

Prolate non-collective-A rare shape phase in high spin state proton emitters $^{141-144}\text{Ho}$ and $^{131-135}\text{Eu}$

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Abstract. Structural transitions in high spin states of deformed odd - Z proton emitters $^{141-144}\text{Ho}$ and $^{131-135}\text{Eu}$ are studied in a theoretical framework using the deformed Nilsson potential and Strutinsky's prescription combined with statistical theory of hot rotating nuclei. These proton emitters are found to exhibit shape transition to a rare prolate non-collective shape phase in excited high spin state which is caused directly by rotation at certain angular momentum values which creates a residual quantum shell effect. This phase exists only in a narrow domain bound by the two spin dependent very low critical temperatures. This unexpected prolate noncollective phase generated by rotation undergoes the expected transition to the oblate noncollective phase at higher angular momentum values. Phenomenon of shape coexistence with prolate and oblate non-collective shapes is also speculated in high spin state ^{131}Eu .

1 Introduction

Shape of an atomic nucleus is governed by the delicate interplay of macroscopic bulk properties of the nuclear matter and the microscopic shell effects which can be profoundly altered by rotation and the nuclear temperature. For several decades it has been well recognized that the temperature and rotation have a profound effect on the intrinsic shape of a nucleus and it undergoes a variety of shape transitions with increasing temperature and angular momentum. The study of such shape-phase transition like phenomena in hot and rotating nuclei has emerged as a very active field of research in contemporary nuclear physics. Experimental probes of the nuclear shape such as Giant Dipole Resonance (GDR) [1, 2] are established as one of the primary tools to understand the dynamics of the hot rotating nuclei. Such nuclei are formed in heavy-ion induced fusion reactions where large amount of dissipated energy E^* is usually accompanied by substantial transfer of angular momentum J from the relative to intrinsic motion, E^* and J being the relevant macroscopic parameters for the description of hot rotating compound nucleus. The theoretical tools presently available for the analysis of nuclear shape transitions are usually based on some mean field approximations like finite temperature Nilsson-Strutinsky Cranking method used by the Dubna group [3], FTHFB used by Goodman [4] and Landau theory by Alhassid et al [5] and statistical theory of hot rotating nuclei by our group initiated by Rajasekaran et al [6, 7].

Here we present our results on the study of structural transitions in high spin states of the deformed odd-Z pro-

ton emitters $^{141-144}\text{Ho}$ and $^{131-135}\text{Eu}$ in a theoretical framework [8] using the deformed Nilsson potential and Strutinsky's prescription [9] combined with statistical theory [6] of hot rotating nuclei. The calculations are performed at very low temperatures to understand the effect of rotation on nearly yrast states which may not be seen at high temperatures because the nuclear deformation is caused by quantum shell effects and increasing the temperature creates thermal excitations which eventually wash out these shell effects driving the equilibrium shape to spherical [10–12] and the nucleus resembles a classical liquid drop.

In view of the growing interest and new experimental advances enabling probe the structure of proton unbound Nilsson orbitals and nuclear deformation beyond proton drip line [13], we focus on exploring shapes and deformations in proton rich rare earth region nuclei Eu and Ho [14] as this region is expected to show rapid shape transitions and large deformations. Prediction of a rare shape phase of prolate non-collective in one of our recent work [8] in proton rich ^{94}Ag at angular momentum range $20-32\hbar$ and 6.7 MeV excitation energy in agreement with the speculation of prolate shape by Mukha et al. [15] has opened up new avenues to explore domains of periodic table with states of such rare shape phases. This prolate noncollective equilibrium phase had not been anticipated before the Ref. [16] and then our work on proton radioactivity from high spin states of proton rich ^{94}Ag [8], which is caused directly by rotation at certain angular momentum values which creates a residual quantum shell effect. This unexpected prolate noncollective phase generated by rotation undergoes the expected transition to the oblate noncollective phase at higher angular momentum values [8]. Such a phase exists only in a narrow domain bound by the two

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spin dependent very low critical temperatures. Observation of shape transition to this rare prolate non-collective shape phase in excited high spin state of proton emitters $^{141-144}\text{Ho}$ and $^{131-135}\text{Eu}$ is the main highlight of the present work.

2 Theoretical Formalism

The hot rotating nucleus is an isolated many-body system with a complex internal structure. At a given temperature and spin, the equilibrium rotational configuration minimizes the nuclear free energy. This equilibrium state is strongly influenced by the effects of temperature and spin on the single particle shell structure, pairing and deformation etc. as shown in our earlier works [7, 17]. Therefore, to determine the shape and deformation of a hot rotating nucleus one should in principle begin with a meanfield approximation with the input parameters excitation energy E^* and angular momentum J and maximize the entropy of the nucleus. However it is much more convenient to replace these extensive variables E^* and J by their intensive partners, the temperature T and the angular velocity of rotation ω which is the standard replacement of micro-canonical description by the canonical one with their usual relations and it should be considered as a part of the mean field approximation with T^{-1} and ω/T regarded as the Lagrange multipliers enforcing the constraints of the given E^* and J as is done in our work.

We treat excited high spin states by using statistical theory which involve the determination of the Grand Partition Function $Q(\alpha, \beta', \gamma')$ of the nuclear system of N neutrons and Z protons

$$Q(\alpha_Z, \alpha_N, \beta', \gamma') = \sum \exp(-\beta' E_i + \alpha_Z Z_i + \alpha_N N_i + \gamma' M_i) \quad (1)$$

where the Lagrangian multipliers α , β' , γ' conserve the particle number, total energy and angular momentum of the system and are fixed by the saddle point equations. The conservation equations in terms of single-particle eigen values for the protons ϵ_i^Z with spin projection m_i^Z and neutrons ϵ_i^N with spin projection m_i^N [18], at a temperature $T(=1/\beta')$ are

$$\langle Z \rangle = \sum n_i^Z = \sum [1 + \exp(-\alpha_Z + \beta' \epsilon_i - \gamma' m_i^Z)]^{-1}, \quad (2)$$

$$\langle N \rangle = \sum n_i^N = \sum [1 + \exp(-\alpha_N + \beta' \epsilon_i - \gamma' m_i^N)]^{-1}, \quad (3)$$

$$\langle E(M, T) \rangle = \sum n_i^Z \epsilon_i^Z + \sum n_i^N \epsilon_i^N \quad (4)$$

$$\langle M \rangle = \sum n_i^Z m_i^Z + \sum n_i^N m_i^N \quad (5)$$

where n_i is the occupation probability. The single particle level scheme is obtained by diagonalisation of the triaxially deformed Nilsson Hamiltonian in the cylindrical basis states [19, 20] with the Hill-Wheeler [21] deformation parameters (β, γ) . The excitation energy of the system is found by

$$E^*(M, T) = E(M, T) - E(0, 0), \quad (6)$$

where $E(0,0)$ is the ground state energy of the nucleus given by

$$E(0, 0) = \sum \epsilon_i^Z + \sum \epsilon_i^N. \quad (7)$$

As illustrated by Moretto [22], the laboratory-fixed z axis can be made to coincide with the body-fixed z' axis and it is possible to identify and substitute M for the total angular momentum J . In the quantum-mechanical limit, the z component M of the total angular momentum $M = M_N + M_Z \rightarrow J + 1/2$, where J is the total angular momentum. The rotational energy E_{rot} is calculated using Eq. (7):

$$E_{rot}(M) = E(M, T) - E(0, T). \quad (8)$$

As $T \rightarrow 0$, E_{rot} corresponds to the yrast energy. The entropy of the system is obtained by

$$S = - \sum [n_i \ln n_i + (1 - n_i) \ln(1 - n_i)] \quad (9)$$

To evaluate deformation and shape of the excited nucleus we calculate excitation energy E^* and entropy S of the hot rotating nuclear system for fixed T and M as a function of β and γ and then incorporate them to the ground state energy calculated using macroscopic-microscopic approach [8] and then minimize free Energy (F) with respect to deformation parameters (β, γ) at temperature T and angular momentum M

$$F(Z, N, T, M, \beta, \gamma) = E_{LDM}(Z, N) + \delta E_{shell}(\beta, \gamma) + E_{def}(\beta, \gamma) + E^*(T, M, \beta, \gamma) - TS(T, M, \beta, \gamma) \quad (10)$$

The Free energy minimization gives the deformation and shape of the hot rotating nucleus. To trace the nuclear shapes and equilibrium deformation we vary the axial deformation parameter β from 0 to 0.4 in steps of 0.01 and the angular deformation parameter γ ranges from -180° (oblate with symmetry axis parallel to the rotation axis) to -120° (prolate with symmetry axis perpendicular to rotation axis) and then to -60° (oblate collective) to 0° (prolate non-collective).

3 Results and Discussion

Spectroscopy of proton emitters $Z \leq 69$ with the discovery of proton radioactivity from ^{131}Eu and ^{141}Ho [14] indicated large deformations as the proton decay rates showed significant deviations from calculations assuming spherical basis. We calculate equilibrium deformations and shapes of these nuclei in the theoretical framework of macroscopic-microscopic approach [8] and as indicated in Ref. [14] the quadrupole deformation obtained is very large. Figure 1 shows β and γ vs. mass number A of proton rich isotopes of Ho and Eu . Ground state shape of ^{131}Eu is predicted to be prolate ($\gamma = -120^\circ$) with $\beta \approx 0.3$. As we move away from proton drip line towards stable valley with increasing N , shape transition from prolate to triaxial is observed. Proton rich Ho isotopes also exhibit triaxial shape with large deformation and move towards oblate shape with β decreasing to very small values when the shell closure $N=82$ approaches. There is a slight deviation in our predicted β values from that of Vreterner et.

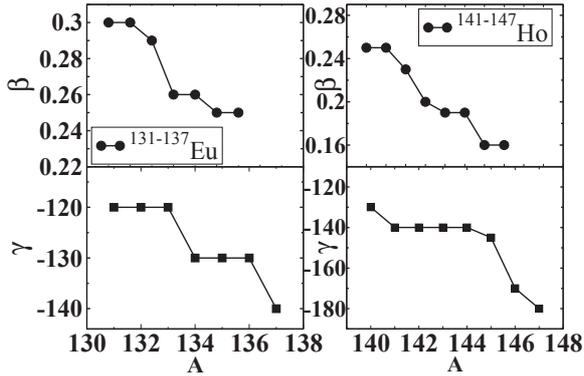


Figure 1. Ground state equilibrium deformation and shape of proton rich isotopes of Eu and Ho

al. [23] which could be due to inclusion of triaxiality in our calculations whereas Vretener et. al. [23] have considered only prolate and oblate shapes and since most nuclei in this region are found to exhibit triaxial shapes justifies the slight deviation in values.

Excited rotational states of Ho and Eu are computed for $M=0$ to $60\hbar$ at very small temperature $T=0.5$ MeV which ensures the states to be nearly yrast and the effects of rotation are predominant. Variation of β with angular momentum M for $^{141-145,149}\text{Ho}$ is shown in figure 2. Proton emitter ^{141}Ho which lies beyond the proton drip line has prolate shape rotating about its symmetry axis at $T=0.5$ MeV and $M=2.5-58.5\hbar$. As N increases, more dominant shape phase is triaxial ($\gamma=10-30^\circ$ for $M=0-38\hbar$) with shape transition to prolate non-collective ($\gamma=0^\circ$) at $M=40-58\hbar$. Quadrupole deformation $\beta \approx 0.2-0.24$ upto $M=30\hbar$, which is close to the speculation of quadrupole deformation $\beta_2 \approx 0.23-0.24$ [24–26] in an experimentally observed proton emitting state of ^{141}Ho at low excitation energy [27]. At ^{149}Ho , prolate non-collective shape phase almost diminishes with deformation becoming almost 0. Temperature also has a profound effect on this rare shape phase as with increasing temperature $T \geq 1.2$ MeV, prolate shape phase starts diminishing and shape transition to well deformed oblate non-collective is seen at $M \geq 40\hbar$ which is the usual classic response of a rotating nucleus. At critical temperature $T=1.7$ MeV, shape is mostly spherical with $\beta \approx 0$. It would be more interesting to evaluate yrast states by incorporating high angular momentum states at zero temperature but it is not possible with our present formalism which makes use of statistical theory. Such states would be soon incorporated in our upcoming works.

Free energy minimization with β and γ is shown for ^{144}Ho . F minima moves from $\gamma=30^\circ$ (triaxial) at $M=28\hbar$ (figure 3) to $\gamma=0^\circ$ (prolate non-collective) at $M=40\hbar$ (figure 4). Odd-A $^{131-135}\text{Eu}$ are highly deformed with $\beta=0.2-0.35$ while rotating with M upto $50\hbar$ at $T=0.5$ MeV. Predominant shape phase is prolate non-collective with shape transition to usual oblate non-collective as shown in figure 5. Phenomena of shape coexistence is speculated in ^{131}Eu at $M=43.5\hbar$ where two F minima are seen at similar

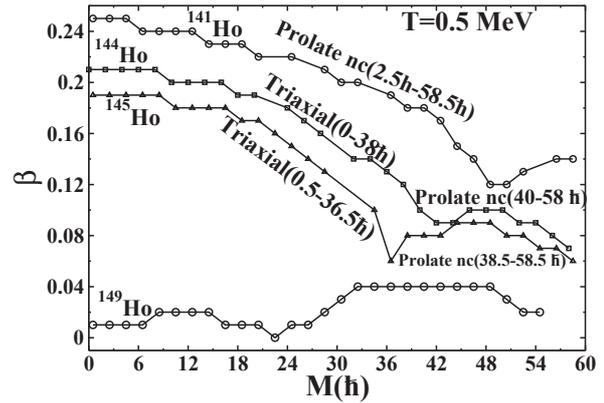


Figure 2. Variation of equilibrium deformation β with angular momentum M at $T=0.5$ MeV for proton rich Ho isotopes

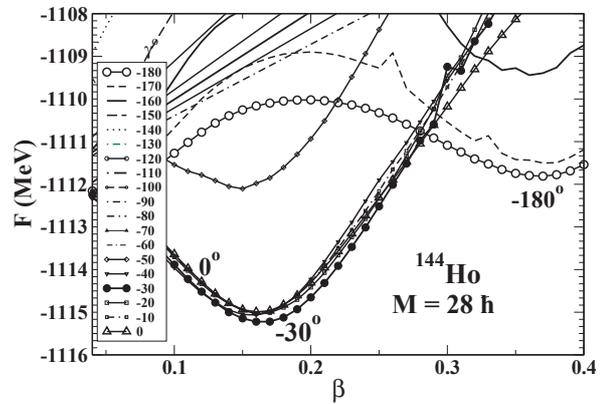


Figure 3. F minimization with respect to β , γ for ^{144}Ho at $M=28\hbar$. F minima is at $\gamma=-30^\circ$ (triaxial shape)

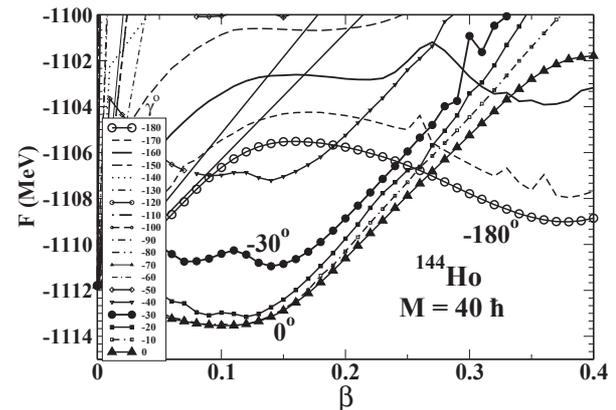


Figure 4. F minimization with β , γ for ^{144}Ho at $M=40\hbar$. Minima is attained at $\gamma=0^\circ$ prolate non-collective shape

energies for shapes corresponding to $\gamma=0^\circ$ (prolate non-collective) and 180° (oblate non-collective). Both shapes seem to coexist at the same energy (see figure 6). This in-

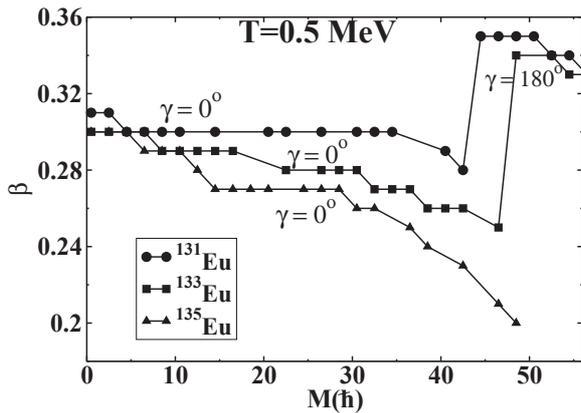


Figure 5. γ variation with angular momentum for $^{131-135}\text{Eu}$. Shape transition from $\gamma = 0^\circ$ (prolate non-collective) to -180° (oblate non-collective phase) can be seen

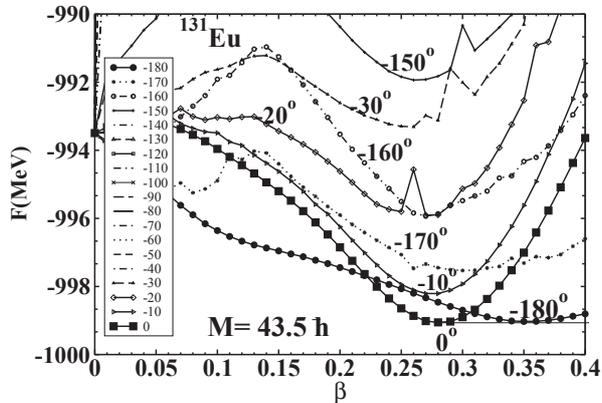


Figure 6. Two F minima at similar energies for shapes corresponding to $\gamma = 0^\circ$ (prolate non-collective) and 180° (oblate non-collective) indicating shape coexistence

interesting observation needs more investigation and would be presented in our subsequent work.

4 Conclusion

A systematic study of search for rare shape phase of prolate non-collective and structural transitions in proton emitters Ho and Eu in ground and excited high spin state with a simple theoretical formalism is presented. Triaxial shape is the dominant shape phase in ground state odd - Z proton emitters $^{141-144}\text{Ho}$ whereas $^{131-133}\text{Eu}$ have prolate shape phase with shape transition to triaxial with increasing N. The rare shape phase of prolate non-collective is seen only in the conditions of certain critical low temperatures, extreme isospin with proton richness and low and

moderate spin values. With increasing temperatures, neutron numbers and spin $\geq 50\hbar$, this shape phase starts disappearing and shape transition to the usual phase of oblate shape with non-collective rotation takes place.

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