

## Study on magnetic field generation and electron collimation in overdense plasmas

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**Abstract.** An analytical fluid model is proposed for artificially collimating fast electron beams produced in interaction of ultraintense laser pulses with specially engineered sandwich structure targets. The theory reveals that in low-density-core structure targets, the magnetic field is generated by the rapid change of the flow velocity of the background electrons in transverse direction (perpendicular to the flow velocity) caused by the density jump. It is found that the spontaneously generated magnetic field reaches as high as 100 MG, which is large enough to collimate fast electron transport in overdense plasmas. This theory is also supported by numerical simulations performed using a two-dimensional particle-in-cell code. It is found that the simulation results agree well with the theoretical analysis.

### 1. INTRODUCTION

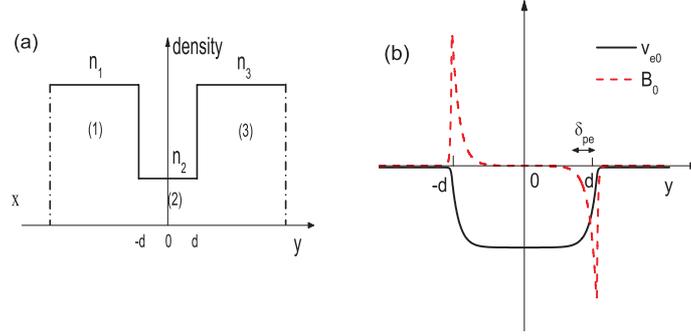
In the fast ignition scheme (FI) [1], a high-efficiency coupling of an ultraintense beam to the compressed core is required to save igniter energy. Crucial issues for the scheme are the efficient generation of well-collimated fast electron beams and their transport from the critical density layer to the compressed fuel. To date, several works have been done regarding the control of the divergence of fast electron beams [2–5]. Robinson et al.'s hybrid-Vlasov-Fokker-Planck simulations [2] have shown some promising results, the fast electrons are collimated in targets exhibiting a high-resistivity-core-low-resistivity-cladding structure analogous to optical waveguides. Recent experiments have also demonstrated this electron collimation in targets with resistivity boundary [3, 4]. In these cases, the magnetic field growth can be derived from the Faraday's law and the Ohm's law as [2]

$$\frac{\partial \mathbf{B}}{\partial t} = c[\eta \nabla \times \mathbf{j}_h + (\nabla \eta) \times \mathbf{j}_h], \quad (1)$$

where  $\eta$  is the resistivity and  $\mathbf{j}_h$  is the fast electron current density. Furthermore, numerical simulations by Zhou et al. [5] showed that low-density-core-high-density-cladding structure target can also generate a mega-Gauss interface magnetic field, which collimates fast electrons even better. However, according to the above equation, the magnetic field generated from the gradient of resistivity and current density should cause divergence of an electron beam, instead of collimating a beam. Therefore, their results clearly indicate that in a low-density-core-high-density-cladding structure target, other mechanisms tend to be dominant for the generation of magnetic field. To date, there still does not exist a

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**Figure 1.** (a) A view of initial plasma density; (b) schematic of the profile of the background electron flow velocity  $v_{e0}(y)$  and the spontaneous magnetic field  $B_0(y)$ .

well-established and satisfactory theory, in particular, to quantitatively predict the magnetic field and to model its dependence on the plasma parameters.

In this paper, an analytical model describing this scenario is presented. The model, describing a uniform fast electron beam propagating in a low-density-core-high-density-cladding structure target, shows clearly the formation of the background return current flow and the generation of the spontaneous magnetic field. The mechanisms governing the generation of magnetic field are studied in detail. Two-dimensional particle-in-cell (PIC) simulations have been run to verify this model. It is shown that the simulation results are consistent with the predictions of our analytical model.

## 2. ANALYTICAL DESCRIPTION OF THE SPONTANEOUSLY GENERATED MAGNETIC FIELD

We consider an ultraintense laser pulse with a large spot size normally irradiating an unmagnetized low-density-core-high-density-cladding structure plasma with density profile as shown in Fig. 1(a), where ions form a fixed and neutralizing background. A fast electron beam is generated at the laser plasma interface. If the spot size is large enough, we can simply assume that an equally infinite and uniform fast electron beam of density  $n_h$  propagates with an average velocity  $v_h$  along the x-axis (longitudinal direction). For purpose of the interface magnetic field excitation study, we assume that the fast electron beam motion is unperturbed. Since the magnetic field can develop in a very short time (of the order of hundred femtoseconds), the combined system of the fast electron beam and the response of the background plasma can be suitably treated by single fluid electron magnetohydrodynamic (EMHD) description. From the electron fluid equations (the continuity equation and force balance equation), we can get the relationship between the self-generated magnetic field and electron flow velocity [6, 7],

$$\mathbf{B} = \frac{c}{e} \nabla \times \mathbf{p}_e, \quad (2)$$

where  $\mathbf{p}_e = m_e \gamma_e \mathbf{v}_{e0}$  is the momentum of the background electron flow and  $\gamma_e = 1/\sqrt{1 - v_{e0}^2/c^2}$  is the relativistic factor. Maxwell's equations for the self-generated electric and magnetic fields,  $\mathbf{E}$ , and  $\mathbf{B}$ , are given by

$$\nabla \times \mathbf{B} = -\frac{4\pi e}{c} (n_{e0} \mathbf{p}_e / m_e \gamma_e + n_h \mathbf{v}_h) + \frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}. \quad (3)$$

Here, for a long beam with pulse length  $l_b \gg v_h/\omega_p$ , where  $v_h$  and  $\omega_p$  are the fast electron beam velocity and background plasma frequency, the displacement current  $\frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}$  is of order  $(v_h/\omega_p l_b)^2 \ll 1$  compared

to the electron current. In the following discussion, the displacement current  $\frac{1}{c} \frac{\partial \mathbf{E}}{\partial t}$  is neglected [6]. Eq. (3) gives

$$\mathbf{p}_e = -\frac{mc\gamma_e}{4\pi en_{e0}} \nabla \times \mathbf{B} - m\gamma_e n_h \mathbf{v}_h / n_{e0}. \quad (4)$$

Substituting Eq. (4) into Eq. (2), we obtain the equations for self-generated magnetic field,

$$\frac{mc^2}{4\pi e^2} \nabla \times \left( \frac{\gamma_e \nabla \times \mathbf{B}}{n_{e0}} \right) + \mathbf{B} = -\frac{mc}{e} \nabla \times \left( \frac{\gamma_e n_h \mathbf{v}_h}{n_{e0}} \right). \quad (5)$$

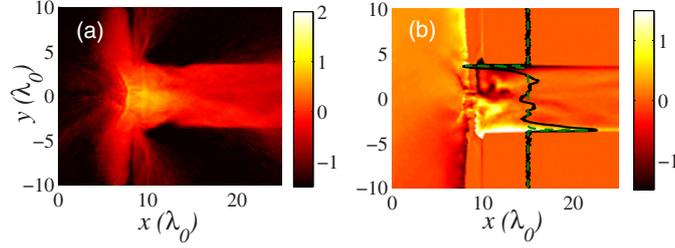
When fast electron beam velocity and background electron density is finite, then the magnetic field  $\mathbf{B} = \nabla \times \mathbf{a}$  is finite. The source is  $\nabla \times \left( \frac{\gamma_e n_h \mathbf{v}_h}{n_{e0}} \right) \approx -\frac{\gamma_e}{n_{e0}^2} \nabla n_{e0} \times \mathbf{j}_h$ , which means the generation of self-generated magnetic field is due to the nonparallel density gradient and fast electron current. For the nonrelativistic case,  $\gamma_e \sim 1$ . It is not easy to solve Eq. (5) directly even though Eq. (5) shows the physics clearly. However, we can still derive the magnetic field from Eqs. (2) and (4). Operating on equation (2) with  $\nabla \times$ , and substituting it into Eq. (4), we obtain the equation for the flow velocity of the background electrons [6, 7]

$$\frac{m_e c^2}{4\pi e^2} \frac{d^2 v_{e0}(y)}{dy^2} = n_{e0} v_{e0}(y) + n_h v_h. \quad (6)$$

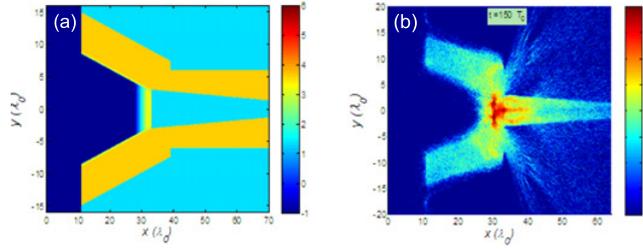
In the present case, the sandwich background plasma has three different regions with ion densities  $n_{i1}$  for region (1)  $y < -d$ ,  $n_{i2}$  for region (2)  $-d \leq y \leq d$ , and  $n_{i3}$  for region (3)  $y > d$ . Since the fast electron beam may be affected by this background plasma at later time, we assume the densities of the fast electron beams are (1)  $n_{h1}$ , (2)  $n_{h2}$ , and (3)  $n_{h1}$ , respectively. From Eq. (6), we know that the fast electron beam is neutralized by the plasma return current everywhere except at the interfaces over a characteristic transverse distance. Together with the continuous boundary conditions at the interfaces, the spontaneous magnetic field  $B_0$  can be solved analytically from Eq. (6),

$$\frac{eB_0}{m_e c} = \frac{dv_{e0}}{dy} = \begin{cases} \frac{1}{(\delta_{pe1} + \delta_{pe2})} \left( \frac{n_{h2} v_{h2}}{n_2} - \frac{n_{h1} v_{h1}}{n_1} \right) \exp \left[ \frac{y+d}{\delta_{pe1}} \right], & y < -d \\ \frac{1}{(\delta_{pe1} + \delta_{pe2})} \left( \frac{n_{h2} v_{h2}}{n_2} - \frac{n_{h1} v_{h1}}{n_1} \right) \left\{ \exp \left[ -\frac{y+d}{\delta_{pe2}} \right] - \exp \left[ \frac{y-d}{\delta_{pe2}} \right] \right\}, & -d \leq y \leq d, \\ \frac{-1}{(\delta_{pe1} + \delta_{pe2})} \left( \frac{n_{h2} v_{h2}}{n_2} - \frac{n_{h1} v_{h1}}{n_1} \right) \exp \left[ -\frac{y-d}{\delta_{pe1}} \right], & y > d \end{cases} \quad (7)$$

where  $\delta_{pej} = c/\omega_{pej}$  ( $j = 1, 2, 3$ ) is the electron skin depth and  $\omega_{pej} = \sqrt{4\pi n_j e^2 / m_e}$  ( $j = 1, 2, 3$ ) is the electron plasma frequency for the background plasma in the  $j$ th region.  $n_j$  and  $v_{hj}$  denote the electron density of the background plasmas and the velocity of the fast electrons, respectively. Here,  $n_3 = n_1$ ,  $n_{h3} = n_{h1}$ , and  $v_{h3} = v_{h1}$  have been used because of the symmetry. From Eq. (7), one can see that the spontaneous magnetic field  $B_0$  exists within a layer of width  $\delta_{pe}$  near the interfaces of different regions,  $y = \pm d$ . We notice that  $B_0$  peaks at the interfaces and is exponentially evanescent to both sides. In the lower density plasma region, the magnetic field tunnels much deeper into the target since the skin depth  $\delta_{pe} \propto n_e^{-1}$ . It should be emphasized that the velocity variation and magnetic field occur even when  $n_h(y)$  is laterally constant, i.e.,  $n_{h1} = n_{h2}$ . The analytical results of background electron flow velocity and the spontaneous magnetic field are drawn on Fig. 1(b).



**Figure 2.** (a) Electron energy density in logarithm scale and (b) spontaneous magnetic fields at  $t = 500$  fs (1 unit  $\simeq 100$  MG).



**Figure 3.** (a) Initial density profile of cone target with a funnel structure; (b) Electron energy density in logarithm scale at time 500 fs.

### 3. NUMERICAL STUDY OF THE GENERATION OF THE MAGNETIC FIELD

In order to describe the generation of magnetic fields in more details, firstly we study low-density-core-high-density-cladding structure targets with the 2D3V PIC code ASCENT [7]. The cladding regions are fully ionized carbon (charge state  $Z_i = 6$ ) with  $m_i/m_e Z_i = 1836 * 12/6$ . The core region is the low density hydrogen plasma. The widths of three regions are:  $8.4\lambda_0$ ,  $7.2\lambda_0$ ,  $8.4\lambda_0$ , respectively. The densities of three regions are:  $200n_c$ ,  $5n_c$ ,  $200n_c$ , respectively. In order to avoid the ruin of the hydrogen plasma by the laser pressure, we place another high density target ( $40n_c$ ) in front of the sandwich target. The p-polarized laser pulse at  $\lambda_0 = 1.06 \mu\text{m}$  wavelength and  $1.5 \times 10^{19} \text{ W/cm}^2$  intensity irradiates the target from the left boundary. The intensity profile is Gaussian in the y direction with a spot size of  $5.0 \mu\text{m}$  (FWHM). The laser rises in  $15T_0$ , after which the laser amplitude is kept constant.

In Fig. 2 (a) the energy density distributions of electrons with energy between  $0.5 \leq E_e [\text{MeV}] \leq 5.0$  at  $t = 500$  fs are plotted. It is clearly seen that the fast electrons generated at the laser plasma interface have a large divergence angle. As time goes on, the electrons are highly collimated after the generation of the magnetic field, few electrons can “leak” out into the high-density-cladding. We also plot the spontaneous magnetic field at time  $t = 500$  fs on Fig. 2(b). Clearly, magnetic field of the order of 100 MG has been generated at the interfaces, which plays a role in collimating fast electrons. Note that the green dashed line is the analytical results from Eq. (7). It is found that the analytical result is consistent with the simulation result (the dark solid line) very well.

In cone-guided fast ignition, the laser energy is firstly deposited into fast electrons in the interaction region at the cone tip [8], which is usually 50 to 100 microns away from the dense core, and the resulting energy flux transports into the dense core. In order to collimate the fast electrons and enhance the laser coupling efficiency, we designed a cone target with a funnel structure, as shown in Fig. 3(a). From the above analysis, we expect that huge magnetic field will be generated at the inner surface of the funnel target, which can collimate fast electrons produced at the cone tip. Fig. 3(b) shows the energy density

distributions of electrons with energy between  $0.5 \leq E_e [MeV] \leq 5.0$  at  $t = 500$  fs. We can see that the fast electrons are well collimated inside the funnel structure. Detailed computations and analysis in this last case are currently in progress, and will be the object of a future publication.

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### References

- [1] M. Tabak *et al.*, Phys. Plasmas **1**, 1626 (1994)
- [2] A.P.L. Robinson *et al.*, Phys. Plasmas **14**, 083105 (2007)
- [3] S. Kar *et al.*, Phys. Rev. Lett. **102**, 055001 (2009)
- [4] B. Ramakrishna *et al.*, Phys. Rev. Lett. **105**, 135001 (2010)
- [5] C. T. Zhou *et al.* Phys. Plasmas **17**, 083103 (2010)
- [6] E. A. Startsev, R. C. Davidson, and M. Dorf, Phys. Plasmas **16**, 092101 (2009)
- [7] H.B. Cai *et al.*, Phys. Rev. E **83**, 036408 (2011)
- [8] H. B. Cai *et al.*, Phys. Rev. Lett. **102**, 245001 (2009)