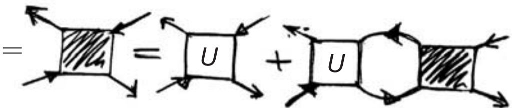
$$= -2\pi i\,a^2\,\delta(\epsilon)\frac{f(\boldsymbol{p} + \boldsymbol{q}/2) - f(\boldsymbol{p} - \boldsymbol{q}/2)}{\omega - \epsilon_{\boldsymbol{p}+\boldsymbol{q}/2} + \epsilon_{\boldsymbol{p}-\boldsymbol{q}/2} + i\,0} + B(p, q)$$

---

for $q \to 0$

$$G(q/2 + p)\,G^h(q/2 - p) \simeq 2\pi i\,a^2\,\delta(\epsilon)\frac{v_{\mathrm{F}}\,\boldsymbol{q}\boldsymbol{n}}{\omega - v_{\mathrm{F}}\boldsymbol{q}\boldsymbol{n} + i\,0}\,\delta(p - p_{\mathrm{F}}) + B(p, q)$$

$$\boldsymbol{n} = \boldsymbol{p}/p$$

$$-i\,T_{\rm ph}(p,p';q) =$$ 

for $|\boldsymbol{p}| \simeq p_{\rm F} \simeq |\boldsymbol{p}'|$ and $|\boldsymbol{q}\boldsymbol{p}| << \omega << \epsilon_{\rm F}$

$$\widehat{T}_{\rm ph}(\boldsymbol{n},\boldsymbol{n}',q) = \widehat{\Gamma}^{\omega}(\boldsymbol{n},\boldsymbol{n}') - \int \frac{d\Omega_{p''}}{4\,\pi}\,\widehat{\Gamma}^{\omega}(\boldsymbol{n},\boldsymbol{n}')\,A(\boldsymbol{n},q)\,\widehat{T}_{\rm ph}(\boldsymbol{n},\boldsymbol{n}',q)$$

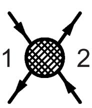
$$A(\boldsymbol{n},q) = a^2\,\frac{m^*\,p_{\rm F}}{\pi^2}\,\frac{v_{\rm F}\,\boldsymbol{q}\boldsymbol{n}}{\omega - v_{\rm F}\boldsymbol{q}\boldsymbol{n} + i\,0}$$

complicated dynamics is here:

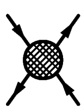
$$\widehat{\Gamma}^{\omega}_{\rm ph}(\boldsymbol{n},\boldsymbol{n}') = \widehat{U}(\boldsymbol{n},\boldsymbol{n}') - \int \frac{d^4p''}{(2\pi)^4\,i}\widehat{U}(\boldsymbol{n},\boldsymbol{n}')\,B(p,q=0)\,\widehat{\Gamma}^{\omega}_{\rm ph}(\boldsymbol{n},\boldsymbol{n}')$$

parameterize

Landau-Migdal parameters

1  2 $= f_{12}(\boldsymbol{n},\boldsymbol{n}') + g_{12}(\boldsymbol{n},\boldsymbol{n}')\,\boldsymbol{\sigma_1}\boldsymbol{\sigma_2}$

extracted from experiment

 $= C_0\,(f + f'\,\boldsymbol{\tau_1}\boldsymbol{\tau_2} + g\,\boldsymbol{\sigma_1}\boldsymbol{\sigma_2} + g'\,\boldsymbol{\sigma_1}\boldsymbol{\sigma_2}\,\boldsymbol{\tau_1}\boldsymbol{\tau_2})$

$$C_0 = \frac{\pi^2}{m_N p_{FN}(n_0)} \simeq 300\ \text{MeV fm}^3 \simeq 0.77 m_\pi^{-2}\ \text{introduced for convenience}$$

$$f(\boldsymbol{n},\boldsymbol{n}') = \sum_l f_l\,P_l(\cos\theta_{\boldsymbol{n}\boldsymbol{n}'}) \qquad g(\boldsymbol{n},\boldsymbol{n}') = \sum_l g_l\,P_l(\cos\theta_{\boldsymbol{n}\boldsymbol{n}'}) \qquad \ldots$$

to be fitted to empirical information (nucleus properties)

effective mass $\quad m^* = m\left(1 + \frac{2}{3}f_1\right)$

compressibility $\quad K = 6\,\frac{p_{\rm F}^2}{m^*}\,(1 + 2\,f_0)$

symmetry energy $\quad E_{\rm sym} = \frac{1}{3}\,\frac{p_{\rm F}^2}{2\,m^*}\,(1 + 2\,f_0')$

[Saperstein, Fayans, et al. 1995, 1998]

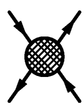
$f \simeq 0,\ f' \simeq 0.5 - 0.6,\ g \simeq 0.05 \pm 0.1,\ g' \simeq 1.1 \pm 0.1$

Nuclear Fermi liquid. Approximations

- Quasiparticle approximation for nucleons, $T \ll \epsilon_{\mathrm{FN}}$.

  Only in this case diagrams with open nucleon lines make sense.
  Otherwise the quasiparticle width is not negligible
  and closed diagram technique has to be used.

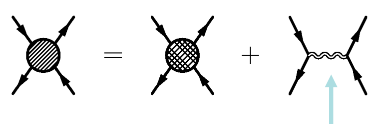
- Reduction of the more local interaction to the point-like interaction



$$= C_0 \left(f_{12} + g_{12} \boldsymbol{\sigma_1} \boldsymbol{\sigma_2} \right),$$

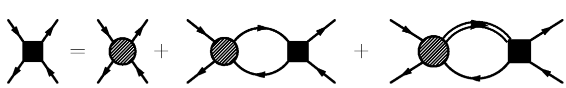
Assumption that the Landau-Migdal parameters, f_{12}, g_{12}, are constants
Is a rough approximation.

- **explicit pionic degrees of freedom**

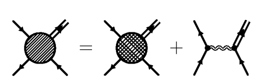


pion with residual (irreducible in NN^{-1} and ΔN^{-1}) s-wave π N interaction and $\pi\pi$ scattering``

- **explicit Δ degrees of freedom**

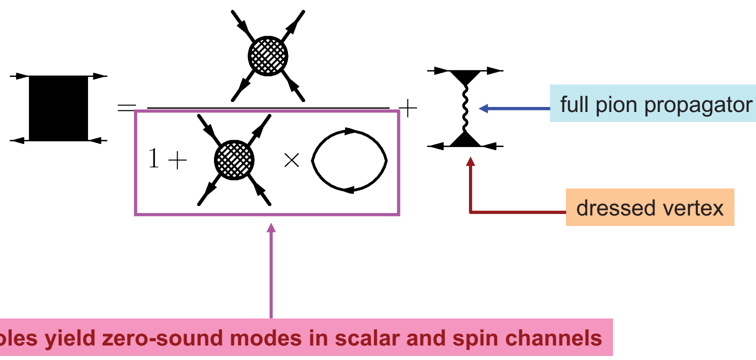


Part of the interaction involving Δ isobar is analogously constructed:



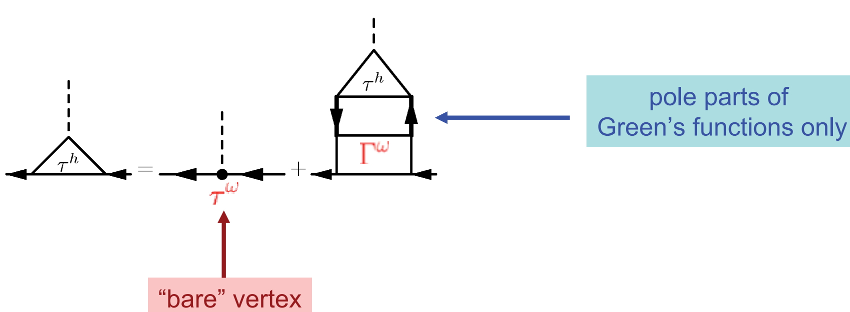
Nuclear Fermi liquid: resummed NN interaction

Graphically, the resummation is straightforward and yields:



Nuclear Fermi liquid: effective coupling

Coupling of the external field to a particle



- "Bare" vertices

"bare" vertex after the Fermi-liquid renormalization $\tau_a^\omega = [1 + \Gamma_0^\omega\, (G_+G_-)^\omega]\, \tau_a^0$

$$V_\mu^{nn} \approx g_V \chi_p^\dagger(p')(1, \boldsymbol{v})\chi_n(p)$$

$$A_\mu^{nn} \approx g_A \chi_p^\dagger(p')(\boldsymbol{\sigma}\cdot\boldsymbol{v}, \boldsymbol{\sigma})\chi_n(p)$$

$$B = (G_+G_-)^\omega = \lim_{q\to 0}\int \frac{2\,d^4p}{(2\pi)^4 i} G_+G_-$$

$$\tau_V^0 - \tau_A^0 = (V_\mu - A_\mu)\, l^\mu$$

weak interactions

$$\hat{\tau}_V^\omega = g_V\left(\tau_{V,0}^\omega\, l_0 - \boldsymbol{\tau}_{V,1}^\omega\, \boldsymbol{l}\right) \qquad \tau_{V,0}^\omega = \frac{e_V}{a}, \qquad \boldsymbol{\tau}_{V,1}^\omega = \frac{e_V}{a}\boldsymbol{v}$$

$$\hat{\tau}_A^\omega = -g_A\left(\boldsymbol{\tau}_{A,1}^\omega \boldsymbol{\sigma}\, l_0 - \tau_{A,0}^\omega\, \boldsymbol{\sigma}\boldsymbol{l}\right) \qquad \tau_{A,0}^\omega = \frac{e_A}{a}, \qquad \boldsymbol{\tau}_{A,1}^\omega = \frac{e_A}{a}\boldsymbol{v}$$

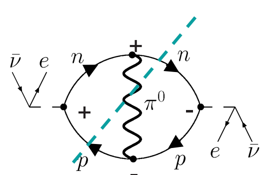
e_A e_V effective charges

$e_V = 1$ $\qquad \omega\, \tau_{V,0}^\omega - \boldsymbol{q}\, \boldsymbol{\tau}_{V,1}^\omega = G^{(\mathrm{pole}),-1}(p + q/2) - G^{(\mathrm{pole}),-1}(p - q/2)$

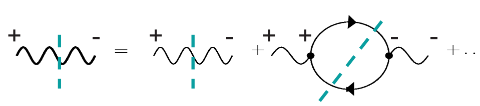
$e_A = 0.8$—0.95 experiment: Gamov-Teller transitions in nuclei $g_A^* \simeq 1$

Optical theorem for modified URCA reactions

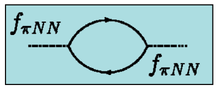
$$\epsilon_\nu^{\mathrm{MU}} = \int \frac{d^3q_e\,(1 - f_e)}{2\,\omega_e\,(2\,\pi)^3}\, \frac{d^3q_{\bar\nu}}{2\,\omega_{\bar\nu}\,(2\,\pi)^3}\, \omega_{\bar\nu}\, [-i\Pi_{\mathrm{MU}}^{-+}(q_e + q_{\bar\nu})]$$



+ 6 other diagrams



Pion spectrum at order $f_{\pi NN}{}^2$



$$D^{-1}(\omega, k) = \omega^2 - m_\pi^2 - k^2 - \Pi_0^R(\omega, k, n) = 0$$

- Perturbation theory in dimensionless parameter $f_{\pi NN} p_{\rm FN} \simeq 2$!
- For $\omega \to 0$, $k \sim p_{\rm FN}$ and for isospin symmetric matter

$$\Pi_0^R \simeq -\alpha_0 - i\beta_0\,\omega,\ \ \alpha_0 \simeq \frac{2\,m_N\,p_F\,k^2\,f_{\pi NN}^2}{\pi^2} > 0,\ \ \beta_0 \simeq \frac{m_N^2\,k\,f_{\pi NN}^2}{\pi} > 0.$$

$$\omega \propto -i\omega^{*\,2}(k_{\rm min})/\beta_0$$

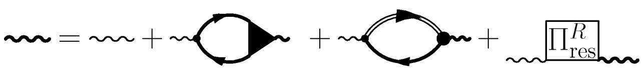
$\phi \sim \exp(-i\,\omega \cdot t)$ grows with time since $\omega^{*\,2}(k_m) = -D^{-1}(0, k_m \simeq p_{\rm F}) < 0$

It would happen already at $n > 0.3\ n_0$

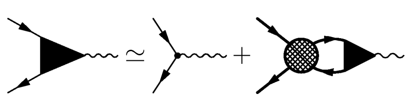
From experiment → no pion condensation in atomic nuclei ($n = n_0$)!

Virtual pion mode

Dyson equation for the full retarded pion Green's function



The πNΔ vertex includes a phenomenological range term.
The full πNN vertex takes into account NN short correlations.



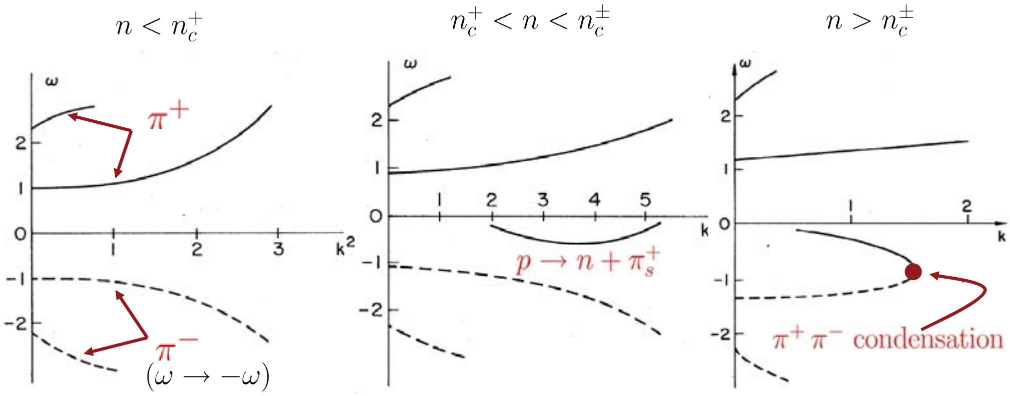
The value of the NN interaction, e.g. in the neutral pion channel, is determined by the full pion propagator at small ω and $k = p_{F,n}$.

$$\omega^{*2}(k) = -(D_\pi^R)^{-1}(\omega = 0, k, \mu_\pi).$$

For $n > n_{c1}$ ($n_{c1} < n_0$) the quantity $\omega^*(k)$ has minimum for $k = k_m \simeq (0.9\text{–}1)\,p_{\rm Fn}$.

The quantity $\omega^*(k_m)$ has the meaning of the *effective pion gap*.

Pion sectra and pion condensation



$n_c^+ \lesssim n_0\,, \quad n_c^\pm \sim (1-3)\,n_0\,, \quad n_c^{\pi^0} \sim (1-3)\,n_0$ [A.B. Migdal, Rev. Mod. Phys. 50 (1978)]

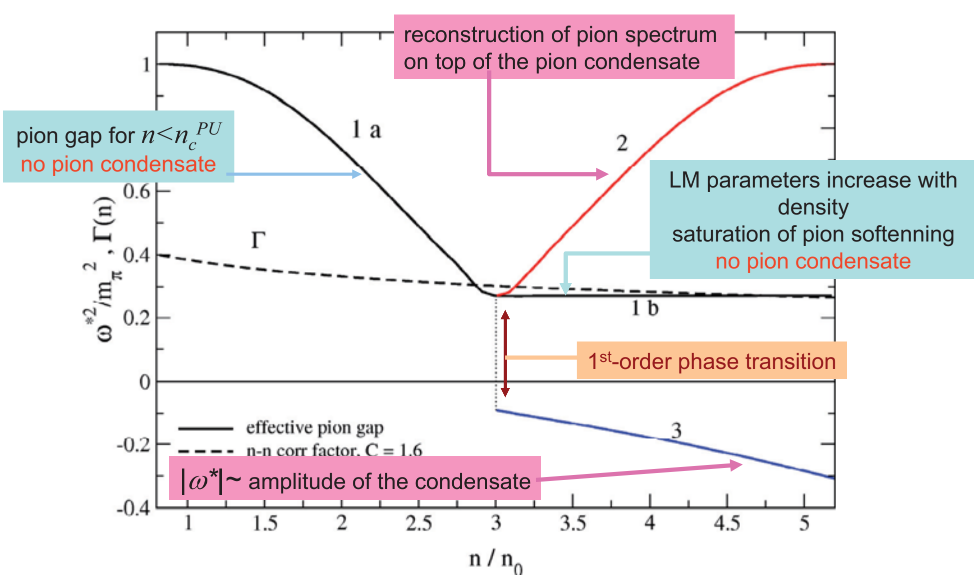
variational calculations [Akmal, Pandharipande, Ravenhall, PRC58 (1998)]:

charged pion condensate: $n_c \simeq 2\,n_0 \quad N = Z$

neutral pion condensate: $n_c \simeq 2\,n_0 \quad N = Z\,, \qquad n_c = 1.3\,n_0 \quad N >> Z$

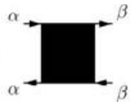
Virtual pion mode

$$\omega^{*2}(k) = -[D_\pi^R(\omega = 0, k, \mu_\pi)]^{-1}$$



Re-summed NN interaction

Re-summed NN interaction in neutral channel:

$\Gamma^R_{\alpha\beta} =$ 

$$= \underbrace{C_0\,\left(\mathcal{F}^R_{\alpha\beta} + \mathcal{Z}^R_{\alpha\beta}\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2)\right)}_{\text{contact terms}} + \underbrace{f^2_{\pi NN}\,\mathcal{T}^R_{\alpha\beta}\,(\boldsymbol{\sigma}_1\cdot\boldsymbol{k})(\boldsymbol{\sigma}_2\cdot\boldsymbol{k})}_{\text{one-pion exchange}}$$

Landau-Migdal parameters

$\alpha,\beta = (n,p)$

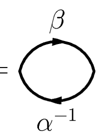
$$\begin{aligned} \mathcal{F}^R_{\alpha\beta} &= f_{\alpha\beta}\gamma(f_{\alpha\beta}) \\ \mathcal{Z}^R_{nn} &= g_{nn}\gamma(g_{nn}) \\ \mathcal{Z}^R_{np} &= g_{np}\gamma(g_{nn}) \end{aligned}$$

$$\begin{aligned} \mathcal{T}^R_{nn} &= \gamma^2(g_{nn})\,D^R_{\pi^0} \\ \mathcal{T}^R_{np} &= -\gamma(g_{nn})\,\gamma_{pp}\,D^R_{\pi^0} \\ \mathcal{T}^R_{pp} &= \gamma^2_{pp}\,D^R_{\pi^0} \end{aligned}$$

correlation functions

$$\begin{aligned} \gamma(x) &= 1/(1-2\,x\,C_0\,\mathcal{L}^R_{nn}) \\ \gamma_{pp} &= (1-4\,g\,C_0\,\mathcal{L}^R_{nn})\,\gamma(g_{nn}) \end{aligned}$$

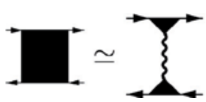
$D_{\pi^0}{}^R$ full retarded Green's function of π^0 calculated with the loop function

$\mathcal{L}_{\alpha\beta} =$ 

[Voskresensky, Senatorov, Sov. J. Nucl. Phys. 45 (1987)]

Re-summed NN interaction

Non-resonance processes: As follows from numerical estimates, the main contribution to NN interaction at $n>n_0$ is given by Modified One-Pion Exchange (MOPE) not by Free One-Pion Exchange (FOPE)



$\propto \dfrac{\Gamma_s^2}{(\omega^*)^2}$

if the spin-isospin channel, ~$(\sigma_1\cdot \boldsymbol{k})\,(\sigma_2\cdot \boldsymbol{k})$, is not forbidden or suppressed by symmetry reasons, small momentum transfer, etc.

Two important ingradients of MOPE interaction:

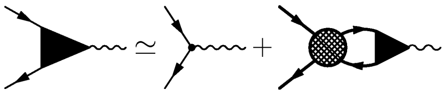
pion softening: pion gap $[\omega^*(n=n_0)]^2 \simeq m_\pi^2$ instead of $m_\pi^2 + p^2_{\mathrm{F},N} \simeq 7\,m_\pi^2$ in FOPE

vertex suppression factor: $\Gamma(n_0) \simeq 0.3$—0.4

➡ Compensation of the repulsion Γ^2 and attraction ω^* for $n = n_0$ but sharp density dependence of the interaction!

Re-summed weak interaction

The weak coupling vertex is renormalized in medium:



wavy line corresponds to weak current

For the β-decay:

$$V_\beta = \frac{G}{\sqrt{2}}\left[\widetilde{\gamma}(f')\, l_0 - g_A\, \widetilde{\gamma}(g')\, \boldsymbol{l\sigma}\right]$$

For processes on the neutral currents $N_1N_2 \to N_1N_2\nu\bar{\nu}$

$$V_{nn} = -\frac{G}{2\sqrt{2}}\left[\gamma(f_{nn})\, l_0 - g_A\, \gamma(g_{nn})\, \boldsymbol{l\sigma}\right]$$

$$V_{pp}^N = \frac{G}{2\sqrt{2}}\left[\kappa_{pp}\, l_0 - g_A\, \gamma_{pp}\, \boldsymbol{l\sigma}\right]$$

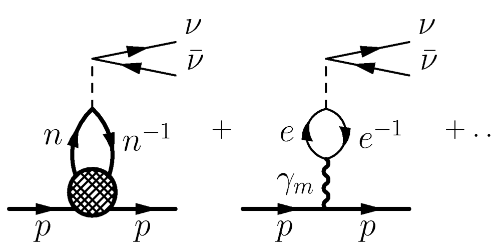
with the correlation functions

$$\kappa_{pp} = c_V - 2 f_{np}\, \gamma(f_{nn})\, C_0\, L_{nn}, \;\; \gamma_{pp} = (1 - 4\, g\, C_0\, L_{nn})\; \gamma(g_{nn}),$$

[Voskresensky, Senatorov, Sov. J. Nucl. Phys. 45 (1987)]

Re-summed weak interaction

Renormalization of the proton vertex (vector part of $V_{pp}^N + V_{pp}^\gamma$) is governed by processes



which are absent in vacuum

The squared matrix element related to the proton vector current term is $\propto c_V^2$ in vacuum, but in medium it is $\propto \kappa_{pp}^2$ (first diagram)

➡ enhancement up to ~ 10—100 times for 1.5—$3n_0$

[Voskresensky, Senatorov, Sov. J. Nucl. Phys (1987)]

Another enhancement factor (up to $\sim 10^2$) comes from the virtual photon (γ_m) dressded in medium (second diagram)

[Voskresensky, Kolomeitsev, Kampfer Sov. JETP, 87 (1998); Leinson, Phys. Lett. B (2000)]

Re-summed weak interaction

Another example of the correlation effect:

In vacuum the branching ratio of the kaon dacays is

$$\frac{\Gamma(K^- \to e^- + \nu_e)}{\Gamma(K^- \to \mu^- + \nu_\mu)} \approx 2.5 \times 10^{-5}$$

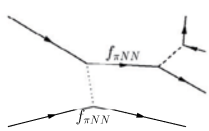
In medium this ratio can be of the order of unity due to Λp^{-1} decays of virtual K^-.

➡ In dependence of the reaction channel, in-medium effects may lead to strong enhancement or suppression of reaction rates

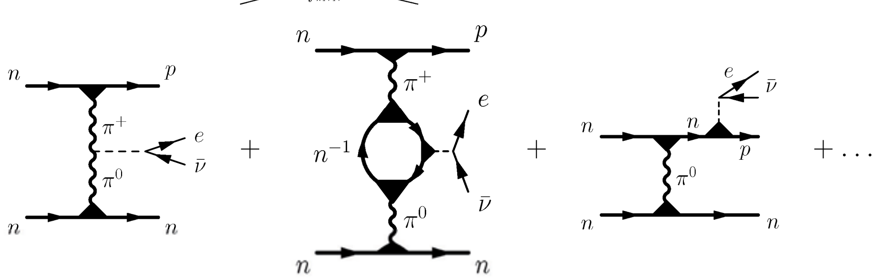
Neglect of these effects may lead to misleading results

Medium effects in two-nucleon processes, MMU

MU(FOPE) :



➡ MMU(MOPE) :



First diagram yields main contribution, second diagram – a less contribution, third diagram (generalizes the MU(FOPE)) yields much less term for $n \gtrsim n_0$.

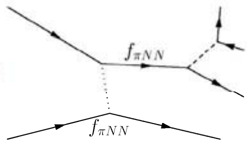
pion softening ➡ enhancement of the rate towards the n_c^{PU}.

$$\frac{\epsilon_\nu[\mathrm{MMU}]}{\varepsilon_\nu[\mathrm{MU}]} \sim 10^3\ (n/n_0)^{10/3} \frac{\Gamma^6(n)}{[\omega^*(n)/m_\pi]^8}.$$

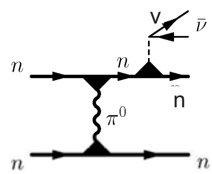
Very strong density dependence

Medium effects in two-nucleon processes, MNB

NB(FOPE) :



MNB(MOPE) :



$+\dots$

pion softening enhancement of the rate towards the $n_c^{\rm PU}$.

$$\frac{\varepsilon_\nu[\rm MNB]}{\varepsilon_\nu[\rm NB]} \sim 10^3\ (n/n_0)^{4/3} \frac{\Gamma^6(n)}{[\omega^*(n)/m_\pi]^3}.$$

A different enhancement factor for the MNB processes compared to MMU.

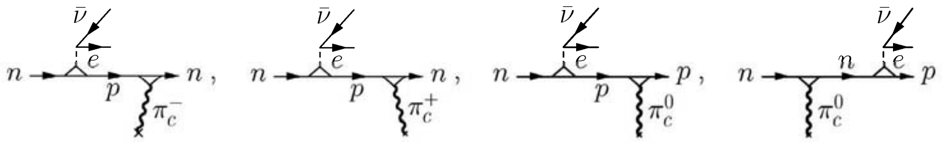
Proper DU processes

They are forbidden up to the density $n_c^{\rm DU}$ when triangle inequality $p_{\rm Fn} < p_{\rm Fp} + p_{\rm Fe}$ begins to fulfill. For traditional EoS like $V18 + \delta v + UIX^*$ DU processes are permitted only for $n > 5\,n_0$.

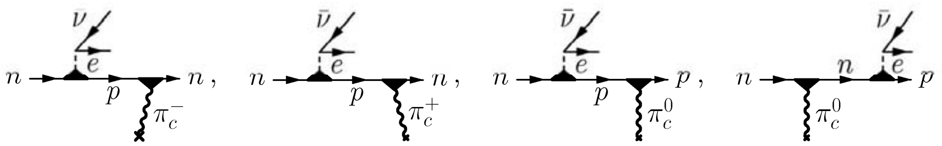
Due to full vertices a factor $\Gamma^2_{\rm w-s}$ in emissivity.
(rather minor modification, since $\omega \simeq p_{{\rm F},e} \gg q \sim T$).

DU-like processes (on condensates)

For $n > n_c^{\rm PU}$ ($M > M_c^{\rm PU}$) PU processes:



with free vertices: $\epsilon_\nu \sim 10^{26}\, T_9^6 (n/n_0)^{1/3}\, \frac{\rm erg}{\rm cm^3\ sec}$ ➡



with full vertices: $\epsilon_\nu \sim 10^{26} \Gamma_s^2 \Gamma_{\rm w-s}^2\, T_9^6 (n/n_0)^{1/3}\, \frac{\rm erg}{\rm cm^3\ sec}$

➡ $\Gamma_s^2 \Gamma_{\rm w-s}^2 \sim 10^{-1} - 10^{-2}$

DU-like processes (on condensates)

- at $n > n_c^{\rm KU} \gtrsim (3–4) n_0$ ($M > M_c^{\rm KU}$)

kaon Urca (KU) processes on charged and neutral kaon condensation may occur.

Kaon condensates can be created in reactions

$$e + X \to K^- + X',\ n + X \to p + K^- + X',\ n + X \to n + \bar{K}^0 + X', \text{ etc.}$$

K^- condensate can arise in *s*-wave [Kaplan, Nelson, PLB175 (1986); Brown, Kubodera, Page, Pizzochero, PRD37 (1988)]

or in p-wave [Kolomeitsev, Voskresensky, Kampfer, NPA588 (1995); Kolomeitsev, Voskresensky, PRC68 (2003)]

➡ extra $\sin^2\theta_C \simeq (0.23)^2$ suppression factor in emissivity,
and a different correlation factor (poorly known constants)

Charged ρ condensate [Voskresensky, Phys.Lett. B392 (1997), Kolomeitsev, Voskresensky, NPA759 (2005)

➡ if ρ-meson is treated as non-Abelian boson
and its effective mass decreases in dense matter (below half of m_ρ)

Neutrino production processes are similar to those for π^- condensate.

Emissivity is estimated as: $\epsilon_\nu^{\rm RU} \sim 10^{26} g_{\rho N}^{*2} \left(\frac{m_N^*}{m_N}\right)^2 \frac{\mu_e}{m_N} \frac{n_{ch}^\rho}{m_\rho^*} \Gamma_s^2 \Gamma_{\rm w-s}^2\, T_9^6,\ \frac{\rm erg}{\rm cm^3\ sec}$

Other resonance processes

[Voskresensky, Khodel, Zverev, Clark, AJ, **533** (2000)]

- Bubble rearrangement of the Fermi sphere in the vicinity of the π^0 condensation critical point. Occurs if necessary condition of stability of the normal state

$$\delta E_0 = \int [\epsilon(p, n(p)) - \mu]\delta n(p) \frac{d^3p}{(2\pi)^3} > 0$$

is violated (when a depression $\epsilon(p, n(p)) - \mu < 0$ forms for $p > p_{\rm F}$ or elevation $\epsilon(p, n(p)) - \mu > 0$, for $p < p_{\rm F}$). Numerical study shows that due to strong attraction in pion channel of NN interact. for $n_{c1} < n < n_c^{\pi^0}$ there arises a bubble (absence of particles) in neutron distrib. for $p_i < p < p_f < p_{\rm F}$ and triangle inequality is fulfilled.

➡ DU-like process $\epsilon_\nu \sim 10^{27} T_9^6$. ➡ T^6 dependence.

- Competing possibility: fermion condensation.

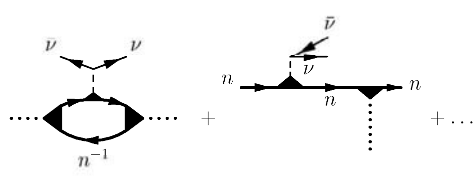
Also new neutron distribution $0 < n(p) < 1$ for $p < p_{\rm F}$ is found in the range $n_{c1} < n < n_c^{\pi^0}$ from condition $\delta E_0/\delta n(p) = \mu$ for $p_i < p < p_f < p_{\rm F}$. It produces $dm_n/dT \propto 1/T$ for $T \to 0$.

➡ DU-like process $\epsilon_\nu \sim 10^{27} T_9^5$ with T^5 dependence, more efficient than DU.

Other resonance processes

There are many other in-medium reaction channels, e.g., with zero sound excitations.

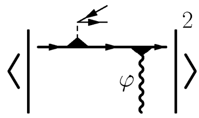
- The most essential contribution comes from the neutral current processes



The dotted line is zero sound quantum of appropriate symmetry. These are resonance processes (second, of DU-type) similar to processes going on condensates. Difference: rates of reactions with zero sounds are $\propto$ to thermal occupations of the corresponding spectrum branches. Contribution of the resonance reactions with zero sounds is suppressed due to a small phase space volume ($q \sim T$), **cf. phonon processes.**

Relations between one-, two-, etc. nucleon processes

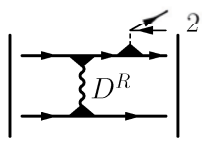
Within the QPA for fermions, the reaction rate with participation of one fermion and the boson is given by [D.V., Senatorov JETP Lett. 40 (1984); JETP 63 (1986)]



For equilibrium system ($T \neq 0$) there is exact relation

$$(<\hat{\varphi}_2^\dagger \hat{\varphi}_1>)(p) = iD^{-+} + |\varphi_c|^2 = \frac{A_B}{\exp(\frac{\omega}{T}) - 1} + |\varphi_c|^2, \quad A_B = -2\mathrm{Im}D^R,$$

A_B contains the δ-function at spectrum branches related to resonance proc. , like zero sound, and also a contrib. $\propto \mathrm{Im}\Pi^R$ of the loop diagram. Within the QPA taking Im part is related to a cut of the diagram. ➡ The same term can be calculated via

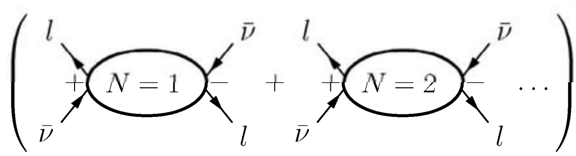


i.e. through two-nucleon process.

Optical theorem again

Perturbative diagrams are irrelevant for calculation of in-medium processes. In general case one should deal with closed diagrams in terms of dressed Green's functions [D.V., Senatorov Sov. Nucl. Phys. 45 (1987); Knoll, D.V. Ann. Phys. 249 (1996)].

$$\frac{dW^{\rm tot}_{\bar\nu l}}{dt} = \frac{(1-n_l)d^3q_{\bar\nu}dq_l^3}{(2\pi)^6\, 4\,\omega_{\bar\nu}\,\omega_l}$$



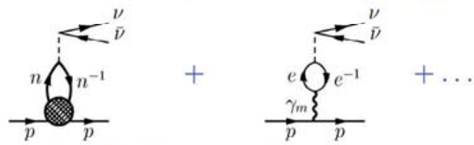
Series in number of full $\{-,+\}$ Green func. (in terms of non-eq. diag. tech.).
Only if QPA for fermions is relevant (i.e. for $T \ll \epsilon_{\rm FN}$) one can do a cut of fermion lines, thus considering diagrams with open fermion legs:

- not the case for pions,
- not the case for the description of initial stage of NS evolution (not small T)
- For $T < T_c$ closed diagrams with two types of fermion Green's functions: normal and anomalous ones.

Superfluidity. DU-like processes. MNPBF processes

Permitted only for $T < T_c$.

Renormalization of the proton vertex (vector part of $V_{pp}^{N} + V_{pp}^{\gamma}$) is governed by processes



forbidden in vacuum. ➡ 10^2 enhancement! [Voskresnsky, Senatorov (1987)]

incorporated in cooling code [Schaab, Voskresensky, Sedrakian, Weber, Weigel, AA (1997); Blaschke, Grigorian, Voskresensky, AA (2001, 2004); Grigorian, Voskresensky, AA (2005)]

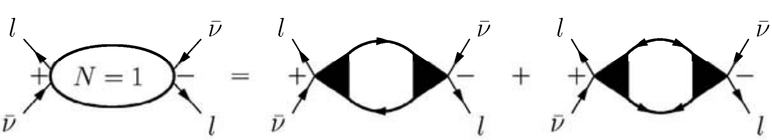
Both for neutrons and protons:

$$\epsilon_\nu \sim 10^{29} \left[\frac{\Delta_{nn}}{\mathrm{MeV}}\right]^7 \left[\frac{T}{\Delta_{nn}}\right]^{1/2} (n/n_0)^{1/3}\, \xi_{ii}^2, \quad \frac{\mathrm{erg}}{\mathrm{cm}^3\,\mathrm{sec}},$$

Δ_{ii} is NN gap, $i = n, p$, $\xi_{ii} = \exp[-\Delta_{ii}/T]$ is superfluid suppression factor.

PBF in optical theorem formalism

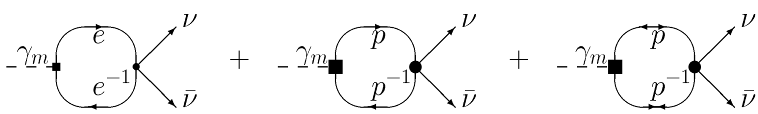
We may explicitly decompose the first term in the series as



➡ PFB processes are of the one-nucleon origin determined by $N = 1$ diagrams. Due to that a huge 10^{29} pre-factor in emissivity.

- QP Green functions and dressed vertices. Due to dressed vertices $\sim 10^2$ factor for MpPFB.

Superfluidity. Massive photon decay



$$\varepsilon_\nu^\gamma \approx 2.6 \cdot 10^{25}\, T_9^{\frac{3}{2}}\, e^{-\frac{m_\gamma}{T}} \left(\frac{m_\gamma}{\mathrm{MeV}}\right)^{\frac{7}{2}} \left(\frac{n}{n_0}\right)^{\frac{8}{3}} \left(1 + \frac{3}{2}\frac{T}{m_\gamma}\right) \; \frac{\mathrm{erg}}{\mathrm{cm}^3\,\mathrm{sec}},$$

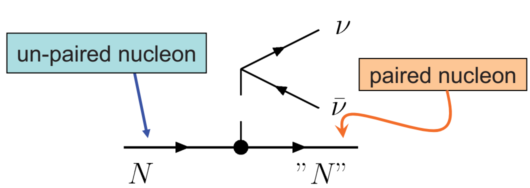
Again, only “normal” correlations were incorporated in vertices

[Voskresensky, Kolomeitsev, Kaempfer, JETP 87 (1998)]

Breaking and Formation of Cooper pairs (PBF)

In superfluid ($T<T_c<0.1$-1 MeV) all two-nucleon processes are suppressed by factor ***exp(-2Δ/T)***

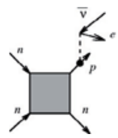
new “quasi”-one-nucleon-like processes (one-nucleon phase space volume) become permitted



Neutrino emission reactions

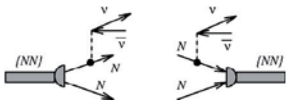
minimal $\quad T < T_{\rm opac} \sim 10^{-1} \div 10^{0}$ MeV

MU:



$$10^{28} \times \left(\frac{T}{\rm MeV}\right)^{8} \left(\frac{n_e}{n_0}\right)^{\frac{1}{3}} \frac{\rm erg}{\rm cm^3\,s} \times e^{-2\Delta/T}$$

PBF:

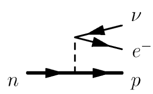


$$10^{28} \times \left(\frac{\Delta}{\rm MeV}\right)^{7} \left(\frac{T}{\Delta}\right)^{\frac{1}{2}} e^{-2\Delta/T} \frac{\rm erg}{\rm cm^3\,s}$$

dominates for $T \ll \Delta$ (at least)

exotic

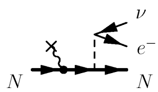
DU:



$$10^{27} \times T_9^6 \left(\frac{n_e}{n_0}\right)^{\frac{1}{3}} \frac{\rm erg}{\rm cm^3\,s} \times e^{-\Delta/T}$$

allowed if $n > n_c^{\rm DU}$

PU:



$$10^{27} \times T_9^6 \frac{|\varphi_c|^2}{m_\pi^2} \frac{\rm erg}{\rm cm^3\,s} \times e^{-\Delta/T}$$

allowed if $n > n_c^{\rm PU}$

1976

THE ASTROPHYSICAL JOURNAL, **205**: 541–544, 1976 April 15

NEUTRINO PAIR EMISSION FROM FINITE-TEMPERATURE NEUTRON SUPERFLUID AND THE COOLING OF YOUNG NEUTRON STARS*

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Received 1975 August 29

its critical temperature T_c for the transition to the superfluid state. At any nonzero temperature $T < T_c$ the neutron fluid has two components: a superfluid condensate and quasiparticle excitations (broken "Cooper pairs"). We calculate the neutrino pair emissivity due to the recombination of broken pairs that then join the condensate.[1] Again, this process depends on the existence of a weak neutral current that permits transitions between states of differing numbers of broken-pair excitations. We find the resulting emissivity to be (in units with $\hbar = c = 1$):

$$\epsilon = \frac{64}{15\pi^3}\left(\frac{G_F}{\sqrt{2}} g_n\right)^2 \nu(0)(kT)^7 F(\beta\Delta) = 4.88 \times 10^{27} T_{10}{}^7 \left(\frac{m^*}{m}\right)^2 (v_n/c) F(\beta\Delta) \quad \text{ergs cm}^{-3}\,\text{s}^{-1}\,. \qquad (1a, b)$$

$10^{20} T_9^7$ *less efficient than MU*

only $\langle V_0(x) l_0(x) V_0(y) l_0^\dagger(y) \rangle$ free vertices only

1986

Sov. J. Nucl. Phys. 45 (3), March 1987

Description of a nuclear interaction in the Keldysh diagram technique and the problem of neutrino luminosity of neutron stars

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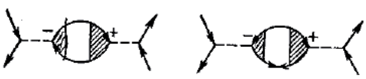
(Submitted 20 February 1986)
Yad. Fiz. 45, 657–669 (March 1987)

IV. GENERALIZATION OF THE OPTICAL-THEOREM FORMALISM IN THE KELDYSH DIAGRAM TECHNIQUE TO SYSTEMS WITH PAIRING. CALCULATION OF THE LUMINOSITY OF THE ONE-NUCLEON PROCESSES $n_{pa} \to n\nu\bar{\nu}$ and $p_{pa} \to p\nu\bar{\nu}$



a b

closed diagram technique

$$\varepsilon_n^{\nu\bar{\nu}} = \frac{4G^2[\gamma(f_{nn}) + 3g_A^2\gamma(g_{nn})]p_{F_n} m_n^* \Delta_n^7}{15\pi^2} I\left(\frac{\Delta_n}{T}\right)$$

$\langle V_0(x) l_0(x) V_0(y) l_0^\dagger(y) \rangle$

$\langle (\boldsymbol{A}\boldsymbol{l}(x))\ (\boldsymbol{A}\,\boldsymbol{l}^\dagger(y)) \rangle$

no contribution for s pairing

$$\epsilon_\nu \sim 10^{28} \left[\frac{\Delta_{nn}}{\mathrm{MeV}}\right]^7 \left[\frac{T}{\Delta_{nn}}\right]^{1/2} (n/n_0)^{1/3}\, \xi_{nn}^2, \quad \frac{\mathrm{erg}}{\mathrm{cm}^3\ \mathrm{sec}}$$

Δ_{nn} is neutron gap and $\xi_{nn}=\exp(-\Delta_{nn}/T)$

One-nucleon phase-space, dominant over MU for T<< 2Δ

1999

Astron. Astrophys. 343, 650–660 (1999)

ASTRONOMY AND ASTROPHYSICS

Neutrino emission due to Cooper pairing of nucleons in cooling neutron stars

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singlet and triplet pairing

$\langle V_0(x) l_0(x) V_0(y) l_0^\dagger(y) \rangle$

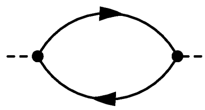
calculated using nucleon wave functions
and Bogolubov transformations

free vertices

relativistic corrections for axial current contribution

Vector current conservation

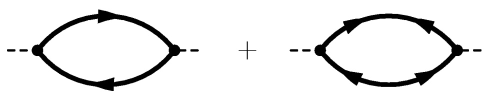
normal matter



free vertices and Green's functions

The WT identity is fulfilled and the current is conserved

superfluid matter



free vertices and non-interacting quasi-particles
with a gapped spectrum $E_p^2 = \epsilon_p^2 + \Delta^2$

Gap appears due to a non-trivial self-energy

Vertex must be modified accordingly. Otherwise the current is not conserved

2006

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Vector current conservation and neutrino emission from singlet-paired baryons in neutron stars

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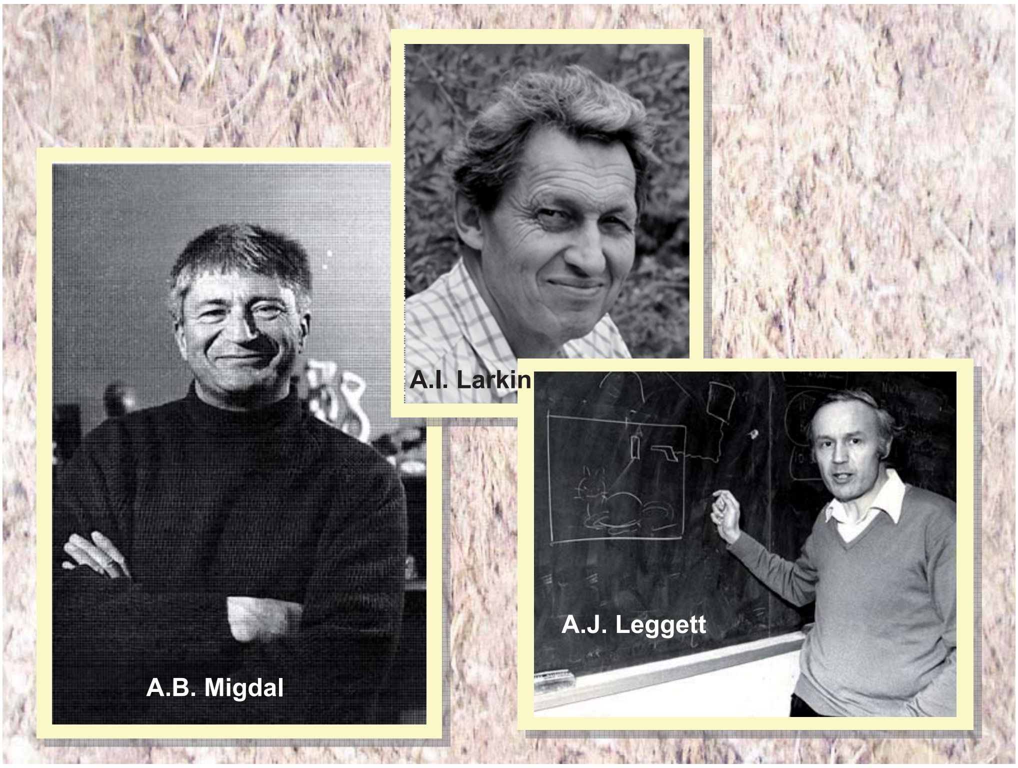
vector current conservation $q_\mu V^\mu = 0$ $\qquad V_0 = \frac{1}{\omega} \boldsymbol{q} \boldsymbol{V}$

$$\langle V_0 V_0 \rangle = \frac{\boldsymbol{q}^2}{\omega^2} \langle \boldsymbol{V} \boldsymbol{V} \rangle \sim O\big(\boldsymbol{q}^2 v_F^2/\omega^2\big)$$ at least !

Conclusion: $\boxed{\text{Thus the neutrino energy losses due to singlet-state pairing of baryons can, in practice, be neglected in simulations of neutron star cooling. This makes unimportant the neutrino radiation from pairing of protons or hyperons.}}$

calculations are done in the limit $\omega << \Delta$, whereas for PFB processes $\omega \sim 2\Delta$



Neutrino emissivity in superfluid Fermi liquid

Consider pure neutron matter at $T<<2\Delta$

$$\varepsilon_{\nu\bar{\nu}} = \frac{G^2}{8}\int \frac{d^3q_1}{(2\pi)^3\,2\,\omega_1}\frac{d^3q_2}{(2\pi)^3\,2\,\omega_2}\,\omega\, \underline{n_{\rm bos}(\omega)}\, 2\,{\rm Im}\sum_{\rm lept.\ spin}\chi(q)$$

produces leading exponential term $\propto e^{-2\Delta/T}$

closed diagrams calculated with Green's functions for $T=0$

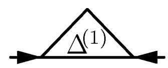
Superfluid system

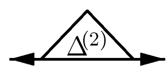
- **Normal Green's functions**

particle $i\,\hat{G} =$  hole $i\,\hat{G}^h(p) =$  .

$$\hat{G}^h(p) = \sigma_2\,[\hat{G}^h(p)]^{\mathrm{T}}\,\sigma_2 = i\,\hat{G}(-p)$$

Number of excitations is not conserved !

amplitude of 2 particle annihilation $-i\,\hat{\Delta}^{(1)} =$ 

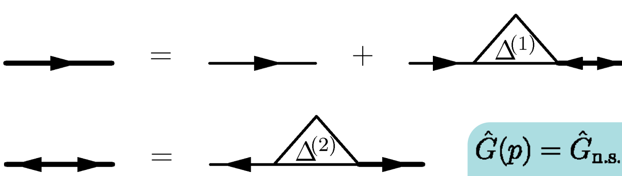
amplitude of 2 particle creation $-i\,\hat{\Delta}^{(2)} =$ 

- **Anomalous Green's functions**

describe transformation particles into holes and vice versa

particle → hole $i\hat{F}^{(1)} =$  hole →particle $i\hat{F}^{(2)} =$ 

Superfluid system



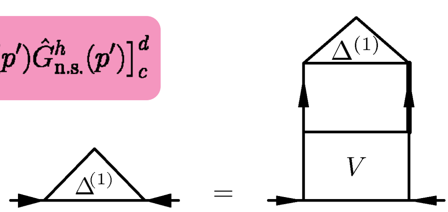
- **Gor'kov equations**

$$\hat{G}(p) = \hat{G}_{\mathrm{n.s.}}(p) + \hat{G}_{\mathrm{n.s.}}(p)\,\hat{\Delta}^{(1)}(p)\,\hat{F}^{(2)}(p)$$

$$\hat{F}^{(2)}(p) = \hat{G}^h_{\mathrm{n.s.}}(p)\,\hat{\Delta}^{(2)}(p)\,\hat{G}(p)$$

- **Gap equation**

$$[\hat{\Delta}^{(1)}]^a_b = \int \frac{d^4p'}{(2\pi)^4 i}\,[\widehat{V}(p,p')]^{ac}_{bd}\,[\hat{G}(p')\hat{\Delta}^{(1)}(p')\hat{G}^h_{\mathrm{n.s.}}(p')]^d_c$$



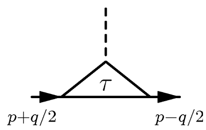
$$[\widehat{V}]^{ac}_{bd} = V_0\,(i\sigma_2)^a_b\,(i\sigma_2)^c_d + V_1\,(i\sigma_2\boldsymbol{\sigma})^a_b\,(\boldsymbol{\sigma}\,i\sigma_2)^c_d$$

attractive interaction in paired particle-particle channel

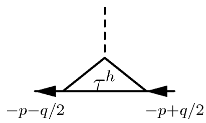
Superfluid system

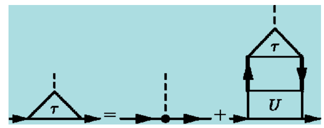
- **Coupling to an external field**

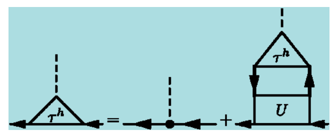
to particle



to hole

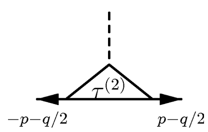




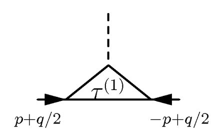


equations for vertices in normal system

In superfluid systems new type of couplings:



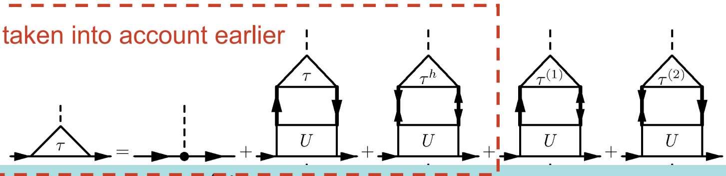
ext. field create 2 particles



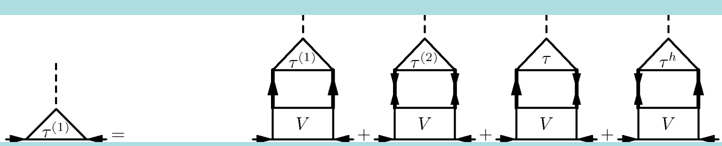
ext. field create 2 holes

Equations for vertices

taken into account earlier



$$\hat\tau^a_b(p,q) - \hat\tau^a_{0b}(p,q) = \int \frac{d^4p'}{(2\pi)^4 i} U^{ac}_{bd}(p,p',q)\Big[\hat G(p'_+)\,\hat\tau(p',q)\,\hat G(p'_-) + \hat F^{(1)}(p'_+)\,\hat\tau^{(h)}(p',q)\,\hat F^{(2)}(p'_-)$$
$$+\hat G(p'_+)\,\hat\tau^{(1)}(p',q)\,\hat F^{(2)}(p'_-) + \hat F^{(1)}(p'_+)\,\hat\tau^{(2)}(p',q)\,\hat G(p'_-)\Big]^d_c$$



$$\tau^{(1)}(p,q) = i\int \frac{d^4p'}{(2\pi)^4} V^{ab}_{cd}(p,p',q)\Big[\hat G(p'_+)\,\hat\tau^{(1)}(p',q)\,\hat G^h(p'_-) + \hat F^{(1)}(p'_-)\,\hat\tau(p',q)\,\hat G(-p'_+)$$
$$+\hat G(p'_-)\,\hat\tau^{(h)}(p',q)\,\hat F^{(1)}(p'_+) + \hat F^{(1)}(p'_-)\,\hat\tau^{(2)}(p',q)\,\hat F^{(1)}(p'_+)\Big]^{cd}$$

Cannot be written in matrix form in Nambu Gor'kov space since U=/=V

Fermi liquid approximation

S wave pairing $\hat{\Delta}^{(1)} = \hat{\Delta}^{(2)} = \Delta\, i\,\sigma_2$ $\hat{F}^{(1)} = \hat{F}^{(2)} = F\, i\sigma_2$

pole parts of the Green's functions

$$G(p) = \frac{a\,(\epsilon + \epsilon_p)}{\epsilon^2 - E_p^2 + i0\,\mathrm{sgn}\epsilon} \qquad F(p) = \frac{-a\,\Delta}{\epsilon^2 - E_p^2 + i0\,\mathrm{sgn}\epsilon}$$

$$\epsilon_p \approx \frac{p^2 - p_{\rm F}^2}{2\,m_N^*} \approx v_{\rm F}\,(p - p_{\rm F}) \longrightarrow$$ "gapped" spectrum $E_p^2 = \epsilon_p^2 + \Delta^2$

Landau-Migdal parameters

particle-hole interaction:

$$[\widehat{\Gamma}^\omega]^{ac}_{bd} = \Gamma^\omega_0(\boldsymbol{n},\boldsymbol{n}')\,\delta^a_b\,\delta^c_d + \Gamma^\omega_1(\boldsymbol{n},\boldsymbol{n}')\,(\boldsymbol{\sigma})^a_b\,(\boldsymbol{\sigma})^c_d.$$

repulsive (stability of Fermi liquid)

particle-particle interaction:

$$[\widehat{\Gamma}^\xi]^{ac}_{bd} = \Gamma^\xi_0(\boldsymbol{n},\boldsymbol{n}')(i\sigma_2)^a_b(i\sigma_2)^c_d + \Gamma^\xi_1(\boldsymbol{n},\boldsymbol{n}')(i\sigma_2\,\boldsymbol{\sigma})^a_b(\boldsymbol{\sigma}\,i\sigma_2)^c_d$$

attractive (to allow pairing of fermions)

Fermi liquid approximation

integration over the internal lines is reduced to the Ferm surface

$$\int \frac{2\,\mathrm{d}^4p}{(2\pi)^4\,i} \simeq \int \frac{\mathrm{d}\Omega_{\boldsymbol{p}}}{4\,\pi} \times \int \mathrm{d}\Phi_p \qquad \int \mathrm{d}\Phi_p = \rho \int_{-\infty}^{+\infty} \frac{\mathrm{d}\epsilon}{2\,\pi\,i} \int_{-\infty}^{+\infty} \mathrm{d}\epsilon_p$$

can be taken explicitly for $T=0$

$$\rho = \frac{m^*\,p_{\rm F}}{\pi^2}$$ density states at Fermi surface

- **Gap equation** for $T \ll 2\,\Delta$

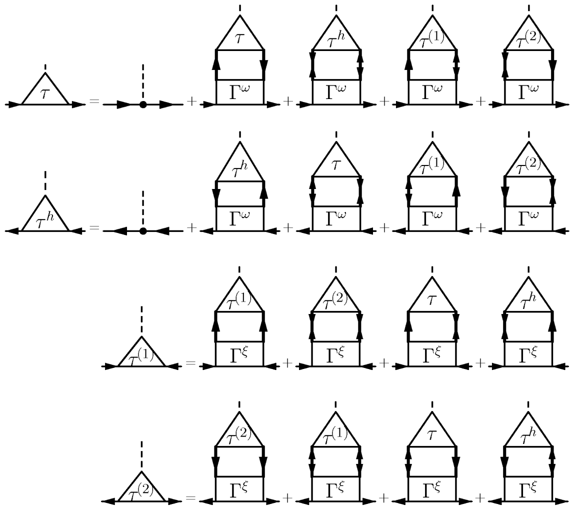
$$\Delta(\boldsymbol{n}) = -A_0\,\langle \underline{\Gamma^\xi_0(\boldsymbol{n},\boldsymbol{n}')}\,\Delta(\boldsymbol{n}')\rangle_{\boldsymbol{n}'} \qquad \langle \dots \rangle_{\boldsymbol{n}} = \int \frac{\mathrm{d}\Omega_{\boldsymbol{n}}}{4\pi}(\dots)$$

$$A_0 = \int \mathrm{d}\Phi_p\, G_{\rm n.s.}(p)\, G^h(p)\,\theta(\xi - \epsilon_p) \approx a^2\,\rho\,\ln(2\xi/\Delta)$$

Γ^ξ in an effective parameterization of a pairing gap

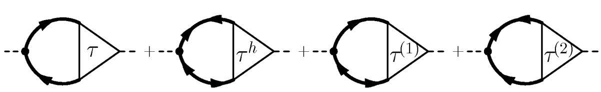
Fermi liquid approximation

- **Coupling to an external field**



Fermi liquid approximation

- **Current-current correlator**

$-i\,\chi =$ 

$$\chi_a \;=\; \mathrm{Tr}\int \frac{d^4p}{(2\,\pi)^4\, i}\hat{\tau}^{\omega}_a \left\{\hat{G}_+\,\hat{\tau}^{\dagger}_a\,\hat{G}_- + \hat{F}^{(1)}_+\,\hat{\tau}^{h\dagger}_a\,\hat{F}^{(2)}_- + \hat{G}_+\,\hat{\tau}^{(1)\dagger}_a\hat{F}^{(2)}_- + \hat{F}^{(1)}_+\,\hat{\tau}^{(2)\dagger}_a\hat{G}_-\right\}$$

$a = V, A$ $\qquad\qquad G_\pm = G(p \pm q/2)$

$$\chi_V(q) = g_V^2 \left\langle \left(l_0 - \boldsymbol{v}\,\boldsymbol{l}\right)\left(l_0^\dagger\,\chi_{V,0}(\boldsymbol{n},q) - \boldsymbol{\chi}_{V,1}(\boldsymbol{n},q)\,\boldsymbol{l}^\dagger\right)\right\rangle_{\boldsymbol{n}}$$

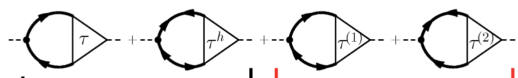
$$\chi_A(q) = g_A^2\, e_A^2 \left\langle \left(l_0\,\boldsymbol{v} - \boldsymbol{l}\right)\left(l_0^\dagger\,\boldsymbol{\chi}_{A,1}(\boldsymbol{n},q) - \chi_{A,0}(\boldsymbol{n},q)\boldsymbol{l}^\dagger\right)\right\rangle_{\boldsymbol{n}}$$

[Kolomeitsev, Voskresensky PRC (2009)]

Solution for correlators

$$\big(\chi_{a,0}(q,\boldsymbol{n}),\boldsymbol{\chi}_{a,1}(q,\boldsymbol{n})\big)$$

$-i\,\chi =$



$$\chi_{a,0}(\boldsymbol{n},q) = \gamma_a(q;P_{a,0})\,\mathcal{L}(\boldsymbol{n},q;P_{a,0})$$

$$\gamma_a^{-1}(q;P) = 1-\Gamma_a^\omega\,\langle\mathcal{L}(\boldsymbol{n},q;P)\rangle_{\boldsymbol{n}}$$

normal Fermi liquid correlations

$\Gamma_V^\omega = \Gamma_0^\omega \qquad \Gamma_A^\omega = \Gamma_1^\omega$

$$\mathcal{L}(\boldsymbol{n},q;P) = L(\boldsymbol{n},q;P) - \frac{\langle O(\boldsymbol{n},q;P)\rangle_{\boldsymbol{n}}}{\langle N(\boldsymbol{n},q)\rangle_{\boldsymbol{n}}}\,M(\boldsymbol{n},q)$$

$$P_{V,0} = -P_{V,1} = -P_{A,0} = P_{A,1} = 1$$

$$L(\boldsymbol{n},q;P) = a^2\,\rho\left[\frac{\boldsymbol{q}\,\boldsymbol{v}}{\omega-\boldsymbol{q}\,\boldsymbol{v}}(1-g(z)) - g(z)\,(1+P)/2\right] \qquad M(\boldsymbol{n},q) = -a^2\,\rho\,\frac{\omega+\boldsymbol{q}\,\boldsymbol{v}}{2\,\Delta}\,g(z)$$

$$N(\boldsymbol{n},q) = a^2\,\rho\,\frac{\omega^2-(\boldsymbol{q}\,\boldsymbol{v})^2}{4\,\Delta^2}\,g(z) \qquad O(\boldsymbol{n},q;P) = a^2\,\rho\left[\frac{\omega+\boldsymbol{q}\,\boldsymbol{v}}{4\,\Delta}+\frac{\omega-\boldsymbol{q}\,\boldsymbol{v}}{4\,\Delta}\,P\right]g(z)$$

$$g(z^2) = \int_{-1/2}^{+1/2}\mathrm{d}x[4\,z^2\,x^2-z^2+1+i\,0]^{-1} \qquad z^2 = \frac{\omega^2-(\boldsymbol{q}\,\boldsymbol{v})^2}{4\,\Delta^2} > 1 \quad \boldsymbol{v} = v_{\mathrm{F}}\,\boldsymbol{n}$$

$$\langle L(\boldsymbol{n},q;+1)\rangle_{\boldsymbol{n}} \simeq -g(\omega^2/\Delta^2)\,\Big(1+O\big(v_{\mathrm{F}}^2\,\boldsymbol{q}^{\,2}/\omega^2\big)\Big) \qquad \langle\mathcal{L}(\boldsymbol{n},q;+1)\rangle_{\boldsymbol{n}} \simeq 0 + O\big(g\,v_{\mathrm{F}}^4\,\boldsymbol{q}^{\,4}/\omega^4\big)$$

$$\boldsymbol{\chi}_{a,1}(\boldsymbol{n},q) = \gamma_a(q;P_{a,1})\,\boldsymbol{v}\,\mathcal{L}(\boldsymbol{n},q;P_{a,1}) + \delta\boldsymbol{\chi}_{a,1}(\boldsymbol{n},q)$$

$$\delta\boldsymbol{\chi}_{a,1}(\boldsymbol{n},q) = \frac{M(\boldsymbol{n},q)}{\langle N(\boldsymbol{n}',q)\rangle_{\boldsymbol{n}'}}\,\langle O(\boldsymbol{n}',q;P_{a,1})\,(\boldsymbol{v}-\boldsymbol{v}\,')\rangle_{\boldsymbol{n}'}$$
$$+\mathcal{L}(\boldsymbol{n},q;P_{a,1})\,\gamma_a(q;P_{a,1})\,\Gamma_a^\omega\,\langle\widetilde{\mathcal{L}}(\boldsymbol{n}',q;P_{a,1})(\boldsymbol{v}\,'-\boldsymbol{v}\,)\rangle_{\boldsymbol{n}'}$$

$$\widetilde{\mathcal{L}}(\boldsymbol{n},q;P) = L(\boldsymbol{n},q;P) - \frac{\langle M(\boldsymbol{n},q)\rangle_{\boldsymbol{n}}}{\langle N(\boldsymbol{n},q)\rangle_{\boldsymbol{n}}}\,O(\boldsymbol{n},q;P)$$

$$P_{V,0} = -P_{V,1} = -P_{A,0} = P_{A,1} = 1$$

Vector current conservation

4-vector $\left(\chi_{V,0}(q,\boldsymbol{n}),\boldsymbol{\chi}_{V,1}(q,\boldsymbol{n})\right)$

$$\mathrm{Im}\,\langle\tau_V^\omega\,\chi_V^\nu\rangle_{\boldsymbol{n}}\,q_\nu = O(f^\omega\,g\,\boldsymbol{q}^{\,6}\,v_{\mathrm{F}}^6/\omega^6)$$

$$\mathrm{Re}\,\langle\tau_V^\omega\,\chi_V^\nu\rangle_{\boldsymbol{n}}\,q_\nu +$$  $$= O(f^\omega\,g\,\boldsymbol{q}^{\,6}\,v_{\mathrm{F}}^6/\omega^6)$$

$$q\sim T,\qquad \omega\sim 2\Delta$$

$$\Gamma_0^{\omega,\xi}=f^{\omega,\xi}/(a^2\rho(n_0))$$

$$\frac{1}{2\,m^*}\psi^\dagger(\boldsymbol{\nabla}-\boldsymbol{V}\,)^2\psi$$

gauging of nucleon kinetic energy

Neutrino emissivity

Total emissivity $\varepsilon_{\nu\nu}=\varepsilon_{\nu\nu,V}+\varepsilon_{\nu\nu,A}$ sum of axial and vector current contributions

$$\varepsilon_{\nu\nu,a}=\frac{G^2\,g_a^{*\,2}}{48\,\pi^4}\int_0^\infty \mathrm{d}\omega\,\omega\,n_{\mathrm{bos}}(\omega)\int_0^\omega \mathrm{d}|\boldsymbol{q}|\boldsymbol{q}^{\,2}\frac{\kappa_a}{a^2}$$

$$\kappa_a=\underbrace{\int\frac{d^3q_1}{2\,\omega_1}\frac{d^3q_2}{2\,\omega_2}\,\delta^{(4)}(q_1+q_2-q)}_{\text{leptonic phase space}}\frac{3}{4\,\pi}\,\mathrm{Im}\sum\ \chi_a(q)$$

- **neutrino emissivity on vector currents**

$$\kappa_V \;=\; \mathrm{Im}\Big[\boldsymbol{q}^{\,2}\,\langle\chi_{V,0}(\boldsymbol{n},q)\rangle_{\boldsymbol{n}} + \langle(\boldsymbol{q}\,\boldsymbol{v}\,)\cancel{\boldsymbol{q}\,\boldsymbol{\chi}_{V,1}}(\boldsymbol{n},q)\rangle_{\boldsymbol{n}} + (\omega^2-\cancel{\boldsymbol{q}^{\,2})\,\langle}\boldsymbol{v}\,\boldsymbol{\chi}_{V,1}(\boldsymbol{n},q)\rangle_{\boldsymbol{n}}$$
$$-\omega\,\langle(\boldsymbol{q}\,\boldsymbol{v}\,)\,\cancel{\chi_{V,0}}(\boldsymbol{n},q)\rangle_{\boldsymbol{n}} - \omega\,\langle\boldsymbol{q}\,\cancel{\boldsymbol{\chi}_{V,1}(\boldsymbol{n}},q)\rangle_{\boldsymbol{n}}\Big]$$

in the limit v_F<<1 naively

→ we obtain the old result

$$\kappa_V = \boldsymbol{q}^{\,2}\,\mathrm{Im}\,\langle\chi_{V,0}(\boldsymbol{n},q)\rangle_{\boldsymbol{n}} = \boldsymbol{q}^{\,2}\,\mathrm{Im}\,\langle L(\boldsymbol{n},q;+1)\rangle_{\boldsymbol{n}} \simeq -\boldsymbol{q}^2\,a^2\,\rho\,\mathrm{Im}\,g\Big(\omega^2/\Delta^2\Big)$$

no vertex corrections !

$$\epsilon_{\nu\nu}^{(0n)} = \frac{4\rho_n\,G^2\Delta_n^7}{15\,\pi^3} I(\frac{\Delta_n}{T}) \qquad I(z) = \int_1^\infty \frac{\mathrm{d}y\,y^5}{\sqrt{y^2-1}} e^{-2zy}\,,$$

- **neutrino emissivity on vector currents**

$$\kappa_V \;=\; \mathrm{Im}\Big[\boldsymbol{q}^{\,2}\,\langle\chi_{V,0}(\boldsymbol{n},q)\rangle_{\boldsymbol{n}} + \langle(\boldsymbol{q}\,\boldsymbol{v}\,)\,\boldsymbol{q}\,\boldsymbol{\chi}_{V,1}(\boldsymbol{n},q)\rangle_{\boldsymbol{n}} + (\omega^2-\boldsymbol{q}^{\,2})\,\langle\boldsymbol{v}\,\boldsymbol{\chi}_{V,1}(\boldsymbol{n},q)\rangle_{\boldsymbol{n}}$$
$$-\omega\,\langle(\boldsymbol{q}\,\boldsymbol{v}\,)\,\chi_{V,0}(\boldsymbol{n},q)\rangle_{\boldsymbol{n}} - \omega\,\langle\boldsymbol{q}\,\boldsymbol{\chi}_{V,1}(\boldsymbol{n},q)\rangle_{\boldsymbol{n}}\Big]$$

exploiting current conservation $\mathrm{Im}\,\langle\tau_V^{\omega}\,\chi_V^{\nu}\rangle_{\boldsymbol{n}}\,q_\nu = 0$

$$\boldsymbol{\kappa_V \;=\; (q^{\,2}-\omega^2)\,\mathrm{Im}\Big\langle\chi_{V,0}(n,q) - v\,\chi_{V,1}(n,q)\Big\rangle_{n} > 0}$$

negative

$$\mathrm{Im}\,\langle\chi_{V,0}(\boldsymbol{n},q)\rangle \;\approx\; -\frac{4\,\boldsymbol{q}^{\,4}v_{\mathrm{F}}^4}{45\,\omega^4}\,a^2\,\rho\,\mathrm{Im}\,g(\frac{\omega^2}{4\Delta^2}) > 0$$

$$\mathrm{Im}\,\langle\boldsymbol{v}\,\boldsymbol{\chi}_{V,1}(\boldsymbol{n},q)\rangle \;\approx\; -\frac{2\,\boldsymbol{q}^{\,2}v_{\mathrm{F}}^4}{9\omega^2}\,a^2\,\rho\,\mathrm{Im}\,g(\frac{\omega^2}{4\Delta^2}) > 0$$

strongly suppressed → $\boldsymbol{\epsilon_{\nu\nu,V}^{(n)} \simeq \frac{4}{81}\,v_{\mathrm{F},n}^4\,\epsilon_{\nu\nu}^{(0n)}}$

- **neutrino emissivity on axial currents**

$$\begin{aligned}\kappa_A \;=\; & \mathrm{Im}\Big[\boldsymbol{q}^{\,2}\langle \boldsymbol{v}\,\boldsymbol{\chi}_{A,1}(\boldsymbol{n},q)\rangle_{\boldsymbol{n}} + (3\,\omega^2 - 2\,\boldsymbol{q}^{\,2})\,\langle \chi_{A,0}(\boldsymbol{n},q)\rangle_{\boldsymbol{n}} \\ & -\; \omega\langle \boldsymbol{q}\,\boldsymbol{\chi}_{A,1}(\boldsymbol{n},q)\rangle_{\boldsymbol{n}} - \omega\,\langle(\boldsymbol{q}\,\boldsymbol{v}\,)\,\chi_{A,0}(\boldsymbol{n},q)\rangle_{\boldsymbol{n}}\Big]\end{aligned}$$

$$\kappa_A \;\approx\; -a^2\,\rho\, v_{\mathrm{F}}^2\,\boldsymbol{q}^{\,2}\left[1 + (1 - \tfrac{2}{3}\tfrac{\boldsymbol{q}^2}{\omega^2}) - \tfrac{2}{3}\right]\mathrm{Im}\,g(\frac{\omega^2}{4\Delta^2})$$

$$\boxed{\epsilon^{(n)}_{\nu\nu,A} \simeq \Big(1 + \frac{11}{21} - \frac{2}{3}\Big)\, v^2_{\mathrm{F},n}\,\epsilon^{(0n)}_{\nu\nu}}$$

contributions from $< A_0 A_0 >$ $< \boldsymbol{A}\boldsymbol{A} >$ cross terms

Emissivity in pair formation breaking reactions

$$R(\mathrm{nPFB}) = \frac{\epsilon^{\mathrm{nPBF}}_{\nu\nu}}{\epsilon^{(0n)}_{\nu\nu}} \simeq \frac{\epsilon^{\mathrm{nPBF}}_{\nu\nu,A}}{\epsilon^{(0n)}_{\nu\nu}} \simeq \frac{6}{7}\,g_A^{*\,2}\,v^2_{\mathrm{F},n} = F_n\,v^2_{\mathrm{F},n}\,.$$

$$\begin{array}{c} n = n_0 = 0.17\ \mathrm{fm}^{-3} \\ m^* = 0.8\ m \end{array} \longrightarrow \begin{array}{c} F_n \simeq 0.9\text{–}1.2 \\ v_{\mathrm{F},n} \simeq 0.36 \end{array} \longrightarrow R \simeq 0.12\text{—}0.15$$

$$\begin{array}{c} n = 2\,n_0 \\ m^* = 0.7\ m \end{array} \longrightarrow R \simeq 0.24\text{—}0.32$$

Main contribution is due to the axial current.

Suppression is of the order ~0.1-- 0.3

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Conclusions ….

THEORY OF SUPERFLUID FERMI LIQUID. APPLICATION TO THE NUCLEUS

- Medium effects are very important in calculation of the emissivity of the neutrino reactions
- Ward identity (vector curent conservation) should be fulfilled for vector current terms.
- Suppression of the emissivity of the PBF processes (1S0 pairing) does not affect qualitative conclusions following from previously done numerical simulations of NS cooling.

probabilities of electromagnetic transitions in nuclei.

I. INTRODUCTION

IN all real many-particle systems the interaction is not small, and therefore in the derivation of quantitative relations one cannot proceed, as is often done, by combining some part of the diagrams of perturbation theory.

superconductors, the Debye temperature). Here one must introduce, in addition to Γ^{ω}, one other function of the angles between the momenta of the quasi-particles, Γ^{ξ}; the spherical harmonic of this function is connected with the width of the energy gap. It is natural to expect that the functions Γ^{ω} and Γ^{ξ} depending on the angles between

Nuclear medium cooling scenario

✔ **Main ingredients**

- **Neutrino production process** including in-medium effects
  Medium effects produce strong density dependence of the production rates
- **Equation of state** of nuclear matter
- **Pairing gaps**

- **Nuclear equation of state** Urbana-Argonne: $A18 + \delta v + UIX^*$:

$$M_{\rm crit}^{\rm DU} \simeq 2\ M_{\odot},\ M_{\rm max} \simeq 2.2\ M_{\odot}\ (n_{\rm cent} \simeq 7\ n_0).$$

but acausal for $n > 4n_0$.

Improvement: Heiselberg, Hjorth-Jensen (HHJ) causal interpolation EoS:

$$M_{crit}^{\rm DU} \simeq 1.839\ M_{\odot},\ \ M_{max} \simeq 1.96\ M_{\odot}\ (n_{cent} \simeq 7\ n_0)$$

- **Pairing gaps**

The gap equation:

$$\Delta(\boldsymbol{p}) = -\int \left[\Gamma_{NN}(\boldsymbol{p},\boldsymbol{p}') + \underline{\Gamma^{\pi}_{NN}(\boldsymbol{p}-\boldsymbol{p}',\omega=\epsilon(\boldsymbol{p}')}\right] \frac{\Delta(\boldsymbol{p}')}{2\epsilon(\boldsymbol{p}')}\frac{d^3p'}{(2\pi)^3}$$

For $^3P_2$ pairing gap tensor force due to a pion exchange is important.

Model I

Numerical calculations by Schwenk and Friman [Phys. Rev. Lett., 92 (2004)], argued for a strong suppression of the $^3P_2$ neutron gaps, down to values <10 keV, as the consequence of the medium-induced spin-orbit interaction.

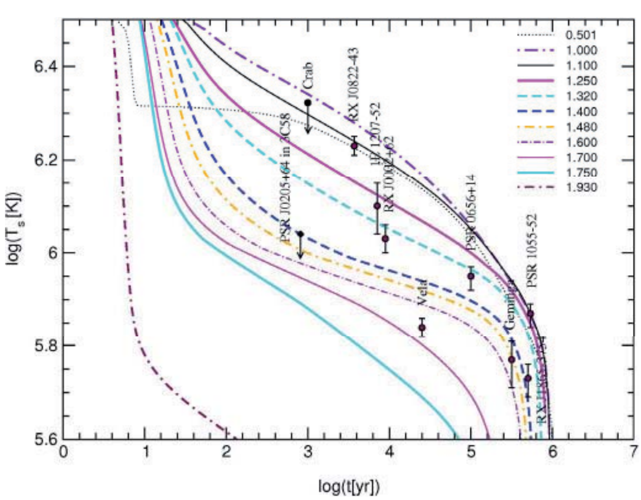
Model II

Khodel et al. [Phys. Rev. Lett.,93 (2004)], assuming a proximity to a 2nd-order phase transition of pion condensation and a strong pion softening, estimated $^3P_2$ neutron pairing gap as large as 1-10~MeV in a broad region of densities.

Both possibilities checked within the ``nuclear medium cooling scenario."

Nuclear medium cooling scenario

[Blaschke, Grigorian, Voskresensky, AA 424 (2004)]

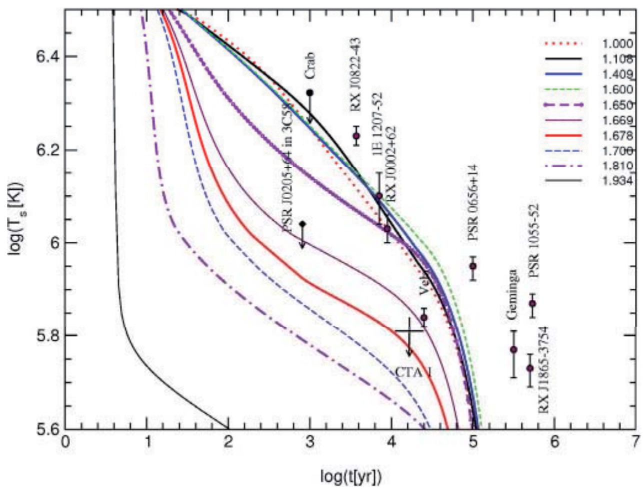


- $^3P_2$ gaps from Schwenk & Friman model.
- This result passed $\log N - \log S$ (population synthesis) controle

[S.Popov et al., A&A (2008)]

Nuclear medium cooling scenario

[Grigorian, Voskresensky, A&A (2005)]



- $^3P_2$ gaps from Khodel et al. model.
- No appropriate description of data.

Conclusions

In-medium effects have to be included otherwise calculations become inconsistent.

Large uncertainties remain due to poor knowledge of properties of dense matter

Comparison with data motivates strong density/(neutron star mass) dependence of medium effects and, thus, different neutron star masses in supernova remnants.