

Top-quark mass measurements using jet rates at LHC

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Abstract. This work presents a new method to measure the top-quark mass in hadronic collisions[1]. The method uses the sensitivity of the $t\bar{t} + 1$ -jet production on the top-quark mass. In detail we study the \mathcal{R} distribution defined as the $t\bar{t} + 1$ -jet normalized cross section differential in the invariant mass of the total system and calculated at NLO accuracy. We prove that the \mathcal{R} distribution has a high sensitivity to the top-quark mass. Furthermore we investigate and quantify the impact of the dominant theoretical and experimental uncertainties. The results obtained show, that the method has the potential to be competitive in precision with established approaches and allows a complementary measurement of the top-quark mass at hadron colliders. We emphasize that in the proposed method the mass parameter is uniquely defined through one-loop renormalization.

1 Introduction

Due to its large mass 173.2 ± 0.9 GeV [2–4] the top quark presents the strongest coupling to the Higgs boson within the framework of the Standard Model (SM). The top quark thus provides an ideal probe for precision tests of the Higgs mechanism and of many alternative explanations of electroweak spontaneous symmetry breaking (EWSB) in Beyond Standard Model (BSM) theories. Therefore, the comparison of the top-quark mass with the mass of the recently observed resonance [5, 6]—assuming that the resonance is the long-sought Higgs boson—can be used to test the validity of the Standard Model [7] and the structure of the electroweak vacuum [8, 9].

Quarks carry color charges responsible for the strong interactions. Due to confinement, we don't see free quarks and there is no pole in the S-matrix. Therefore quark masses are not observables by themselves, they are just parameters on the underlying theory (i.e SM). Precise values of these parameters depend on the renormalization scheme used to define them. The most commonly used definitions are the pole mass, m_q^{pole} , and the running mass, $\bar{m}_q(\mu_r)$.

New observables for a precise top quark mass measurement should fulfil some basic requirements: not only a good sensitivity to the mass and a good experimental accessibility are needed. The observable should also be theoretically calculable keeping non-perturbative corrections small. In addition the calculation should use a well defined mass scheme fixed at least through NLO calculations to distinguish between different mass definitions (i.e. m_q^{pole} and $\bar{m}_q(\mu_r)$).

The top-quark mass is presently inferred by kinematical reconstruction of the invariant mass of its decay prod-

ucts (see e.g., Ref. [2]) or by its relation to theoretical predictions like the inclusive top-quark pair production cross section [11]. The mass determination from kinematical reconstruction measurements reaches precisions of ~ 1 GeV. Mass determinations using the inclusive cross section Ref. [11] presents a weak sensitivity to the top-quark mass: $\frac{\Delta\sigma}{\sigma} \sim -5 \frac{\Delta m_t}{m_t}$. However we emphasize that in this measurement the renormalization scheme is unambiguously defined in difference with the determination based on the kinematical reconstruction.

In this work we advocate a new method to measure the top-quark mass in high energetic hadron collisions at the LHC, looking for a high precision measurement with a unambiguously renormalization scheme defined. For that, the mass dependence of the production of top-quark pairs in association with an additional jet is exploited.

2 Top-quark mass measurements with $t\bar{t} + 1$ -jet events

2.1 Top-quark pair production in association with a hard jet at NLO accuracy in QCD

The NLO QCD corrections for $t\bar{t} + 1$ -jet + X have been presented in Refs. [13, 14] for Tevatron and LHC (14 TeV) conditions where inclusive and differential cross sections were studied. Also tools to match $t\bar{t} + 1$ -jet at NLO with parton shower algorithms have been developed and tested in Refs. [15, 25]. The various studies mentioned above show, that the theoretical description of the process $t\bar{t} + 1$ -jet based on predictions at NLO accuracy in QCD is well under control.

m_t^{pole} [GeV]	$\sigma_{t\bar{t}+1\text{-jet}}$ [pb]	
	LO	NLO
	$p_T(\text{jet}) > 50 \text{ GeV}, \eta(\text{jet}) < 2.5$	
160	66.727(5)	60.04(8)
165	57.615(4)	52.25(9)
170	49.910(3) ⁺³⁰ ₋₁₇	45.45(6) ⁺¹ ₋₆
172.5	46.508(3) ⁺²⁸ ₋₁₅	42.37(6) ⁺¹ ₋₆
175	45.372(3)	39.46(6)
180	37.800(2)	34.73(5)

Table 1. The $t\bar{t} + 1\text{-jet} + X$ cross section using LO and NLO calculations [13, 14] for proton-proton collisions at 7 TeV and for different m_t^{pole} values. Jets are defined using the anti-kt algorithm [17] with $R=0.4$ as implemented in the FASTJET package [18]. The additional jet is required to have $p_T > 50 \text{ GeV}$ and $|\eta| < 2.5$. The uncertainty due to the limited statistics of the numerical calculation is indicated in parenthesis affecting the last digit. The scale uncertainty is also shown for some top-quark mass values. The CTEQ6.6 [19] (CT09MC1 [20]) PDF set has been used to obtain the NLO (LO) results.

In addition to what has been done in Refs. [13, 14] we have calculated the $t\bar{t} + 1\text{-jet}$ cross section using different masses for proton proton collisions at 7 TeV. In this study and in the following sections, the pole mass scheme has been chosen. To estimate the theoretical uncertainties due to the truncation of the perturbative series the renormalization (μ_r) and factorization (μ_f) scales have been varied between $2 \cdot m_t^{\text{pole}}$ and $m_t^{\text{pole}}/2$.

To study the uncertainties due to the limited knowledge of the proton substructure different PDF sets have been used and compared. More specifically, we used the PDF sets CTEQ6.6 and MSTW2008nlo90cl [21]. For a top-quark mass of $m_t^{\text{pole}} = 170 \text{ GeV}$ we find, for example,

$$\sigma_{t\bar{t}+1\text{-jet}}^{\text{NLO, MSTW08}} = 49.21 \text{ pb}. \quad (1)$$

2.2 The observable

We propose to study normalized differential distributions, since inclusive cross sections do not lead to any significant improvement: the sensitivity of $\sigma_{t\bar{t}+1\text{-jet}}$ is very similar to the inclusive $t\bar{t}$ cross section and, in general, measuring inclusive cross sections is experimentally challenging. We need to focus on specific kinematical configurations where an enhanced sensitivity can be obtained. A natural observable to look at is the differential distribution of the $t\bar{t} + 1\text{-jet}$ cross section with respect to the invariant mass $s_{t\bar{t}j}$ of the final state. More precisely, we study the dimensionless distribution:

$$\mathcal{R}(m_t^{\text{pole}}, \rho_s) = \frac{1}{\sigma_{t\bar{t}+1\text{-jet}}} \frac{d\sigma_{t\bar{t}+1\text{-jet}}}{d\rho_s}(m_t^{\text{pole}}, \rho_s), \quad (2)$$

where ρ_s is defined as $\rho_s = \frac{2m_0}{\sqrt{s_{t\bar{t}j}}}$ with a fixed value for $m_0 = 170 \text{ GeV}$. Due to the normalization, different uncertainties may cancel between numerator and denominator.

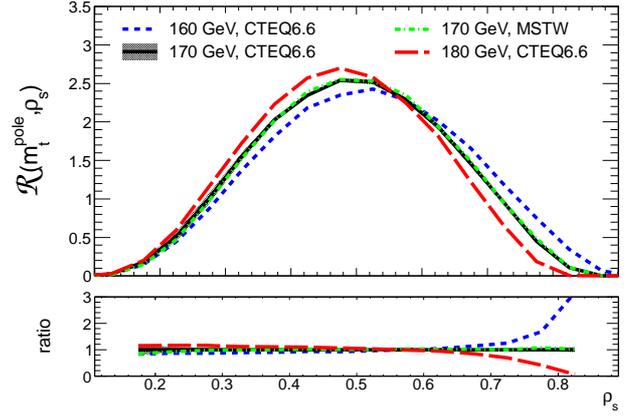


Figure 1. The $\mathcal{R}(m_t^{\text{pole}}, \rho_s)$ distribution calculated at NLO accuracy for different masses $m_t^{\text{pole}} = 160, 170$ and 180 GeV . For $m_t^{\text{pole}} = 170 \text{ GeV}$ the scale and PDF uncertainties evaluated as discussed in the text are shown. The ratio with respect to the result for $m_t^{\text{pole}} = 170 \text{ GeV}$ is shown in the lower plot.

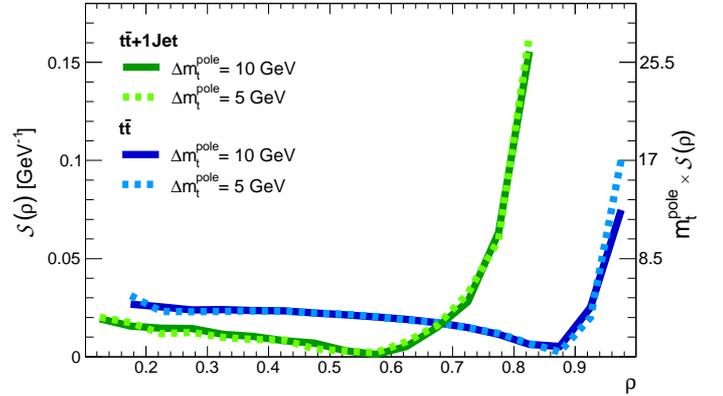


Figure 2. The sensitivity $\mathcal{S}(\rho_s)$ of \mathcal{R} with respect to the top-quark mass as defined in Eq. (3).

This distribution has been calculated for different top-quark pole masses. The results are shown in the Fig. 1. This figure shows that for large ρ_s the sensitivity to m_t^{pole} is high. For ρ_s close to one it is expected that the production of heavier quark masses is suppressed compared with lighter masses. For the high energy region (ρ_s close to zero) one would naively expect a loss of sensitivity as we observe in the figure. We can also observe that at $\rho_s \sim 0.55$ the three curves cross due to the fact that they are normalized to one.

To quantify the sensitivity we studied

$$\mathcal{S}(\rho_s) = \sum_{\Delta = \pm 5 - 10 \text{ GeV}} \frac{|\mathcal{R}(170 \text{ GeV}, \rho_s) - \mathcal{R}(170 \text{ GeV} + \Delta, \rho_s)|}{2|\Delta| \mathcal{R}(170 \text{ GeV}, \rho_s)} \quad (3)$$

The result for \mathcal{S} is shown in Fig. 2. It can be seen in this figure that values up to 25 are reached for $m_t^{\text{pole}} \times \mathcal{S}$ at $\rho_s \sim 0.8$ which means that a one per cent change in the mass translates into a 25 per cent of change in \mathcal{R} . Fig. 2

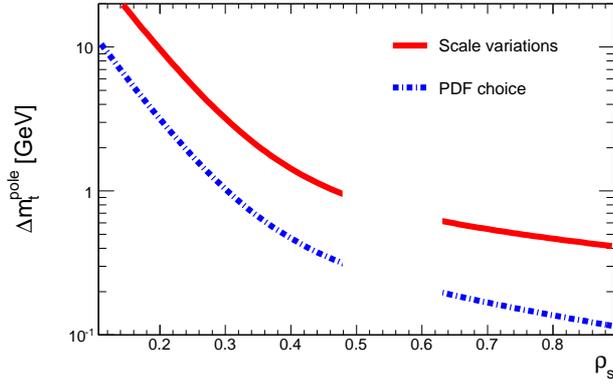


Figure 3. This plot shows the expected impact of scale (magenta line) and PDF (blue dashed line) uncertainties on the measured top-quark mass value. The region where \mathcal{R} is essentially insensitive to the top-quark mass is not shown.

also shows the results for the case that \mathcal{R} is defined for $t\bar{t}$ inclusive final state (with $\rho = 2m_0/\sqrt{s_{t\bar{t}}}$). Only in the very extreme region of large ρ – where reliable theoretical calculation are challenging and also experimental uncertainties may become larger – similar values of \mathcal{S} are obtained.

To estimate the impact of different uncertainties we show, in Fig. 3, the quantities:

$$\frac{\Delta\mathcal{R}_\mu/\mathcal{R}(170\text{ GeV}, \rho_s)}{\mathcal{S}(\rho_s)} \quad \text{and} \quad \frac{\Delta\mathcal{R}_{\text{PDF}}/\mathcal{R}(170\text{ GeV}, \rho_s)}{\mathcal{S}(\rho_s)} \quad (4)$$

where $\Delta\mathcal{R}_\mu$ and $\Delta\mathcal{R}_{\text{PDF}}$ are the scale and PDF uncertainties of $\mathcal{R}(170\text{ GeV}, \rho_s)$. We do not show the region around $\rho_s \approx 0.6$ because of the vanishing sensitivity. Fig. 3 shows that the main source of uncertainty is due to the scale variation while the impact of the PDF uncertainties is much smaller.

We are thus lead to the conclusion that from the theoretical perspective \mathcal{R} provides an interesting alternative to existing methods for top-quark mass measurements.

3 Experimental viability study

We study now the stable final state particles originated from typical $t\bar{t} + 1\text{-jet} + X$ events as produced in proton-proton interactions at 7 TeV center-of-mass energy. The results presented here illustrates qualitatively the viability and the potential of the method. The top-quark decay and hadronization are taken into account leading to complicated event topologies similar to those reconstructed in real experiments. In the Monte Carlos studies we use only publicly available tools and do not make any reference to a particular LHC experiment.

This study only considers the so called semi-leptonic decay channel which assumes that one of the two W boson decays leptonically (we just consider electron or muon channels) whereas the remaining W boson decays hadronically. This semi-leptonic channel has a very good balance between efficient identification and event rate, since

roughly 8/27 of all top-quark pair events decay semi-leptonically. These final state configurations have a particular signature: the presence of one high- p_T lepton, high missing transverse energy because of the presence of the neutrino, two jets originating from the b quarks and at least two additional jets.

Selected events are required to fulfil the presence of all these objects enumerated above and, in addition, topological constraints are applied to the event: the invariant masses of the two non b -jets are required to be compatible with the mass of the W boson within 20% and the two reconstructed top-jet systems are required to have similar masses within a range of precision also around 20%. The missing energy has to be compatible with a neutrino which, together with the identified lepton can both be attributed to originate from the decay of the W boson. Finally, an extra jet, not associated to the t or the \bar{t} is identified. It should be noticed that the quantity ρ_s considered depends on the invariant mass of all jets and fermions and is, therefore, independent of misidentified jet associations.

After the reconstruction of the \mathcal{R} observable several experimental uncertainties have been studied: different modelling of color reconnection; different events generator and fragmentation models (POWHEG [15, 24] with Pythia’s parton shower[32] versus MC@NLO [28] with Herwig’s parton shower [29, 30]) ; the impact of a wrongly reconstructed jet energy; the unfolding procedure to correct the \mathcal{R} distribution to the perturbative partonic level; the backgrounds, mainly W +jets; and the statistical error depending on the collected luminosity.

In the approach advocated here we do not expect that color reconnection plays an important role since the observable itself does not rely on a precise momentum reconstruction. The momenta enter only through the jet algorithm which determines whether an event passes the jet selection cuts. We expect only a very weak effect of color reconnection on the jet algorithm. Apart from this, color reconnection could in principle also affect the determination of $s_{t\bar{t}j}$, however since $s_{t\bar{t}j}$ is an inclusive quantity we do not expect a major effect here. Furthermore a ‘wrong’ $s_{t\bar{t}j}$ would only affect events at the bin boundaries and it is not unlikely that migrations at the left boarder and the right boarder will compensate to a large extend. To assess the impact of color reconnection on \mathcal{R} we used both Pythia6 and Pythia8 and compared the situation of using color reconnection in the respective default setup to the situation where color reconnection is completely switched off. Evidently, the aforementioned procedure gives a rather extreme estimate of the uncertainty. We conclude that even in the worst case the uncertainty is below 400 MeV using reasonable values of ρ_s . The results are summarized in Fig. 4.

The impact of the event generator and the fragmentation model has been studied by the comparison between POWHEG with Pythia and MC@NLO with Herwig. We observe that the observable is very stable and the value of m_t^{pole} which is extracted using one or the other model is found to remain stable within 0.20 ± 0.20 GeV for the high range of ρ_s .

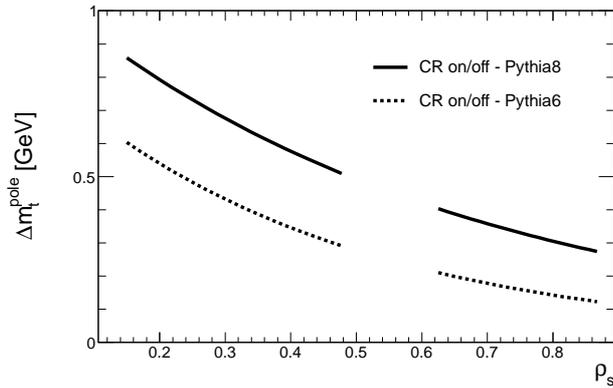


Figure 4. Impact of color reconnection on the top-quark mass determination using \mathcal{R} .

The uncertainty due to the reconstructed jet energy is usually attributed to the uncertainty on the jet energy scale (JES). To estimate these effects the energy of the jets has been changed by 3% up and down before the construction of \mathcal{R} . Then, the observable has been constructed and the top-quark mass has been extracted. In this way, an uncertainty of $\sim 1\text{GeV}$ on the extracted top-quark mass has been estimated. A better compromise between efficiency and resolution is possible though it would imply tools which are detector specific.

The observable proposed in this article allows to unfold the data and reproduce the original \mathcal{R} distribution at parton level through an unfolding method independent of the MC top-quark mass. This method is based on the shape of the distribution and has been evaluated to have an uncertainty of $\sim 0.3\text{GeV}$ on the top-quark mass extraction from \mathcal{R} . This uncertainty comes from the finite statistics of the MC generators.

It is estimated that, after apply the topological constraints described above, a $\sim 5 - 10\%$ of the reconstructed events will come from background signal (from single-top and W +jet topologies). Finally, assuming a 1% of efficiency of selection for these topologies and an integrated luminosity of $5, fb^{-1}$ for 7 TeV p-p collisions, a statistical uncertainty of 1.4 GeV has been estimated.

It is important to mention that a real analysis using data and detector specific tools is needed to understand the exact value of the uncertainties affecting the determination of m_t^{pole} but we can estimate that a total error of 1 GeV is achievable.

Table 2. Summary of the experimental uncertainties studied

Source of uncertainty	Result
Color Reconnection	$\Delta m_t^{\text{pole}} < 0.25 \text{ GeV}$
Event Gen.	$\Delta m_t^{\text{pole}} \sim 0.2 \pm 0.2 \text{ GeV}$
Jet reconstruction	$\Delta m_t^{\text{pole}} \sim 0.8 \text{ GeV}$
Unfolding	$\Delta m_t^{\text{pole}} \sim 0.3 \text{ GeV}$
Backgrounds	$\sim 5\%$ of the events
Statistics	$\Delta m_t^{\text{pole}} \sim 1.4 \text{ GeV}$ for $5 fb^{-1}$

4 Conclusions

The sensitivity to the top-quark mass of a differential distribution with respect $\sim 1/s_{\bar{t}t}$ is exploited. In this analysis the renormalization scheme of the top-quark mass is uniquely defined. The method presents a high mass sensitivity and low uncertainties coming from uncalculated higher order corrections and from the parton distribution functions. The theoretical uncertainties are estimated to be below 1 GeV. Finally, the preliminary study of the experimental viability shows that precisions of $\sim 1 \text{ GeV}$ are achievable. This study has not been addressed to any particular experiment, so a further analysis using real data and a detector specific framework is needed.

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