

Rotation and alignment of high- j orbitals in transfermium nuclei

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Abstract. The structure of nuclei with $Z \sim 100$ is investigated systematically by the Cranked Shell Model (CSM) with pairing correlations treated by a Particle-Number Conserving (PNC) method. In the PNC method, the particle number is conserved and the Pauli blocking effects are taken into account exactly. By fitting the experimental single-particle spectra in these nuclei, a new set of Nilsson parameters (κ and μ) is proposed. The experimental kinematic moments of inertia and the band-head energies are reproduced quite well by the PNC-CSM calculations. The band crossing, the effects of high- j intruder orbitals and deformation are discussed in detail.

1 Introduction

The exploration of the island of stability with high mass and charge, i.e., the region of superheavy elements (SHE), has been one of the fundamental questions in natural science. Great experimental progress has been made in synthesizing the superheavy elements. Up to now, superheavy elements with $Z \leq 118$ have been synthesized via cold and hot fusion reactions [1–3]. However, due to the extremely low production cross-sections, these experiments can rarely reveal the detailed spectroscopic information. One indirect way is to study lighter nuclei in the deformed region with $Z \approx 100$ and $N \approx 152$, which are the heaviest systems accessible in present in-beam experiments (see Refs. [4–6] and references therein). The strongly downsloping orbitals originating from the spherical subshells active in the vicinity of the predicted shell closures come close to the Fermi surface of transfermium nuclei due to deformation effect. The rotational properties of transfermium nuclei will be strongly affected by these spherical orbitals. The proton $1/2^- [521]$ orbital is of particular interest since it stems from the spherical $2f_{5/2}$ orbital. The spin-orbit interaction strength of $2f_{5/2} - 2f_{7/2}$ partner governs the size of the possible $Z = 114$ shell gap. The Cranked Shell Model (CSM) with the pairing correlations treated by a Particle-Number Conserving (PNC) method [7, 8] is used to study the rotational and single-particle properties of $Z \sim 100$ nuclei.

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2 Theoretical framework

The Cranked Shell Model Hamiltonian of an axially symmetric nucleus in the rotating frame is expressed as:

$$H_{\text{CSM}} = H_0 + H_P = \sum_n (h_{\text{Nil}} - \omega j_x)_n + H_P(0) + H_P(2), \quad (1)$$

where h_{Nil} is the Nilsson Hamiltonian[11], $-\omega j_x$ is the Coriolis force with the cranking frequency ω about the x axis (perpendicular to the nuclear symmetry z axis). H_P is the pairing including monopole and quadrupole pairing correlations,

$$H_P(0) = -G_0 \sum_{\xi\eta} a_{\xi}^{\dagger} a_{\bar{\xi}}^{\dagger} a_{\bar{\eta}} a_{\eta}, \quad (2)$$

$$H_P(2) = -G_2 \sum_{\xi\eta} q_2(\xi) q_2(\eta) a_{\xi}^{\dagger} a_{\bar{\xi}}^{\dagger} a_{\bar{\eta}} a_{\eta}, \quad (3)$$

with $\bar{\xi}$ and $\bar{\eta}$ being the time-reversal states of a Nilsson state ξ and η , respectively. The quantity $q_2(\xi) = \sqrt{16\pi/5} \langle \xi | r^2 Y_{20} | \xi \rangle$ is the diagonal element of the stretched quadrupole operator, and G_0 and G_2 are the effective strengths of monopole and quadrupole pairing interactions, respectively.

In our calculation, $h_0(\omega) = h_{\text{Nil}} - \omega j_x$ is diagonalized firstly to obtain the cranked Nilsson orbitals. Then, H_{CSM} is diagonalized in a sufficiently large Cranked Many-Particle Configuration (CMPC) space to obtain the yrast and low-lying eigenstates. Instead of the usual single-particle level truncation in common shell-model calculations, a cranked many-particle configuration truncation (Fock space truncation) is adopted which is crucial to make the PNC calculations for low-lying excited states both workable and sufficiently accurate [9, 10]. The eigenstate of H_{CSM} is expressed as:

$$|\psi\rangle = \sum_i C_i |i\rangle, \quad (4)$$

where $|i\rangle$ is a cranked many-particle configuration (an occupation of particles in the cranked Nilsson orbitals) and C_i is the corresponding probability amplitude.

The angular momentum alignment $\langle J_x \rangle$ of the state $|\psi\rangle$ is given by:

$$\langle \psi | J_x | \psi \rangle = \sum_i |C_i|^2 \langle i | J_x | i \rangle + 2 \sum_{i<j} C_i^* C_j \langle i | J_x | j \rangle. \quad (5)$$

The kinematic moment of inertia is $J^{(1)} = \langle \psi | J_x | \psi \rangle / \omega$.

3 Results and discussions

The Nilsson parameters (κ, μ) proposed in Refs. [11, 12] cannot well describe the experimental level schemes of transfermium nuclei while it is optimized to reproduce the experimental level schemes for the rare-earth and actinide nuclei. By fitting the experimental single-particle levels in the odd- A nuclei with $Z \approx 100$, we obtained a new set of Nilsson parameters κ and μ (see Table 1) which are dependent on the main oscillator quantum number N as well as on the orbital angular momentum l [13, 14].

The CMPC space in the work of Ref. [13] is constructed in the proton $N = 4, 5, 6$ shells and the neutron $N = 6, 7$ shells. The dimensions of the CMPC space for the nuclei with $Z \approx 100$ are about 1000 both for protons and neutrons. Figure 1 gives the experimental and calculated moments of inertia

Table 1. Nilsson parameters κ and μ proposed for the nuclei with $Z \approx 100$. Taken from Ref. [13, 14].

N	l	κ_p	μ_p	N	l	κ_n	μ_n
4	0,2,4	0.0670	0.654				
5	1	0.0250	0.710	6	0	0.1600	0.320
	3	0.0570	0.800	2		0.0640	0.200
	5	0.0570	0.710	4,6		0.0680	0.260
6	0,2,4,6	0.0570	0.654	7	1,3,5,7	0.0634	0.318

of excited 1-qp bands in the odd- Z Bk, Es, and Md isotopes (taken from Ref. [13]). The data are well reproduced by the PNC calculations. Only one signature band was observed in ^{251}Md . We calculated $J^{(1)}$'s for two signature partner bands which vary smoothly with frequency in ^{251}Md .

In order to investigate the effect of the proton $N = 7$ shell on the rotational properties of the transfermium nuclei, the proton $N = 7$ shell is included to construct the CMPC space [15, 16]. Figure 2 shows the experimental and calculated kinematic moment of inertia $J^{(1)}$ of the $1/2^-$ [521] band in ^{251}Md . We find that the $1/2^-$ [770] orbital plays an important role in the rotational properties of ^{251}Md . Since the position of the $1/2^-$ [770] orbital is very sensitive to the deformation [17], we calculate ^{251}Md for $\varepsilon_2 = 0.28$ and 0.255 with and without the proton $N = 7$ shell, respectively [16]. There is no signature splitting when the proton $N = 7$ shell is not included [see Figure 2 (a)]. When the effect of the proton $N = 7$ shell is considered, the signature splitting occurs at $\hbar\omega \approx 0.225$ for $\varepsilon_2 = 0.28$ and $\hbar\omega \approx 0.275$ for $\varepsilon_2 = 0.255$, respectively [see Figure 2 (b)]. When $\varepsilon_2 = 0.28$, a sharp backbending of the $\alpha = -1/2$ band is shown at $\hbar\omega \approx 0.225\text{MeV}$ while the $\alpha = +1/2$ bands varies smoothly in the whole observed frequency range. The backbending of the $\alpha = -1/2$ band is due to the band crossing between the $1/2^-$ [521] ($\alpha = -1/2$) and $1/2^-$ [770] ($\alpha = -1/2$) configurations at $\hbar\omega \approx 0.255\text{MeV}$. The similar situation is hold for $\varepsilon_2 = 0.255$, but the band crossing occurs at higher frequency $\hbar\omega \approx 0.275\text{MeV}$.

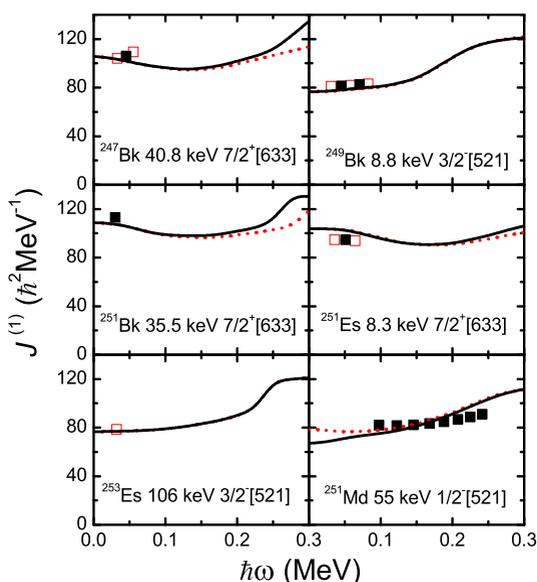


Figure 1. The experimental and calculated MOI's $J^{(1)}$ of the excited 1-qp bands in odd- Z Bk, Es, and Md isotopes. The data are taken from Refs. [5, 6] and references therein. The experimental MOI's are denoted by full squares (signature $\alpha = +1/2$) and open squares (signature $\alpha = -1/2$), respectively. The calculated MOI's by the PNC method are denoted by solid lines (signature $\alpha = +1/2$) and dotted lines (signature $\alpha = -1/2$), respectively. The effective pairing interaction strengths for both protons and neutrons for all these odd- N nuclei are, $G_n = 0.30\text{ MeV}$, $G_{2n} = 0.02\text{ MeV}$, $G_p = 0.25\text{ MeV}$, and $G_{2p} = 0.01\text{ MeV}$. Taken from Ref. [13].

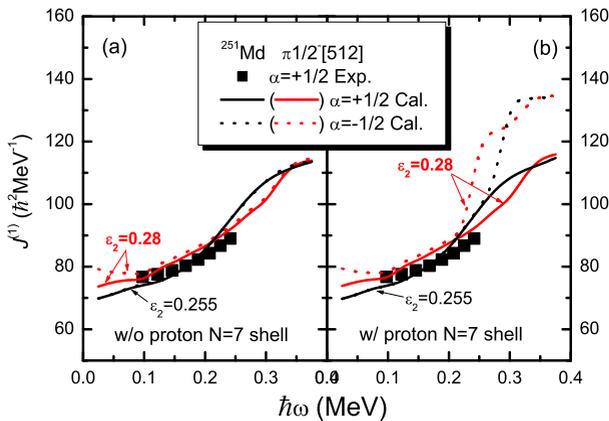


Figure 2. Experimental and calculated kinematic moment of inertia $J^{(1)}$ of the $\pi 1/2^- [521]$ band in ^{251}Md . The experimentally observed $1/2^- [521](\alpha = +1/2)$ band is denoted by solid square. The calculated $J^{(1)}$ are denoted by solid ($\alpha = +1/2$) and dashed lines ($\alpha = -1/2$), respectively. The calculated $J^{(1)}$ for $\varepsilon_2 = 0.28$ and $\varepsilon_2 = 0.255$ are denoted by red and black lines, respectively.

4 Summary

The rotational bands in the nuclei with $Z \approx 100$ are investigated by using a Cranked Shell Model (CSM) with the pairing correlations treated by a Particle-Number Conserving (PNC) method. In the PNC-CSM method, the blocking effects are taken into account exactly. By fitting the experimental single-particle spectra in these nuclei, a new set of Nilsson parameters (κ and μ) is proposed. The experimentally observed variations of moment of inertia for these nuclei with the frequency ω are reproduced very well by the PNC-CSM calculations. The high- j intruder proton orbital $\pi 1j_{15/2} (1/2^- [770])$ plays an important role in the sharp backbending of the $1/2^- [521](\alpha = -1/2)$ band for ^{251}Md .

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