

EXPERIMENTAL VERIFICATION OF NUMERICAL OPTIMIZATION OF A PLUNGER FOR GLASS PRESSING

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Abstract: *The main aim of these experiments is to verify the results of numerical optimization described in [1], [2], [3] and [4]. The results of measuring the classical plunger with its drilled internal cavity and the optimized plunger with its sparked-out inner cavity and applied flow control body are presented in this article. The temperature distributions across both plunger surfaces are estimated by values measured at four points placed 5 millimetres under the plunger surfaces.*

1. INTRODUCTION

Two main problems appear during glass pressing on the surface of a moulded piece. If the surface of the plunger is too hot at the moment of separation of the moulded piece and the plunger, then the glass melt adheres to the device and deformation of the moulded piece follows. Conversely, if the surface of the plunger is too cold, then small fire cracks appear on the surface of the moulded piece, which means poorer quality of production. For these reasons, it is convenient to achieve a constant distribution of temperature across the surface of the moulding device at the moment of separation of the plunger from the moulded piece.

2. FORMULATION OF THE PROBLEM

We consider the union of four planar regions $\Omega = \Omega_0 \cup \Omega_1 \cup \Omega_2 \cup \Omega_3$, which represents a planar cross section of the system mould, glass, plunger, canal of the plunger and flow control body.

Region Ω_0 represents the plunger, region Ω_1 a cooled glass piece, region Ω_2 the cooling canal inside the plunger, where cooling water flows, and region Ω_3 represents the mould. Furthermore, we denote Γ_1 as the boundary between plunger Ω_0 and moulded piece Ω_1 and Γ_2 as the boundary between plunger Ω_0 and plunger cavity Ω_2 . Γ_2 has a shape received as a result of the optimization problem described in [3]. We denote Γ_3 as part of the boundary connecting the system mould, moulded piece and plunger with presser, Γ_4 part of the axis of symmetry, Γ_5 part of the boundary formed by the outward surface of the flow control body, which has a shape which it received as a result of the second step of the optimization problem described in [4]. Γ_6 is the denotation for part of the boundary between moulded piece Ω_1 and mould Ω_3 and Γ_7 is the outward boundary of the mould, which is surrounded by an external environment.

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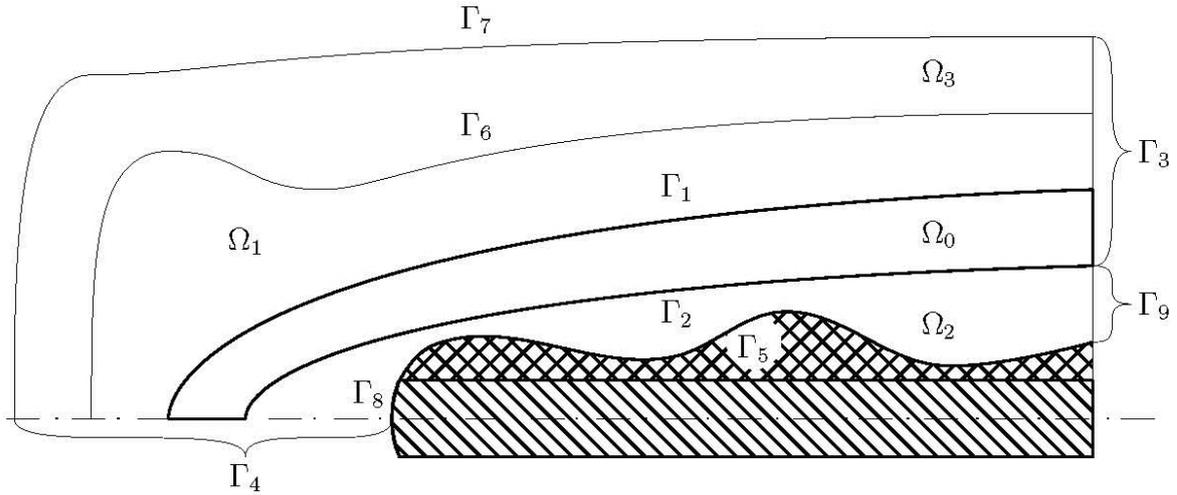


Fig. 1 Scheme of the system mould, glass piece, plunger, canal of plunger and flow control body.

Γ_8 denotes part of the boundary where cooling water comes into the cooling canal of the plunger, and Γ_9 part of the boundary where water exits.

In the regions Ω_0 (plunger) and Ω_3 (mould), the conduction of heat without an inner heat source is described by the equation

$$-\frac{k}{\rho} \Delta \vartheta = 0 . \quad (1)$$

In the region Ω_1 (glass piece), we use the equation for the conduction of heat with inner heat source q_1 in the form

$$-\frac{k}{\rho} \Delta \vartheta = q_1 . \quad (2)$$

In the region Ω_2 (plunger canal in which flows cooling water) the conduction of heat is described by the energy equation in the form

$$c_v \text{grad} \vartheta \cdot \mathbf{u} - \frac{k}{\rho} \Delta \vartheta = 2\nu |D(\mathbf{u})|^2 , \quad (3)$$

where ϑ denotes temperature on the absolute temperature scale and \mathbf{u} is the velocity of flowing water satisfying the equation of continuity

$$\text{div} \mathbf{u} = 0 \quad (4)$$

and Navier-Stokes equation in the form

$$\mathbf{u} \cdot \text{grad} \mathbf{u} - \frac{\mu}{\rho} \Delta \mathbf{u} = \mathbf{f} - \frac{1}{\rho} \text{grad} p . \quad (5)$$

In the model we assume transit conditions for contact between two bodies, where one of them changes its mass because of the influence of solidification, on boundaries Γ_1 and Γ_6 , respectively, in the form

$$(k_1 \frac{\partial \vartheta_1}{\partial n})^+ - (k_0 \frac{\partial \vartheta_0}{\partial n})^- = \beta_1 \quad \text{on } \Gamma_1 \quad (6)$$

and

$$(k_1 \frac{\partial \vartheta_1}{\partial n})^+ - (k_3 \frac{\partial \vartheta_3}{\partial n})^- = \beta_6 \quad \text{on } \Gamma_6 . \quad (7)$$

The boundary conditions of the third type for contact between a body and its environment at boundaries Γ_2 and Γ_7 , respectively, are

$$-(k_0 \frac{\partial \vartheta_0}{\partial n})^- = \alpha(\vartheta_0|_{\Gamma_2} - \vartheta_2|_{\Gamma_2}) \quad \text{on } \Gamma_2 \quad (8)$$

and

$$-(k_3 \frac{\partial \vartheta_3}{\partial n})^- = \alpha(\vartheta_3|_{\Gamma_7} - \vartheta_4) \quad \text{on } \Gamma_7 , \quad (9)$$

where ϑ_i denotes temperature in Ω_i ($i=0,2,3$) and ϑ_4 the temperature of the environment.

In article [3] the problems of shape optimization were formulated, where the shape of boundary Γ_2 and Γ_5 , respectively, such that the cost functional

$$\mathfrak{S}^S(\Gamma_2) = \|\vartheta(\Gamma_2)|_{\Gamma_1} - K\|_{0,r,\Gamma_1}^2 \quad (10)$$

and

$$\mathfrak{S}^P(\Gamma_5) = \|\vartheta(\Gamma_5)|_{\Gamma_1} - K\|_{0,r,\Gamma_1}^2 \quad (11)$$

take the minimum value of the set of admissible design functions.

The experiment suggested itself as a result of the numerical results of optimization problems. The distribution of temperature in the classical plunger with a drilled internal cavity was compared with the distribution of temperature in the optimized plunger with its sparked-out inner cavity and applied flow control body. For details see the article of M. Starý [5], where the course of the experiment with a detailed description of the testing equipment are described.

Measurements were evaluated at four points placed 5 millimetres below the surface. Point B1 is located on the lower end of the axis of the plunger (5mm below the surface), point B4 at a distance of 50 millimetres above a plane perpendicular to the plunger axis and level with the vertex, point B7 at a distance of 125 millimetres (from the aforementioned plane) and point B10 at a distance of 200 millimetres.

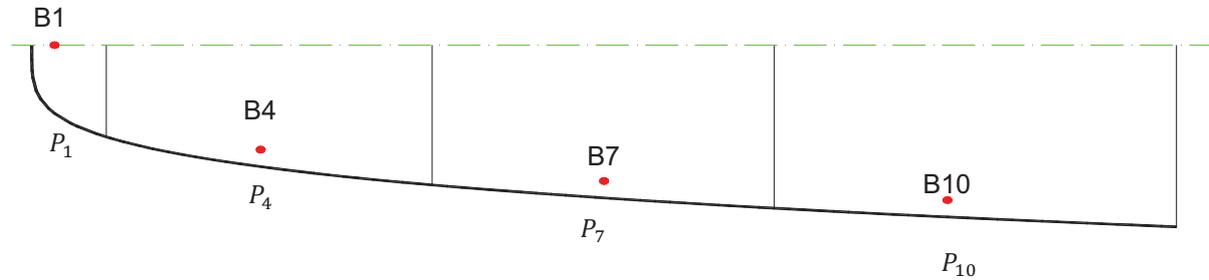


Fig. 2 Schema of the location of measurement points with areas corresponding to each point.

Surface corresponding to point Bi	P_1	P_4	P_7	P_{10}	Total P
Area of corresponding surface [m ²]	0,003	0,016	0,021	0,029	0,069

Tab. 1 Areas of surfaces corresponding to single measurement points.

The results of five measurements of the original plunger construction (Plunger I) and the results of five measurements of the plunger with the optimized cooling cavity and applied flow control body (Plunger II) are summarized in tab. 2. Stagnation temperatures after six hours at these points are in the first four lines, the range of measured temperatures is on the fifth line.

The average temperatures computed as a weighted arithmetical mean

$$\bar{\vartheta}_l = \frac{1}{P} (\vartheta_1 \cdot P_1 + \vartheta_4 \cdot P_4 + \vartheta_7 \cdot P_7 + \vartheta_{10} \cdot P_{10}) , \quad (12)$$

where the weights are areas of surfaces corresponding to single measurement points (see fig. 2 and tab. 1), are on the sixth line of tab. 2.

The arithmetical mean of average temperatures from five measurements

$$K = \frac{\sum_{l=1}^5 \bar{\vartheta}_l}{5} \quad (13)$$

is introduced in the seventh line.

In the last line of tab. 2, the numerical approximation of cost functional (11) in the form

$$\mathfrak{S}^{Exp} = (\vartheta_{B1} - K)^2 \cdot P_1 + (\vartheta_{B4} - K)^2 \cdot P_4 + (\vartheta_{B7} - K)^2 \cdot P_7 + (\vartheta_{B10} - K)^2 \cdot P_{10} \quad (14)$$

is computed.

	Plunger I					Plunger II				
Experiment	1.	2.	3.	4.	5.	1.	2.	3.	4.	5.
ϑ_{B1} [°C]	113,4	113,3	113,0	112,9	112,5	108,9	105,1	104,7	105,8	101,7
ϑ_{B4} [°C]	104,3	105,8	106,8	106,1	106,2	105,0	102,2	103,1	104,7	100,7
ϑ_{B7} [°C]	101,9	99,3	100,0	98,7	98,2	109,5	107,9	109,3	109,1	107,8
ϑ_{B10} [°C]	96,7	96,2	96,9	95,7	95,0	105,9	105,2	106,6	105,5	104,9
Δ_{\max} [°C]	16,7	17,1	16,1	17,2	17,5	4,5	5,7	6,2	4,4	7,1
$\bar{\vartheta}_1$	100,8	100,1	100,8	99,8	99,3	106,9	105,3	106,5	106,4	104,7
K	100,2					106,0				
\mathfrak{S}^{Exp}	1,21	1,50	1,51	1,67	1,89	0,30	0,33	0,38	0,24	0,60

Tab. 2 Stagnation temperatures for 5 experiments for each plunger.

The main target of the conducted optimization was to achieve uniform temperature across the entire surface of the plunger.

We are testing the null hypothesis about the correspondence of mean values of the outputs of the cost functional of experimental data obtained from both plungers against the alternative, namely, that the mean value of the outputs of the cost functional on the optimized plunger is statistically significantly less than the mean value of the outputs of the cost functional from the original plunger. We denote μ_1 the mean value of outputs of the cost functional \mathfrak{S}^{Exp} on plunger I and μ_2 the mean value of outputs of \mathfrak{S}^{Exp} on plunger II.

A single tail test about the correspondence of mean values was used, so

$$H_0 : \mu_1 = \mu_2, \quad (15)$$

$$H_1 : \mu_1 > \mu_2. \quad (16)$$

Testing statistic

$$T = \frac{\bar{\mathfrak{S}}_1^{Exp} - \bar{\mathfrak{S}}_2^{Exp}}{\sqrt{\frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} \sqrt{\frac{1}{n_1} + \frac{1}{n_2}}}} = 3,9857, \quad (17)$$

where $\bar{\mathfrak{S}}_1^{Exp}$ and $\bar{\mathfrak{S}}_2^{Exp}$, respectively, denote the arithmetical average of data from sample 1 and sample 2, n_1 and n_2 the size of sample 1 and sample 2, s_1^2 and s_2^2 the standard deviation of data from sample 1 and sample 2.

3. CONCLUSION

The results allow us to reject the null hypothesis even at significance level $\alpha=0,005$. That means that we can say with 99,5% probability that the results obtained from plunger II are better than the results obtained from plunger I.

We can calculate a 95% confidence interval for the difference of mean values $\mu_1 - \mu_2$ in the form (0,821; 1,514). This allows us to say that the mean value μ_2 is about 0,821 less than the mean value μ_1 with a probability of 97,5%.

4. ACKNOWLEDGEMENT

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