

Wormholes and Time-Machines in Nonminimally Coupled Matter-Curvature Theories of Gravity

Orfeu Bertolami^{1,a} and Ricardo Zambujal Ferreira^{2,b,c}

¹*Departamento de Física e Astronomia, Faculdade de Ciências, Universidade do Porto, Rua do Campo Alegre 687, 4169-007 Porto, Portugal*

²*CP3-Origins & Danish Institute for Advanced Study DIAS, University of Southern Denmark, Campusvej 55, DK-5230 Odense M, Denmark*

Abstract. In this work we show the existence of traversable wormhole and time-machine solutions in a modified theory of gravity where matter and curvature are nonminimally coupled. Those solutions present a nontrivial redshift function and exist even in the presence of ordinary matter which satisfies the dominant energy condition.

1 Introduction

General Relativity (GR) is one of the great pillars of modern physics and it is based on the idea that the curvature of space-time is dynamically tied up with the matter distribution. GR has been well tested in most of its foundations [1] and it accounts for all gravitational phenomena locally, and also globally, if one admits dark components. Indeed, GR can account for the cosmological observations only if one assumes that the Universe is predominantly dominated composed by two yet unknown constituents: dark energy ($\Omega_{DE} \simeq 73\%$) and dark matter ($\Omega_{DM} \simeq 23\%$). This implies that only 4% of the content of the Universe is baryonic.

Despite its extraordinary agreement with the observations, it is believed that GR cannot be regarded as the final theory (see e.g. Ref. [2] for a survey).

The first modifications to GR were suggested by Weyl, in 1919 [3], and Eddington, in 1923 [4]. However, the motivation for modifying GR became more acute when it was realized that it is not compatible with Quantum Mechanics.

At this point there are two possible alternatives. One can consider that there are really new and unknown dark constituents dominating the Universe or one assumes that the observed features with only ordinary matter are deviations from GR [5].

A prototype model of the first alternative is the standard Λ CDM that assumes the existence of a cosmological constant to account for dark energy and weak-interacting particles, arising from extensions to the Standard Model, to account for dark matter.

The second approach, which will be adopted in this contribution, consists in assuming that the need of invisible components to account for the observations, actually signal the need to extend GR.

^ae-mail: orfeu.bertolami@fc.up.pt

^be-mail: ferreira@cp3.dias.sdu.dk

^cSpeaker

There are various ways to generalize GR. In this work we will consider an extension of the so-called $f(R)$ theories [6].

2 Nonminimally coupled matter-curvature theories of gravity

In $f(R)$ theories one replaces the scalar curvature R in the gravitational action by an arbitrary function of R . There are two main motivations for doing so. First, this is the simplest way of modifying the action and, second, this replacement can account for the main features of higher order theories of gravity. However, our attention will be focused on an extension of $f(R)$ theories which has received a great deal of attention recently. In this extension matter and curvature are nonminimally coupled, that is NMC modified theories of gravity.

This approach does possess some new and interesting features. In NMC theories, besides the change in the gravitational sector like in $f(R)$ theories, one also couples the Lagrangian density of matter nonminimally with the curvature by an arbitrary function of the Ricci scalar [7]:

$$S = \int \left[\frac{1}{2k} f_1(R) + (1 + \lambda f_2(R)) \mathcal{L}_M \right] \sqrt{-g} d^4x. \quad (1)$$

NMC theories have striking properties such as the nonconservation of the energy momentum tensor, which leads to deviation from geodesic motion of test particles [7], putative deviations from hydrodynamic equilibrium of stars [8], and the breaking of the degeneracy of the Lagrangian densities, which in GR give rise to the energy-momentum tensor of perfect fluids [9].

Moreover, these models allow for the mimicking of dark matter profiles in galaxies [10] and clusters of galaxies [11], as well as of dark energy at cosmological scales [12]. It was also shown that one can obtain somewhat more natural conditions for preheating in inflationary models [13] and that the coupling between curvature and matter is equivalent, under conditions, to an effective pressure which leads, in the weak field limit, to a generalized Newtonian potential [14]. It has also been shown that NMC between curvature and matter can mimic, for a suitable matter distribution, a cosmological constant [15].

In this work we review the results of Ref. [16], where it was investigated the possibility of generating closed timelike curves (CTCs) in this specific modified theory of gravity.

3 Wormholes and Time-Machines

A CTC is a closed worldline in space-time. This apparently simple feature has disturbing consequences as such a curve can globally violate causality [17]. A related issue concerns the fact that microscopic systems are described by laws which are invariant under time reversal. This means that one can start with some initial conditions and evolve forward in time or, equivalently, start from future conditions and evolve the system towards the past. Nevertheless, this feature is not shared by the macroscopic world as the second law of thermodynamics provides a well defined time direction.

However, the possibility of curving time in GR implies that causality might be violated. In fact, several CTCs solutions are known in GR, the first one discovered by van Stockum in 1937 [18]. One can ask whether CTCs can be avoided by any physical mechanism or if CTCs are possible without paradoxes. Currently, there is no complete and satisfactory answer to this issue and this has led to conjectures such as the well known Chronology Protection Conjecture of Stephen Hawking [19] (see the contribution of Orfeu Bertolami in this volume for a more detailed discussion).

Nevertheless, if one allows for time-travel or exchange of information between the past and future, one has to ensure some properties besides the creation of CTCs. In Ref. [20] a detailed analysis of the

conditions to allow for an interstellar journey of beings through a traversable wormhole is presented and in a subsequent work (Ref. [21]) it was shown how to convert traversable wormholes into time-machines. It is known that such conversion is not unique [22].

In GR, in order to construct time-machine solutions, exotic types of matter, which violate energy conditions such as the Null Energy Condition (NEC), $T_{\mu\nu}k^\mu k^\nu < 0$, are required. Furthermore, a quantum analysis of matter configurations under these conditions show that CTCs become unstable due to several effects like the Casimir effect, gravitational back-reaction and others [22]. Hence, time-travel turns out to be most likely impossible.

In the context of NMC theories, wormhole geometries have been studied for the particular case of trivial red-shift functions [23, 24], the function that defines the g_{00} component of the metric. In the work of Ref. [16] these results were extended and connected with the possibility of time-travel through traversable wormholes. The main issue in NMC theories is that the NEC violation close to the wormhole throat concerns the effective energy-momentum tensor (c.f. Ref. [25]), that is $T_{\mu\nu}^{eff} k^\mu k^\nu < 0$, where k^μ is a null vector which, for simplicity, is chosen to be radial $k^\mu = k^0 \left(1, \sqrt{\frac{-g_{00}}{g_{rr}}}, 0, 0\right)$ and

$$T_{\mu\nu}^{eff} = \frac{1}{F_1} \left[\left(\Delta_{\mu\nu} - \frac{R}{2} g_{\mu\nu} \right) F_1(R) + \frac{1}{2} g_{\mu\nu} f_1(R) + 2\lambda (\Delta_{\mu\nu} - R_{\mu\nu}) \mathcal{L}_M F_2(R) + (1 + \lambda f_2(R)) T_{\mu\nu}^{(m)} \right], \quad (2)$$

with $\Delta_{\mu\nu} = \nabla_\mu \nabla_\nu - g_{\mu\nu} \square$.

Moreover, using results from the literature, we demonstrate that time-machines can be created, under conditions, from wormholes in static spherically symmetric space-times described by the metric,

$$ds^2 = -e^{2\Phi(r)} dt^2 + \frac{dr^2}{1 - \frac{b(r)}{r}} + r^2 (d\theta^2 + \sin^2 \theta d\phi^2), \quad (3)$$

where $\Phi(r)$ and $b(r)$ are arbitrary functions, usually referred to as redshift function and shape function, respectively.

4 Results

Thus, in the context of NMC modified theories of gravity, we sought for those solutions in the simplest case of a nonminimal coupling where $f(R) = R$ [16]. The field equations were then solved for a perfect fluid threading the wormhole with isotropic pressure and two different energy densities. One constant and localized given by $\rho(r) = \rho_0 \Theta(r - r_2)$, where r_2 is some arbitrary scale, and another exponentially decaying given by $\rho_2(r) = \frac{\rho_0 r_0}{r} e^{-\frac{r-r_0}{\sqrt{2\lambda}}}$, where r_0 is the wormhole throat. For these energy densities, the shape function was obtained. Then, the system of equations was solved for the redshift function and the pressure in two limits, $r \rightarrow r_0$ and $r \rightarrow \infty$, in order to ensure the violation of the NEC and a suitable asymptotic behavior.

In the first case, wormhole solutions were obtained with a constraint on the coupling parameter: $\lambda > 1/(2\rho_0)$. Moreover, these solutions can be obtained even for ordinary matter if $\rho_0 > \frac{1}{2\lambda} \left(1 + \frac{r_0}{\sqrt{2\lambda + r_0^2}}\right)$. However, the localized and constant energy density $\rho_1(r)$ has an intrinsic problem as it implies an unavoidable discontinuity at an arbitrary scale. Hence, although wormhole solutions are found, these are not traversable since the discontinuity leads to unphysical regimes.

Concerning the second case, for the energy density $\rho_2(r)$, wormhole solutions were obtained for a physical region of parameters (ρ_0, r_0, λ) . Furthermore, it was also found a region in the parameter space where these solutions are well defined and satisfy the Dominant Energy Condition for ordinary matter ($\rho > 0$). In this case, discontinuities, horizons and ill defined functions are avoided. Moreover, since the solution is obtained for ordinary matter, it is stable.

5 Conclusions

Thus, we can conclude that well behaved CTCs and wormholes solutions are possible in NMC models of gravity. Furthermore, these solutions can be transformed in traversable wormholes which allow for time-travel of ordinary matter if some quantitative conditions, both for the wormhole and for the acceleration which yields the time shift, are satisfied.

It is relevant to stress that, as expected, the two wormhole solutions obtained do not include the case $\lambda = 0$, which corresponds to GR. Thus, the crucial point in generating the wormhole solutions lies in the presence of this nonminimal coupling between the curvature and the matter.

References

- [1] O. Bertolami and J. Páramos, arXiv:1212.2177 (2012).
- [2] O. Bertolami, arXiv:1112.2048 (2011).
- [3] H. Weyl, *Annalen Phys.* **59**, 101-133 (1919).
- [4] A. S. Eddington, *The Mathematical Theory of Relativity*, (University Press, Cambridge, 1923).
- [5] O. Bertolami, *Int. J. Mod. Phys. D* **16**, 2003 (2007).
- [6] T. P. Sotiriou and V. Faraoni, *Rev. Mod. Phys.* **82**, 451 (2010).
- [7] O. Bertolami, C. G. Boehmer, T. Harko, and F. S. Lobo, *Phys. Rev. D* **75**, 104016 (2007).
- [8] O. Bertolami and J. Páramos, *Phys. Rev. D* **77**, 084018 (2008).
- [9] O. Bertolami, F. S. Lobo, and J. Páramos, *Phys. Rev. D* **78**, 064036 (2008).
- [10] O. Bertolami and J. Páramos, *JCAP* **3**, 009 (2010).
- [11] O. Bertolami, P. Frazão, and J. Páramos, arXiv:1111.3167 (2011).
- [12] O. Bertolami, P. Frazão, and J. Páramos, *Phys. Rev. D* **81**, 104046 (2010).
- [13] O. Bertolami, P. Frazão, and J. Páramos, *Phys. Rev. D* **83**, 044010 (2011).
- [14] O. Bertolami and A. Martins, *Phys. Rev. D* **85**, 024012 (2012).
- [15] O. Bertolami and J. Páramos, *Phys. Rev. D* **84**, 064022 (2011).
- [16] O. Bertolami and R. Z. Ferreira, *Phys. Rev. D* **85**, 104050 (2012).
- [17] O. Bertolami and F. S. Lobo, *NeuroQuantol.* **7**, 1 (2009).
- [18] W. J. van Stockum, *Proc. Roy. Soc. Edinburgh* **57**, 135-154 (1937).
- [19] S. Hawking, *Phys. Rev. D* **46**, 603-611 (1992).
- [20] M. Morris and K. Thorne, *Am. J. Phys.* **56**, 395 (1988).
- [21] M. Morris, K. Thorne, and U. Yurtsever, *Phys. Rev. Lett.* **61**, 1446 (1988).
- [22] M. Visser, *Phys. Rev. D* **28**, 554 (1993).
- [23] N.M. Garcia and F. S. Lobo, *Phys. Rev. D* **82**, 104018 (2010).
- [24] N.M. Garcia and F. S. Lobo, *Class. Quant. Grav.* **28**, 085018 (2011).
- [25] O. Bertolami and M. C. Sequeira, *Phys. Rev. D* **79**, 104010 (2009).