

## The three shapes of $^{32}\text{Mg}$

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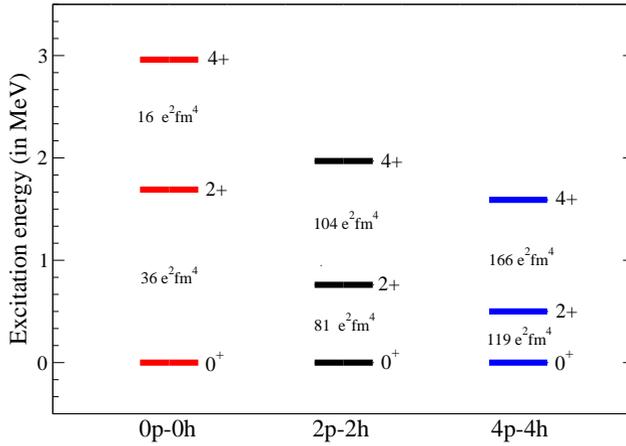
**Abstract.** The  $N=20$  and  $N=28$  "islands of inversion" are described by large scale shell model calculations which mix configurations with different  $N\hbar\omega$  or equivalently with different number of particles promoted from the *sd*-shell to the *pf*-shell. We pay particular attention to the properties of the states at fix  $N\hbar\omega$  which turn out to be the real protagonists of the physics at  $N=20$ . In particular we study the appearance of spherical, deformed and superdeformed structures in  $^{32}\text{Mg}$  at fixed  $0p-0h$ ,  $2p-2h$  and  $4p-4h$  configurations. Finally we submit that the islands of deformation at  $N=20$  and  $N=28$  merge in the Magnesium isotopes

### 1 Introduction

At the neutron rich edge, the structure of the spherical mean field is at variance with the usual one at the stability line. The reason is that, at the stability line, the  $T=0$  channel of the nucleon-nucleon interaction has a stronger weight relative to the  $T=1$  channel than it has when the neutron excess is very large. If the shell gaps get reduced, intruder configurations, usually two neutron excitations across the neutron closure, take advantage of the availability of open shell protons to build highly correlated states that are more bound than the closed shell configuration. Thus, the shell closure is said to have vanished. Although it was known since long that the ground state parity of  $^{11}\text{Be}$  was at odds with the naive shell model picture [1], this fact was overlooked until much later, in connection with the discovery of halo nuclei with  $N = 8$ . Studies of charge radii, atomic masses and nuclear spectra in the Mg and Na isotopic chains did show that a region of deformation exists around  $N = 20$  below  $^{34}\text{Si}$ . Key experimental references are gathered in [2–5]. Since then, a lot of experimental and theoretical work has ensued. Early mean field calculations suggested that deformation was responsible for the excess of binding of  $^{31}\text{Na}$  [6], but at this stage to get a deformed minimum required the inclusion of "ad hoc" rotational corrections. In the framework of the shell model, the deformation in the region was soon associated with the dominance of two-particle -two-hole ( $2p - 2h$ ) excitations across the  $N = 20$  shell gap between the normally occupied neutron  $d_{3/2}$  orbit and the valence  $f_{7/2}$  and  $p_{3/2}$  orbits [7]. These configurations were dubbed intruders since they do not obey the normal filling of the standard spherical mean field.

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**Figure 1.** (Color online) The low energy spectra and  $B(E2)$ 's of the 0p-0h, 2p-2h and 4p-4h configurations in  $^{32}\text{Mg}$ .

## 2 The physics at fixed $N\hbar\omega$

What is the driving force behind the abrupt changes leading to the appearance of these "islands of inversion"? What makes these intruder states special? That they need to be highly correlated in order to compensate for the energy loss associated to the breaking of the normal filling of the spherical mean field. Obviously, small gaps are easier to overcome, thus a reduction of the neutron magic gaps at the very neutron rich edge is good news for the intruders. The mechanisms need not to be the same in the different regions. For instance in  $^{11}\text{Li}$  the intruder is mostly pairing boosted while in  $^{11}\text{Be}$  the quadrupole interaction is more important. In the other three neutron rich regions,  $N=20$ ,  $N=28$  and  $N=40$ , the quadrupole interaction is the main player. Let us concentrate in the  $N=20$  case. Compared to the configurations with closed  $N=20$ , the intruders (np-nh) have neutrons in open  $sd$  and  $pf$ -shell orbits and in some cases protons in open  $sd$ - shell orbits. This favors the efficient build up of correlations by the neutron-proton quadrupole interaction when the open orbits are the appropriate ones. And whose are these is dictated by the different variants of  $SU(3)$ .

Let's make these statement quantitative in a few selected cases. In this section all the calculations are performed at fixed  $N\hbar\omega$  with an extension of the interaction  $SDPF-U$  [8],  $SDPF-U-MIX$  [9], which makes it possible to mix configurations with different number of particles promoted from the  $sd$ -shell to the  $pf$ -shell. We only include neutron excitations from the  $sd$  to the  $pf$ -shell without any other truncation. We have verified that the effect of the proton excitations to the  $pf$ -shell is negligible. The results for the low energy levels of  $^{32}\text{Mg}$  are presented in Figure 1. We can follow the evolution from the semimagic 0p-0h result, with a high excited  $2^+$  and a low  $B(E2)$  to a rotational-like 2p-2h whose  $B(E2)$  corresponds to  $\beta=0.4/0.5$  and finally to a perfect rigid rotor 4p-4h with  $E(4^+)/E(2^+)=3.2$  and a very large  $B(E2)$  that corresponds to a super-deformed structure. Most important for our aims is that the gains in energy due to the correlations are very different in the 0p-0h, 2p-2h and 4p-4h

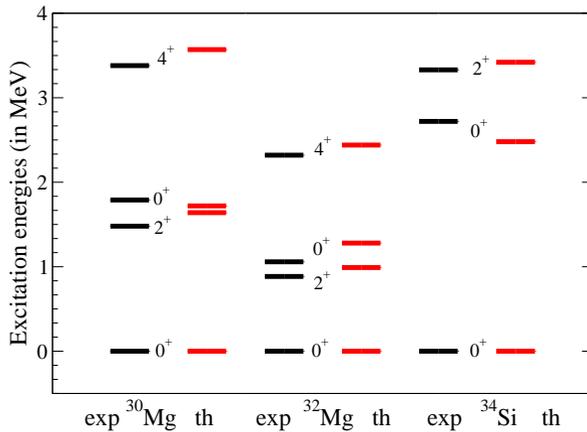
spaces; 1.5 MeV, 12.5 MeV and 21 MeV respectively. These huge correlation energies may eventually overcome the spherical mean field gaps. In fact this is the case in  $^{32}\text{Mg}$ . With `SDPF-U-MIX` the lowest 2p-2h  $0^+$  state is about 400 keV below with the lowest  $0^+$  of the 4p-4h space and 700 keV below the  $0^+$  of the 0p-0h configuration. This near degeneracy of the 2p-2h and 4p-4h bandheads is not a spurious manifestation of our spherical mean field not producing the right *sd* – *pf* gap, but due to the fact that the energy gain per particle promoted to the *pf*-shell, is the same for both configurations. We want to stress again the fact that, in favorable circumstances like these, the gain in correlation energy of the intruders can beat the spherical mean field.

The configuration with four neutrons in the *pf*-shell and two neutron holes in the *sd*-shell maximizes the quadrupole moment and, *a fortiori* the quadrupole correlation energy. Therefore one should expect the 2p-2h configurations to be also dominant in  $^{34}\text{Mg}$ . On the contrary the 0p-0h configurations should start taking over in  $^{36}\text{Mg}$ . This would establish the limit of the N=20 "island of inversion" at, say, A=35. However, our calculations show that in  $^{36}\text{Mg}$  the  $0f_{7/2}$  orbit is strongly depopulated in favor of the  $1p_{3/2}$  orbit. This indicates that both islands of inversion at N=20 and N=28 have actually merged in a single one.

### 3 Landing at the island of inversion; $^{30}\text{Mg} \rightarrow ^{32}\text{Mg}$ and $^{34}\text{Si} \rightarrow ^{32}\text{Mg}$

There are two courses to land at the "island of inversion" by the  $^{32}\text{Mg}$  shore : through the isotopic and the isotonic chains. Both are of paramount importance for the understanding of the rich variety of structural changes which take place in the region. Adding two neutrons to  $^{30}\text{Mg}$  provokes the inversion of the normal and intruder configurations which are shifted by nearly 3 MeV in  $^{32}\text{Mg}$ . In the isotonic course the transition is even more abrupt as has been recently shown in a GANIL experiment [10]: by removing two protons from  $^{34}\text{Si}$ , the intruder (deformed) state is shifted down by about 4 MeV with respect to the spherical one to become the ground state of  $^{32}\text{Mg}$ . We compare the experimental data with the shell model results in Figure 2. The calculations include configurations with up to 6 neutrons on the *pf*-shell when necessary for convergence. In  $^{30}\text{Mg}$  and  $^{34}\text{Si}$  the ground states are dominantly (>80%) 0p-0h and the first excited  $0^+$ 's dominantly 2p-2h. They differ in the structure of the lowest  $2^+$  which is 0p-0h in  $^{30}\text{Mg}$  and 2p-2h in  $^{34}\text{Si}$ . More details on this last nucleus can be found in reference [10]. The structure of the  $0^+$  states in  $^{32}\text{Mg}$  is extremely singular; the ground state is indeed dominated by the 2p-2h, with large 4p-4h mixing, while the excited  $0^+$  is a mixture of 0p-0h, and 4p-4h components. The precise amounts of the mixing depend critically on the relative location of the three fixed  $N\hbar\omega$  band-heads. The  $2^+$  has a structure similar to the ground state. In addition, the calculated spectroscopic quadrupole moments of the  $2^+$  and  $4^+$  states and the  $B(E2)$ 's in the yrast band are compatible with a single intrinsic state with  $Q_0 \approx 65 \text{ e fm}^2$ . In  $^{34-40}\text{Mg}$  the calculated  $2^+$  and  $4^+$  excitation energies remain rather constant at about 0.65 MeV and 2.0 MeV respectively, in good agreement with the available experimental data. The E2 properties are compatible with  $Q_0 \approx 70 \text{ e fm}^2$  both in experiment and theory. These comparisons provide clear evidence of the merging of the N=20 and N=28 islands of inversion/deformation. The same merging does occur in the Neon and Sodium chains, even if their N=28 isotones most probably lay beyond the neutron drip line. The large occupation of the  $1p_{3/2}$  orbit approaching the drip line may favor the appearance of neutron halos, as experimentally hinted in  $^{33}\text{Ne}$ .

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**Figure 2.** (Color online) Comparison between experiment and theory for the most important low lying states in  $^{30}\text{Mg}$ ,  $^{32}\text{Mg}$  and  $^{34}\text{Si}$

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