

Finite Theories predictions vs. the Discovery of a Higgs-like Boson at the LHC

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Abstract. Finite Unified Theories (FUTs) have proven very successful so far. In particular, they predicted the top quark mass one and half years before its experimental discovery, while around five years ago confronting their predictions with the values of the top and bottom quark masses at the time, as well as with other low-energy experimental results, a light Higgs-boson in the mass range $\sim 121 - 126$ GeV was predicted, in striking agreement with the recent discovery of a Higgs like state at ATLAS and CMS. FUTs are $N = 1$ supersymmetric Grand Unified Theories, which can be made all-loop finite based on the principle of reduction of couplings, which in turn provides them with a large predictive power. Here we review a FUT model based on $SU(5)$ as gauge group. It is worth noting that this model naturally predicted a relatively heavy spectrum with the coloured supersymmetric particles above 1.5 TeV, consistent with the non-observation of those particles at the LHC, as well as a large $\tan\beta$. Recently, restricting further the parameter space of this FUT model according to the discovery of a Higgs-like state and B -physics observables, we found predictions for the rest of the Higgs masses and s -spectrum.

1 Introduction

In the successes of the Standard Model (SM) it has recently been added the discovery at the LHC [1, 2] of a state that can be interpreted as the long expected Higgs boson of the SM. However, the plethora of free parameters in the SM suggests that it cannot be considered as the Theory of Elementary Particles, but rather as a low energy limit of a more fundamental one. The celebrated Minimal Supersymmetric Standard Model (MSSM) [3–5] was widely guessed as being the next step towards the construction of a more fundamental theory. Despite the huge amount of free parameters of the MSSM it was expected that it could be realised in a constrained form called CMSSM [6] with only five free parameters. The

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recent results from LHC exclude certain regions of the CMSSM and seem to suggest the existence of a rather heavy spectrum in case this version of the MSSM is actually realized in nature [7, 8].

A strategy in searching for a fundamental theory possibly at the Planck scale has been developed in refs. [9–28], whose basic ingredients are GUTs and supersymmetry, but its consequences certainly go beyond the known ones [12–18]. The method consists of hunting for renormalization group invariant (RGI) relations holding below the Planck scale, which in turn are preserved down to the GUT scale. An impressive aspect of the RGI relations is that one can guarantee their validity to all-orders in perturbation theory by studying the uniqueness of the resulting relations at one-loop, as was proven in the early days of the programme of reduction of couplings [19, 20]. Even more remarkable is the fact that it is possible to find RGI relations among couplings that guarantee finiteness to all-orders in perturbation theory [21–23].

The Gauge–Yukawa unification scheme, based in RGI relations applied in the dimensionless couplings of supersymmetric GUTs, such as gauge and Yukawa couplings, had noticeable successes by predicting correctly the top quark mass in the finite [12, 13] and in the minimal $N = 1$ supersymmetric $SU(5)$ GUTs [14]. Finite Unified Theories are $N = 1$ supersymmetric GUTs which can be made finite to all-loop orders, including the soft-SUSY breaking sector (for reviews and detailed refs. see [18, 24–27]), which involves parameters of dimension one and two. Taking into account the restrictions resulting from the low-energy observables, it was possible to extend the predictive power of the RGI method to the Higgs sector and the SUSY spectrum. The Higgs boson mass thus predicted [28]

$$M_h \simeq 121 - 126 \text{ GeV} \quad (1)$$

is in agreement with the recent discovery of a Higgs-like state at the LHC [1, 2]. As further features a heavy SUSY spectrum and large values of $\tan\beta$ (the ratio of the two vacuum expectation values of the Higgs fields) were found [28].

In these proceedings, we review an $SU(5)$ -based finite SUSY model and its predictions, taking into account the restrictions resulting from the low-energy observables [28]. Only one model survives all the phenomenological constraints. Then we extend our previous analysis by imposing more recent constraints resulting from the bounds on $\text{BR}(B_s \rightarrow \mu^+\mu^-)$. Moreover, as the crucial new ingredient we interpret the newly discovered particle at ~ 126 GeV as the lightest MSSM Higgs boson and we analyse which constraints imposes the measured value of the Higgs boson mass on the predictions of the SUSY spectrum.

2 Finiteness

Finiteness can be understood by considering a chiral, anomaly free, $N = 1$ globally supersymmetric gauge theory based on a group G with gauge coupling constant g . The superpotential of the theory is given by

$$W = \frac{1}{2} m^{ij} \Phi_i \Phi_j + \frac{1}{6} C^{ijk} \Phi_i \Phi_j \Phi_k, \quad (2)$$

where m^{ij} (the mass terms) and C^{ijk} (the Yukawa couplings) are gauge invariant tensors and the matter field Φ_i transforms according to the irreducible representation R_i of the gauge group G . All the one-loop β -functions of the theory vanish if the β -function of the gauge coupling $\beta_g^{(1)}$, and the anomalous dimensions of the Yukawa couplings $\gamma_i^{j(1)}$, vanish, i.e.

$$\sum_i \ell(R_i) = 3C_2(G), \quad \frac{1}{2} C_{ipq} C^{j pq} = 2\delta_i^j g^2 C_2(R_i), \quad (3)$$

where $\ell(R_i)$ is the Dynkin index of R_i , and $C_2(G)$ is the quadratic Casimir invariant of the adjoint representation of G . These conditions are also enough to guarantee two-loop finiteness [29]. A striking fact is the existence of a theorem [21–23] that guarantees the vanishing of the β -functions to all-orders in perturbation theory. This requires that, in addition to the one-loop finiteness conditions (3), the Yukawa couplings are reduced in favour of the gauge coupling to all-orders (see [27] for details). Alternatively, similar results can be obtained [30–32] using an analysis of the all-loop NSVZ gauge beta-function [33, 34].

Next consider the superpotential given by (2) along with the Lagrangian for soft supersymmetry breaking (SSB) terms

$$-\mathcal{L}_{\text{SB}} = \frac{1}{6} h^{ijk} \phi_i \phi_j \phi_k + \frac{1}{2} b^{ij} \phi_i \phi_j + \frac{1}{2} (m^2)_i^j \phi^{*i} \phi_j + \frac{1}{2} M \lambda \lambda + \text{h.c.}, \quad (4)$$

where the ϕ_i are the scalar parts of the chiral superfields Φ_i , λ are the gauginos and M their unified mass, h^{ijk} and b^{ij} are the trilinear and bilinear dimensionful couplings respectively, and $(m^2)_i^j$ the soft scalars masses. Since we would like to consider only finite theories here, we assume that the gauge group is a simple group and the one-loop β -function of the gauge coupling g vanishes. We also assume that the reduction equations admit power series solutions of the form

$$C^{ijk} = g \sum_n \rho_{(n)}^{ijk} g^{2n}. \quad (5)$$

According to the finiteness theorem of refs. [21–23, 35], the theory is then finite to all orders in perturbation theory, if, among others, the one-loop anomalous dimensions $\gamma_i^{j(1)}$ vanish. The one- and two-loop finiteness for h^{ijk} can be achieved through the relation [36]

$$h^{ijk} = -MC^{ijk} + \dots = -M\rho_{(0)}^{ijk} g + O(g^5), \quad (6)$$

where \dots stand for higher order terms.

In addition it was found that the RGI SSB scalar masses in Gauge-Yukawa unified models satisfy a universal sum rule at one-loop [37]. This result was generalized to two-loops for finite theories [24], and then to all-loops for general Gauge-Yukawa and finite unified theories [38]. From these latter results, the following soft scalar-mass sum rule is found [24]

$$\frac{(m_i^2 + m_j^2 + m_k^2)}{MM^\dagger} = 1 + \frac{g^2}{16\pi^2} \Delta^{(2)} + O(g^4) \quad (7)$$

for i, j, k with $\rho_{(0)}^{ijk} \neq 0$, where $m_{i,j,k}^2$ are the scalar masses and $\Delta^{(2)}$ is the two-loop correction

$$\Delta^{(2)} = -2 \sum_l [(m_l^2 / MM^\dagger) - (1/3)] \ell(R_l), \quad (8)$$

which vanishes for the universal choice, i.e. when all the soft scalar masses are the same at the unification point. This correction also vanishes in the model considered here.

3 Finite $SU(5)$ Unified Theories

Finite Unified Theories (FUTs) have always attracted interest for their intriguing mathematical properties and their predictive power. One very important result is that the one-loop finiteness conditions (3) are sufficient to guarantee two-loop finiteness [39]. A classification of possible one-loop finite

models was done by two groups [40–42]. The first one and two-loop finite $SU(5)$ model was presented in [43], and shortly afterwards the conditions for finiteness in the soft SUSY-breaking sector at one-loop [44] were given. In [45] a one and two-loop finite $SU(5)$ model was presented where the rotation of the Higgs sector was proposed as a way of making it realistic. The first all-loop finite theory was studied in [12, 13], without taking into account the soft breaking terms. Finite soft breaking terms and the proof that one-loop finiteness in the soft terms also implies two-loop finiteness was done in [36]. The inclusion of soft breaking terms in a realistic model was done in [46] and their finiteness to all-loops studied in [47], although the universality of the soft breaking terms lead to a charged LSP. This fact was also noticed in [48], where the inclusion of an extra parameter in the boundary condition of the Higgs mixing mass parameter was introduced to alleviate it. The derivation of the sum-rule in the soft SUSY breaking sector and the proof that it can be made all-loop finite were done in refs. [24] and [38] respectively, allowing thus for the construction of all-loop finite realistic models.

From the classification of theories with vanishing one-loop gauge β -function [40], one can easily see that there exist only two candidate possibilities to construct $SU(5)$ GUTs with three generations. These possibilities require that the theory should contain as matter fields the chiral supermultiplets $\mathbf{5}$, $\bar{\mathbf{5}}$, $\mathbf{10}$, $\bar{\mathbf{5}}$, $\mathbf{24}$ with the multiplicities (6, 9, 4, 1, 0) or (4, 7, 3, 0, 1), respectively. Only the second one contains a $\mathbf{24}$ -plet which can be used to provide the spontaneous symmetry breaking (SB) of $SU(5)$ down to $SU(3) \times SU(2) \times U(1)$. For the first model one has to incorporate another way, such as the Wilson flux breaking mechanism to achieve the desired SB of $SU(5)$ [12, 13]. Therefore, for a self-consistent field theory discussion we would like to concentrate only on the second possibility.

The particle content of the model we will study consists of the following supermultiplets: three ($\bar{\mathbf{5}} + \mathbf{10}$), needed for each of the three generations of quarks and leptons, four ($\bar{\mathbf{5}} + \mathbf{5}$) and one $\mathbf{24}$ considered as Higgs supermultiplets. When the gauge group of the finite GUT is broken the theory is no longer finite, and we will assume that we are left with the MSSM.

Therefore, a predictive Gauge-Yukawa unified $SU(5)$ model which is finite to all orders, in addition to the requirements mentioned already, should also have the following properties:

1. One-loop anomalous dimensions are diagonal, i.e., $\gamma_i^{(1)j} \propto \delta_i^j$.
2. The three fermion generations, in the irreducible representations $\bar{\mathbf{5}}_i, \mathbf{10}_i$ ($i = 1, 2, 3$), should not couple to the adjoint $\mathbf{24}$.
3. The two Higgs doublets of the MSSM should mostly be made out of a pair of Higgs quintet and anti-quintet, which couple to the third generation.

In the following we discuss an all-order finite model, which can be obtained from the class of models suggested in Refs. [49, 50] with a modification to suppress non-diagonal anomalous dimensions [24]. Another FUT model has also been studied previously [28, 53], but the one considered here, which we will label “Best FUT model”, is the one that complies with all the low-energy phenomenological constraints that we apply.

3.1 Best FUT model

The superpotential which describes the model before the reduction of couplings takes place is of the form [12, 13, 24, 43, 45]

$$W = \sum_{i=1}^3 \left[\frac{1}{2} g_i^u \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \mathbf{10}_i \bar{\mathbf{5}}_i \bar{H}_i \right] + g_{23}^u \mathbf{10}_2 \mathbf{10}_3 H_4 \quad (9)$$

$$+ g_{23}^d \mathbf{10}_2 \bar{\mathbf{5}}_3 \bar{H}_4 + g_{32}^d \mathbf{10}_3 \bar{\mathbf{5}}_2 \bar{H}_4 + \sum_{a=1}^4 g_a^f H_a \mathbf{24} \bar{H}_a + \frac{g^\lambda}{3} (\mathbf{24})^3 ,$$

where H_a and \bar{H}_a ($a = 1, \dots, 4$) stand for the Higgs quintets and anti-quintets.

After the reduction of couplings the symmetry of the superpotential W (9) is enhanced. The superpotential has now a $Z_4 \times Z_4 \times Z_4$ symmetry, with the following superpotential

$$W_B = \sum_{i=1}^3 \left[\frac{1}{2} g_i^u \mathbf{10}_i \mathbf{10}_i H_i + g_i^d \mathbf{10}_i \bar{\mathbf{5}}_i \bar{H}_i \right] + g_{23}^u \mathbf{10}_2 \mathbf{10}_3 H_4$$

$$+ g_{23}^d \mathbf{10}_2 \bar{\mathbf{5}}_3 \bar{H}_4 + g_{32}^d \mathbf{10}_3 \bar{\mathbf{5}}_2 \bar{H}_4 + g_2^f H_2 \mathbf{24} \bar{H}_2 + g_3^f H_3 \mathbf{24} \bar{H}_3 + \frac{g^\lambda}{3} (\mathbf{24})^3 , \quad (10)$$

For this model the non-degenerate and isolated solutions to $\gamma_i^{(1)} = 0$ give us:

$$(g_1^u)^2 = \frac{8}{5} g^2 , (g_1^d)^2 = \frac{6}{5} g^2 , (g_2^u)^2 = (g_3^u)^2 = (g_{23}^u)^2 = \frac{4}{5} g^2 ,$$

$$(g_2^d)^2 = (g_3^d)^2 = (g_{23}^d)^2 = (g_{32}^d)^2 = \frac{3}{5} g^2 , \quad (11)$$

$$(g^\lambda)^2 = \frac{15}{7} g^2 , (g_2^f)^2 = (g_3^f)^2 = \frac{1}{2} g^2 , (g_1^f)^2 = (g_4^f)^2 = 0 ,$$

and from the sum rule we obtain [24]:

$$m_{H_u}^2 + 2m_{\mathbf{10}}^2 = M^2 , m_{H_d}^2 - 2m_{\mathbf{10}}^2 = -\frac{M^2}{3} , m_{\mathbf{5}}^2 + 3m_{\mathbf{10}}^2 = \frac{4M^2}{3} , \quad (12)$$

i.e., in this case we have only two free parameters $m_{\mathbf{10}} \equiv m_{\mathbf{10}}$, and M for the dimensionful sector.

As already mentioned, after the $SU(5)$ gauge symmetry breaking we assume we have the MSSM, i.e. only two Higgs doublets. This can be achieved by introducing appropriate mass terms that allow to perform a rotation of the Higgs sector [12, 13, 43, 45, 51], in such a way that only one pair of Higgs doublets, coupled mostly to the third family, remains light and acquire vacuum expectation values. To avoid fast proton decay the usual fine tuning to achieve doublet-triplet splitting is performed. Notice that, although similar, the mechanism is not identical to minimal $SU(5)$, since we have an extended Higgs sector.

Thus, after the gauge symmetry of the GUT theory is broken we are left with the MSSM, with the boundary conditions for the third family given by the finiteness conditions, while the other two families are basically decoupled.

We will now examine the phenomenology of such an all-loop Finite Unified theory with $SU(5)$ gauge group, where the reduction of couplings has been applied only on the third generation of quarks and leptons. An extension to three families, and the generation of quark mixing angles and masses in Finite Unified Theories has been addressed in [52], where several examples are given. These extensions are not considered here.

3.2 Restrictions from low-energy observables

Since the gauge symmetry is spontaneously broken below M_{GUT} , the finiteness conditions do not restrict the renormalization properties at low energies, and all it remains are boundary conditions on the gauge and Yukawa couplings (11), the $h = -MC$ relation (6), and the soft scalar-mass sum rule (12) at M_{GUT} . Thus we examine the evolution of these parameters according to their RGEs up to two-loops for dimensionless parameters and at one-loop for dimensionful ones with the relevant boundary conditions. Below M_{GUT} their evolution is assumed to be governed by the MSSM. We further assume a unique SUSY breaking scale M_{SUSY} (which we define as the geometrical average of the stop masses) and therefore below that scale the effective theory is just the SM. This allows to evaluate observables at or below the electroweak scale. We discuss first the third generation of quark masses that are leading to the strongest constraints on the model under investigation. Next we apply the recent B physics and Higgs-boson mass constraints. We discuss briefly the anomalous magnetic moment of the muon.

We now present the comparison of the predictions of the model with the experimental data, see ref. [28, 53] for more details, starting with the heavy quark masses. In fig.1 we show the predictions for the top pole mass, m_t , and the running bottom mass at the scale M_Z , $m_b(M_Z)$, as a function of the unified gaugino mass M , for the cases $\mu < 0$ and $\mu > 0$, for the model under consideration. The running bottom mass is used to avoid the large QCD uncertainties inherent for the pole mass. In the evaluation of the bottom mass m_b , we have included the corrections coming from bottom squark-gluino loops and top squark-chargino loops [54]. We compare the predictions for the running bottom quark mass with the experimental value [55]

$$m_b(M_Z) = 2.83 \pm 0.10 \text{ GeV} . \quad (13)$$

One can see that the value of m_b depends strongly on the sign of μ due to the above mentioned radiative corrections involving SUSY particles. The values for $\mu > 0$ are above the central experimental value, with $m_b(M_Z) \sim 4.0 - 5.0$ GeV. For $\mu < 0$, on the other hand, there is overlap with the experimentally measured values, $m_b(M_Z) \sim 2.5 - 2.8$ GeV. Therefore, the experimental determination of $m_b(M_Z)$ clearly selects the negative sign of μ .

Now we turn to the top quark mass. The prediction for the top quark mass is ~ 172 GeV, as shown in the lower plot of fig. 1. Comparing this prediction with the experimental value [56]

$$m_t^{\text{exp}} = (173.2 \pm 0.9) \text{ GeV} \quad (14)$$

and recalling that the theoretical value for m_t may suffer from a correction of $\sim 4\%$ [18, 25, 57], we see that clearly model is in agreement with it. In addition the value of $\tan\beta$ is found to be $\tan\beta \sim 48$.

We now analyse the impact of further low-energy observables on the model, and we concentrate only on the case $\mu < 0$. As additional constraints we consider the following observables: the rare b decays $\text{BR}(b \rightarrow s\gamma)$ and $\text{BR}(B_s \rightarrow \mu^+\mu^-)$. More details and a complete set of references can be found in ref. [28].

For the branching ratio $\text{BR}(b \rightarrow s\gamma)$, we take the experimental value estimated by the Heavy Flavour Averaging Group (HFAG) [58–60]

$$\text{BR}(b \rightarrow s\gamma) = (3.55 \pm 0.24_{-0.10}^{+0.09} \pm 0.03) \times 10^{-4}, \quad (15)$$

where the first error is the combined statistical and uncorrelated systematic uncertainty, the latter two errors are correlated systematic theoretical uncertainties and corrections respectively. For the branching ratio $\text{BR}(B_s \rightarrow \mu^+\mu^-)$, we take the value given by the Heavy Flavour Averaging Group (HFAG) is [61]

$$\text{BR}(B_s \rightarrow \mu^+\mu^-) \lesssim 4.5 \times 10^{-9} \quad (16)$$

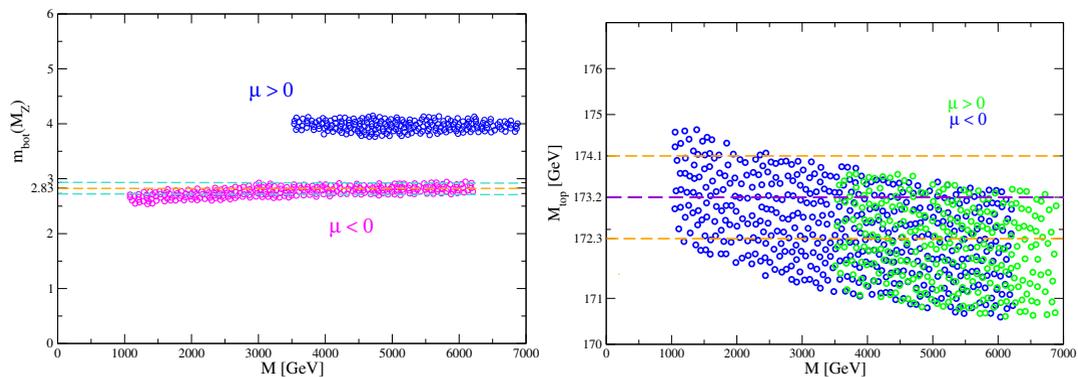


Figure 1. The bottom quark mass at the Z boson scale (left) and top quark pole mass (right) are shown as function of M .

at the 95% C.L. [62]. A first measurement at the $\sim 3\sigma$ level of $\text{BR}(B_s \rightarrow \mu^+\mu^-)$ was published by the LHCb collaboration [63] recently. The value is given as $\text{BR}(B_s \rightarrow \mu^+\mu^-) = (3.2_{-1.2}^{+1.4}(\text{stat})_{-0.3}^{+0.5}(\text{syst})) \times 10^{-9}$, i.e. the upper limit at the 95% CL is slightly higher than what we used as an upper limit. Furthermore, no combination of this new result with the existing limits exists yet. Consequently, as we do not expect a sizable impact of this very new measurement on our results, we stick for this analysis to the simple upper limit.

The prediction of the lightest Higgs boson mass, as obtained with FeynHiggs [64–67], as a function of M , the unified gaugino mass, is shown in Fig. 2, where the B physics constraints are already taken into account. One can see that the lightest Higgs boson mass range is in

$$M_h \sim 121 - 126 \text{ GeV}, \quad (17)$$

where the uncertainty roughly corresponds to variations of the soft scalar masses. To this value one

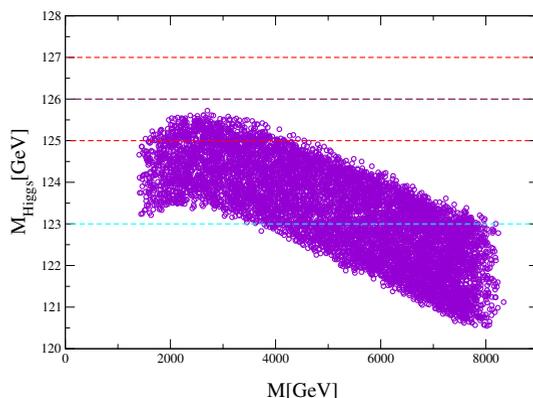


Figure 2. The lightest Higgs mass, M_h , as function of M with $\mu < 0$, see text.

has to add ± 2 GeV coming from unknown higher order corrections [66]. We have also included a

small variation, due to threshold corrections at the GUT scale, of up to 5% of the FUT boundary conditions. Interpreting the light Higgs boson as the Higgs-like state discovered at the LHC [1, 2], we can impose a constraint on our results to the Higgs mass of

$$M_h \sim 126.0 \pm 1 \pm 2 \text{ GeV} , \quad (18)$$

where ± 1 comes from the experimental error and ± 2 corresponds to the theoretical error, and see how this affects the SUSY spectrum. Constraining the allowed values of the Higgs mass this way puts a limit on the allowed values of the unified gaugino mass, as can be seen from Fig. 2. The red lines correspond to the pure experimental uncertainty and restrict $2 \text{ TeV} \lesssim M \lesssim 5 \text{ TeV}$. The blue line includes the additional theory uncertainty of $\pm 2 \text{ GeV}$. Taking this uncertainty into account no bound on M can be placed. However, a substantial part of the formerly allowed parameter points are now excluded. This in turn restricts the lightest observable SUSY particle (LOSP), which turns out to be the light scalar tau [53].

The resulting SUSY masses for the model are rather large. The lightest SUSY particle mass starts around 600 GeV, with the rest of the spectrum being very heavy. A numerical example of the lighter part of the spectrum is shown in Table 1.

The full particle spectrum of the model with $\mu < 0$, again compliant with quark mass constraints and the B -physics observables, is shown in Fig. 3. Including the Higgs mass constraint favours the lower parts of the parameter space. However, even neglecting the theory uncertainty on M_h (right plot in Fig. 3) permits SUSY masses which would remain unobservable at the LHC, the ILC or CLIC. Including the theory uncertainties, even higher masses are permitted, further weakening the discovery potential of the LHC and future e^+e^- colliders. The coloured supersymmetric particles are above $\sim 1.8 \text{ TeV}$ in agreement with the non-observation of those particles at the LHC [68–70].

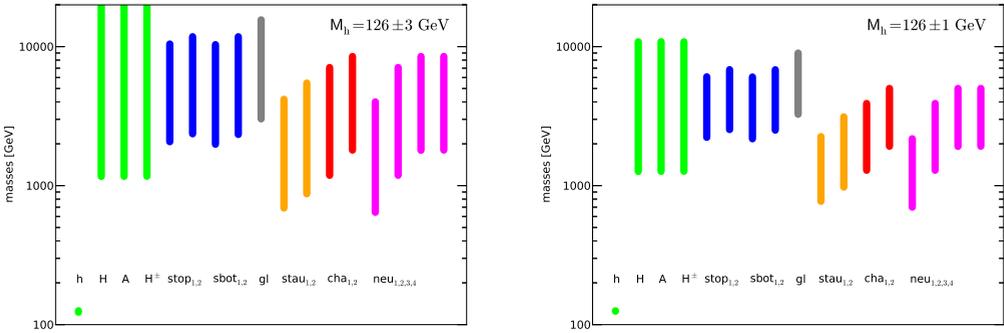


Figure 3. The particle spectrum of the best FUT model with $\mu < 0$, where the points shown are in agreement with the quark mass constraints and the B -physics observables. The light (green) points on the left are the various Higgs boson masses. The dark (blue) points following are the two scalar top and bottom masses, followed by the lighter (gray) gluino mass. Next come the lighter (orange) scalar tau masses. The darker (red) points to the right are the two chargino masses followed by the lighter shaded (pink) points indicating the neutralino masses.

We note that with such a heavy SUSY spectrum the anomalous magnetic moment of the muon, $(g - 2)_\mu$ (with $a_\mu \equiv (g - 2)_\mu/2$), gives only a negligible correction to the SM prediction. The comparison of the experimental result and the SM value (based on the latest combination using e^+e^- data) [71]

$$a_\mu^{\text{exp}} - a_\mu^{\text{theo}} = (28.7 \pm 8.0) \times 10^{-10}. \quad (19)$$

Mbot(M_Z)	2.74	Mtop	174.1
Mh	125.0	MA	1517
MH	1515	MH $^\pm$	1518
Stop1	2483	Stop2	2808
Sbot1	2403	Sbot2	2786
Mstau1	892	Mstau2	1089
Char1	1453	Char2	2127
Neu1	790	Neu2	1453
Neu3	2123	Neu4	2127
Mgluino	3632	tan β	47.2

Table 1. A representative light spectrum with $\mu < 0$, compliant with the B physics constraints. All masses are in GeV.

would disfavor the best FUT model with $\mu < 0$ [72, 73]. However, since the results would be very close to the SM result, we cannot exclude the model on this fact alone.

4 Conclusions

A number of proposals and ideas in high-energy physics have matured with time and have survived after careful theoretical studies and confrontation with experimental data. These include part of the original GUTs ideas, mainly the unification of gauge couplings and, separately, the unification of the Yukawa couplings, a version of fixed point behaviour of couplings, and certainly the necessity of SUSY as a way to take care of the technical part of the hierarchy problem. On the other hand, a very serious theoretical problem, namely the presence of divergencies in Quantum Field Theories (QFT), although challenged by the founders of QFT [74–76], was mostly forgotten in the course of developments of the field partly due to the spectacular successes of renormalizable field theories, in particular of the SM. However, fundamental developments in Theoretical Particle Physics are based in reconsiderations of the problem of divergencies and serious attempts to solve it. These include the motivation and construction of string and non-commutative theories, as well as $N = 4$ supersymmetric field theories [77, 78], $N = 8$ supergravity [79–83] and the AdS/CFT correspondence [84]. It is a thoroughly fascinating fact that many interesting ideas that have survived various theoretical and phenomenological tests, as well as the solution to the UV divergencies problem, find a common ground in the framework of $N = 1$ Finite Unified Theories, which we have described in the previous sections. From the theoretical side they solve the problem of UV divergencies in a minimal way. On the phenomenological side, since they are based on the principle of reduction of couplings (expressed via RGI relations among couplings and masses), they provide strict selection rules in choosing realistic models which lead to testable predictions. The celebrated success of predicting the top-quark mass [12–15, 17, 85] is now extended to the predictions of the Higgs masses and the supersymmetric spectrum of the MSSM [28, 86]. The predicted mass of the lightest Higgs boson turns out to be naturally in agreement with the discovery of a Higgs-like state at the LHC. Identifying the lightest Higgs boson with the newly discovered state, and taking into account the latest values for the other low-energy constraints, we restrict further the allowed parameter space of this model with $\mu < 0$. We reviewed how this reduction of parameter space impacts the predictions of the SUSY spectrum. It turns out that the resulting spectrum is rather heavy and largely unobservable at the LHC. The same holds for the LC. Only at $\sqrt{s} = 3$ TeV some uncoloured particles might be in the reach.

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