

Exotic and conventional mesons from lattice

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Abstract. Recent results on meson spectroscopy from lattice QCD are reviewed. The emphasis is on interesting states near thresholds like Z_c^+ , $X(3872)$ and $D_s^0(2317)$. Another focus is on the meson resonances in light, strange and charm sector, where the resonance masses as well as the strong decay widths are extracted from the lattice.

1 Introduction

Lattice QCD aims at describing hadrons and interactions between them based on first-principle Quantum Chromodynamics. It is based on the evaluation of correlation functions by means of Feynman path integral of QCD action on discretized and finite Euclidean space-time. I review recent results on spectrum for the exotic and conventional mesons.

2 The discrete spectrum from lattice and information encoded in it

The physics information on a meson (below, near or above threshold) is commonly extracted from the discrete energy spectrum in lattice QCD. The physical system for given quantum numbers is created from the vacuum $|\Omega\rangle$ using interpolator O_j^\dagger at time $t = 0$ and the system propagates for time t before being annihilated by O_i . To study a meson state with given J^P one can use $O \simeq \bar{q}\Gamma q$, $(\bar{q}\Gamma_1 q)_{\vec{p}_1}(\bar{q}\Gamma_2 q)_{\vec{p}_2}$, $[\bar{q}\Gamma_1 \bar{q}][q\Gamma_2 q]$ with desired quantum numbers. After the spectral decomposition the correlators are expressed in terms of the energies E_n of eigenstates $|n\rangle$ and their overlaps Z_j^n

$$C_{ij}(t) = \langle \Omega | O_i(t) O_j^\dagger(0) | \Omega \rangle = \sum_n Z_i^n Z_j^{n*} e^{-E_n t}, \quad Z_j^n \equiv \langle \Omega | O_j | n \rangle. \quad (1)$$

The most widely used method to extract the discrete spectrum E_n and overlaps from the correlation matrix $C_{ij}(t)$ is the generalised eigenvalue method $C(t)u^{(n)}(t) = \lambda^{(n)}(t)C(t_0)u^{(n)}(t)$ [1, 2]. The energies E_n are extracted from the exponential behavior of the eigenvalues $\lambda^{(n)}(t) \propto e^{-E_n t}$ at large t .

All physical eigenstates with given quantum numbers appear as energy levels in principle. These can be "one-meson" states (for example χ_{c1} in 1^{++} charmonium channel), "two-meson" states (for example $D\bar{D}^*$) and the multi-meson states (for example $J/\psi\pi\pi$). In reality the eigenstates are mixtures of these Fock components. Three- and more-meson states have never been taken into account in the study of the meson spectroscopy yet. The major step during the preceding years came from treating

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two-meson states rigorously. These have discrete spectrum due to periodic boundary condition on finite lattice. If the two mesons do not interact, then momenta of each meson is $\vec{p} = \frac{2\pi}{L}\vec{N}$ with $\vec{N} \in N^3$, and the energies of $M_1(\vec{p})M_2(-\vec{p})$ are $E^{n,i} = E_1(p) + E_2(p)$ with $E_{1,2}(p) = (m_{1,2}^2 + p^2)^{1/2}$. The energies E_n extracted from the lattice are slightly shifted in presence of the interaction and the shift provides rigorous information on the scattering matrix, as discussed bellow. In experiment, two-meson states correspond to the two-meson decay products with a continuous energy spectrum.

The mass of a hadron well below strong decay threshold is simply $m = E|_{\vec{p}=0}$. The masses of resonances and near-threshold bound-states have to be inferred from the infinite-volume scattering matrix of the one-channel (elastic) or multiple-channel (inelastic) scattering. The bound states correspond to the poles of scattering matrix on the real axis below threshold, while the resonances masses and widths are extracted from the Breit-Wigner type fits of the corresponding cross-section or phase shift. The bound states and narrow resonances typically manifest themselves as levels that appear in addition to the expected discrete two-meson levels.

The most widely used approach for extracting the infinite-volume scattering matrix from the finite volume E_n is based on Lüscher's seminal work and its generalizations. In the case of elastic scattering between two hadrons with zero-total momenta, the energy $E_n = (m_1^2 + p^2)^{1/2} + (m_2^2 + p^2)^{1/2}$ renders momenta p of each meson in the region outside the interaction. The infinite volume phase shift at that energy is given by the Lüscher's relation $\delta_l(p) = \text{atan}[\sqrt{\pi}pL/2Z_{00}(1; (pL/2\pi)^2)]$ if the partial wave l dominates the scattering [3]. This is a favorable case where one equation determines one unknown $\delta_l(E_n)$ for each energy level E_n . The generalizations of this relation to multiple partial waves, non-zero total momenta, twisted boundary conditions, coupled-channel scattering and three-particle systems have also been derived in a series of papers recently. For each energy level E_n this generally leads to one (determinant) equation with several unknown $\delta_l^a(E_n^{cm})$ and the rigorous extraction becomes much more challenging. In this case the analysis may rely on certain parametrizations of the scattering matrix as a function of E^{cm} , which may render otherwise unsolvable problem tractable. It is encouraging that the Hadron Spectrum Collaboration presented the first simulation of two-coupled channel system $K\pi - K\eta$ and extracted the poles corresponding to strange mesons relying on the parametrization of the scattering matrix [4].

The overlaps $Z_i^n = \langle \Omega | O_i | n \rangle$ provide wealth of information about the composition of each lattice eigenstate $|n\rangle$. This information has been used so far mostly as a qualitative guidance on the importance of various Fock components. It remains an open question how to use this rich source of information to rigorously extract physics information on the physical states (especially for smeared quarks) and analytic considerations in this direction may prove fruitful.

3 Mesons well below threshold

Well below strong decay threshold there are no multi-hadron states, and the mass of a single hadron is extracted from $m = E|_{\vec{p}=0}$ extrapolated to $L \rightarrow \infty$, $a \rightarrow 0$ and $m_q \rightarrow m_q^{phys}$. Particular care has to be taken concerning discretization errors related to heavy quarks and complementary methods lead to compatible results in the continuum limit.

Many precision results are available for a number of years. The continuum and chiral extrapolations of low-lying charmonia were, for example, addressed by Fermilab/MILC [5] and HPQCD/MILC [6] collaborations recently. The resulting splittings between ground-state masses in different channels as well as spin-averaged masses of $2S$ and $1S$ charmonia are in good agreement with experiment. In all simulations of charmonia and other hidden charm channels reported here, the charm-quark annihilation contribution is omitted (while possible Wick contractions with $u/d/s$ annihilation are taken into account) and it is indeed OZI suppressed in experiment. The rigorous treatment of charm annihilation

presents an unsolved problem due to the mixing with a number of light hadron channels and the noise in the disconnected diagrams.

The η and η' can strongly decay only to the three-meson states, therefore they are very narrow and can be treated using standard technique to a good approximation. Their masses as well as the flavour mixing angle were determined as a function of m_π by ETMC collaboration [7], recovering experimental values in the chiral limit.

4 Excited mesons within a single-hadron approach

The great majority of hadrons lie near or above strong decay threshold. Yet most of them have been treated until recently based on a single-hadron approximation. For meson states this entails (i) using only quark-antiquark interpolating fields $O \simeq \bar{q}q$ for mesons, (ii) assuming that all energy levels correspond to “one-particle” states and (iii) that the mass of the excited resonance equals $m = E$. These are strong assumptions for the resonances, which are not asymptotic states. The approach also ignores the effect of the threshold on near-threshold states.

The most extensive light isoscalar [8], D , D_s [9] and $\bar{c}c$ [10] spectrum was extracted by the Hadron Spectrum Collaboration (HSC) on $N_f = 2 + 1$ anisotropic configurations with $m_\pi \simeq 400$ MeV. The mixing angle between $(\bar{u}u + \bar{d}d)/\sqrt{2}$ and $\bar{s}s$ components for isoscalar mesons mass was also calculated and the mixing is found small for most of the states [8]. The continuum J^{PC} was reliably identified using advanced spin-identification method. An impressive number of excited states was extracted in each channel with a good accuracy in spite of the disconnected contribution for isoscalars. States are identified with members of $\bar{q}q$ multiplets nS , nP , nD and nF based on overlaps $\langle O_i | n \rangle$, where interpolators are chosen to resemble multiplet members. There are several remaining states which are identified as hybrids: they do not fit $\bar{q}q$ multiplets and show strong overlap with $O \simeq \bar{q}F_{\mu\nu}q$.

5 Near-threshold mesons (beyond single-hadron approach)

Most of the exciting states found by experiments are located near thresholds, for example $X(3872)$, $Z_c^+(3900)$, $Z_b^+(10610)$, $Z_b^+(10650)$, $D_s^0(2317)$ and $\Lambda(1405)$. The quarkonium-like states, which lie near threshold and above threshold, are listed in Tables 10 and 12 of a review by Brambilla et al. [11]. Identifying whether these states arise from QCD or not, and what is their nature, presents an exciting and important challenge to the lattice community.

Indeed most of the effort in the hadron spectroscopy during past few years went in going beyond the single-hadron approximation and taking into account two-hadron eigenstates rigorously. Note that majority of the studies focus on the (elastic) energy region near threshold, where the methods may be tractable at present, but one can not expect spectra of highly excited multiplets from rigorous approach soon.

Z_c^+ : Several charged-charmonia with quark content $\bar{c}c\bar{d}u$ were discovered recently in experiment. Most notably these are $Z^+(4430)$ with $J^P = 1^+$ discovered by Belle and confirmed by LHCb, and $Z_c^+(3900)$ with unknown J^P discovered by BESIII and confirmed by Belle and CLEOc [11]. In is important to note that $Z_c^+(3900)$ was found in $J/\psi \pi$ invariant mass only through $e^+e^- \rightarrow Y(4460) \rightarrow (J/\psi \pi^+) \pi^-$. No resonant structure in $J/\psi \pi^+$ was seen in $\bar{B}^0 \rightarrow (J/\psi \pi^+) K^-$ by BELLE [12], in $\bar{B}^0 \rightarrow (J/\psi \pi^+) \pi^-$ by LHCb [13] or in $\gamma p \rightarrow (J/\psi \pi^+) n$ by COMPASS [14]. This might indicate that the peak seen in $Y(4460)$ decay might not be of dynamical origin [15, 16].

The first search for $Z_c^+(3900)$ on the lattice considered $J/\psi \pi$ and $D\bar{D}^*$ scattering and no Z_c^+ candidate was found [17]. The $D\bar{D}^*$ scattering was considered in [18] and the authors conclude that they do not find the state either.

The most extensive lattice search for Z_c^+ with mass below 4.2 GeV in the channel $I^G(J^{PC}) = 1^+(1^{+-})$ is performed in [19]. The major challenge is presented by the two-meson states $J/\psi \pi$, $\psi_{2S} \pi$, $\psi_{1D} \pi$, $D\bar{D}^*$, $D^*\bar{D}^*$, $\eta_c \rho$ that are inevitably present in this channel in addition to potential Z_c^+ candidates. The spectrum of eigenstates is extracted using a number of meson-meson and diquark-antidiquark interpolating fields. All the expected two-meson states are found but no additional candidate for Z_c^+ [19]. It is also illustrated how a simulation incorporating low-lying two-mesons states seems to render a Z_c^+ candidate [20], which is however not robust after further two-meson states around 4.2 GeV are implemented [19]. It is concluded that the experimental Z_c^+ candidates with $I^G(J^{PC}) = 1^+(1^{+-})$ and a mass below 4.07 GeV are either very broad or most likely not dominated by the $[\bar{c}\bar{d}]_{\bar{3}_c}[cu]_{\bar{3}_c}$ Fock component.

D_{s0}^* , D_{s1} : The quark models expected $D_{s0}^*(2317)$ and $D_{s1}(2460)$ above DK and D^*K thresholds, but they were experimentally found slightly below them. The first lattice QCD simulations that take the effect of these thresholds into account used DK and D^*K interpolating fields in addition to the $\bar{s}c$ [21, 22]. The position of thresholds is almost physical in this $N_f = 2 + 1$ simulation with nearly physical $m_\pi \simeq 156$ MeV. The $D^{(*)}K$ phase shift is extracted from each energy level and then parametrized in the region close to threshold using effective range formula. The large negative scattering length is an indication for the presence of the bound states. The effective range expansion renders the position of the poles in $S \propto (\cot \delta - i)^{-1}$ related to $D_{s0}^*(2317)$ and $D_{s1}(2460)$ close to the experimental masses. The summary of the resulting D_s spectrum for these two states as well as other D_s states is summarized in Fig. 9 of [22] for two values of pion masses.

$X(3872)$: A candidate for the charmonium(like) state $X(3872)$ is found 11 ± 7 MeV below the $D\bar{D}^*$ threshold for $J^{PC} = 1^{++}$, $I=0$, $N_f=2$ and $m_\pi \simeq 266$ MeV [23]. This is the first lattice simulation that establishes a candidate for $X(3872)$ in addition to χ_{c1} and the nearby scattering states $D\bar{D}^*$ and $J/\psi \omega$. The large and negative $a_0 = -1.7 \pm 0.4$ fm for $D\bar{D}^*$ scattering is one indication for a shallow bound state $X(3872)$. The mass of X is determined from the position of the pole in S matrix which is obtained by interpolating $D\bar{D}^*$ scattering phase shift near threshold. The established $X(3872)$ has a large overlap with $\bar{c}c$ as well as $D\bar{D}^*$ interpolating fields [23]. The single-hadron approach using just $\bar{c}c$ interpolators renders only one level near DD^* threshold just like in previous simulations. In this case one can not reliably establish whether this level is related to $X(3872)$ or $D(0)\bar{D}^*(0)$.

In the $I = 1$ channel, only the $D\bar{D}^*$ and $J/\psi \rho$ scattering states are found, and no candidate for $X(3872)$ [23]. This is in agreement with a popular interpretation that $X(3872)$ is dominantly $I = 0$, while its small $I = 1$ component arises solely from the isospin breaking and is therefore absent in the simulation with $m_u = m_d$.

6 Rigorous treatment of hadronic resonances

The rigorous treatment of a resonance in an elastic channel $M_1 M_2$ amounts to determination of the discrete spectrum including two-meson states, determination of the scattering phase shift from each energy level and making a Breit-Wigner type fit of the phase shift as described in Section 2. The only hadron resonance studied in this way until recently is $\pi\pi \rightarrow \rho \rightarrow \pi\pi$, which has been simulated by a number of lattice collaborations until now (see for example [24]). In the following I summarize results for other channels, where only pioneering steps have been made.

K^* , κ , K_0^* and K_2 : K^* mesons and in particular the $K^*(892)$ were frequently addressed in lattice simulations, but always ignoring that the $K^*(892)$ decays strongly. The simulation [25] presents the

first extraction of the masses and widths for the K^* resonances by simulating $K\pi$ scattering in p -wave with $I = 1/2$. A Breit-Wigner fit of the phase renders a $K^*(892)$ resonance mass $m^{lat} = 891 \pm 14$ MeV and the $K^*(892) \rightarrow K\pi$ coupling $g^{lat} = 5.7 \pm 1.6$ compared to the experimental values $m^{exp} \approx 892$ MeV and $g^{exp} = 5.72 \pm 0.06$, where g parametrizes the $K^* \rightarrow K\pi$ width. Mixing of s and p -wave is taken into account when extracting the phase shift around the $K^*(1410)$ and $K_2^*(1430)$ resonances. This gives an estimate of the $K^*(1410)$ resonance mass $m^{lat} = 1.33 \pm 0.02$ GeV compared to $m^{exp} = 1.414 \pm 0.0015$ GeV assuming the experimental $K^*(1410) \rightarrow K\pi$ coupling.

The first simulation of two-coupled channel system $K\pi - K\eta$ was presented just around the time of this meeting [4]. The scattering matrix in complex plane was parametrized and the parameters were extracted using the fit to the finite volume spectrum via Lüscher-type method. The poles corresponding κ and $K^*(892)$ are found below $K\pi$ threshold for the employed $m_\pi \approx 400$ MeV, while $K_0^*(1430)$ and K_2 are found as resonances above threshold.

D_0^* and D_1 : The first rigorous simulation of a hadronic resonance that contains charm quarks addresses the broad scalar $D_0^*(2400)$ and the axial $D_1(2430)$ charmed-light mesons, which appear in $D\pi$ and $D^*\pi$ scattering [26]. The simulation is done for $N_f = 2$ and $m_\pi \approx 266$ MeV. The resonance parameters are obtained using a Breit-Wigner fit to the elastic phase shifts. The resulting $D_0^*(2400)$ mass is 351 ± 21 MeV above the spin-average $\frac{1}{4}(m_D + 3m_{D^*})$, in agreement with the experimental value of 347 ± 29 MeV above. The resulting $D_0^* \rightarrow D\pi$ coupling $g^{lat} = 2.55 \pm 0.21$ GeV is close to the experimental value $g^{exp} \leq 1.92 \pm 0.14$ GeV, where g parametrizes the width $\Gamma \equiv g^2 p^*/s$. The results for $D_1(2430)$ are also found close to the experimental values; these are obtained by appealing to the heavy quark limit, where the neighboring resonance $D_1(2420)$ is narrow.

The charmed scalar meson puzzle wonders why the strange $D_{s0}^*(2317)$ and the non-strange $D_0^*(2400)$ charmed scalar mesons have a mass within 1 MeV of each other experimentally, while one would naively expect a larger splitting $O(m_s)$. The question is whether this near degeneracy is due to the strange meson being pushed down or the non-strange one being pushed up. This puzzle can be addressed by considering the lattice results for $D_0^*(2400)$ [26], which is found as a resonance in $D\pi$, and $D_{s0}^*(2317)$ [21], which is found as a pole below DK . Both masses are found close to experiment. This favors the interpretation that the near degeneracy is a consequence of strange meson being pushed down due to DK threshold. On the other hand, the interpretation that $D_0^*(2400)$ is pushed up due to tetra quark Fock component $\bar{u}\bar{s}sc$ is disfavored since $N_f = 2$ simulation [26] renders its mass close to the experiment without any strange content in valence or sea sectors.

a_1 and b_1 : The light axial-vector resonances $a_1(1260)$ and $b_1(1235)$ are explored for $N_f = 2$ by simulating the corresponding scattering channels $\rho\pi$ and $\omega\pi$ [27]. Interpolating fields $\bar{q}q$ and $\rho\pi$ or $\omega\pi$ are used to extract the s -wave phase shifts for the first time. It is assumed that ρ and ω are stable, which is justified in the energy region of interest for the employed parameters $m_\pi \approx 266$ MeV and $L \approx 2$ fm. A Breit-Wigner fit of the phase shift gives the $a_1(1260)$ resonance mass $m_{a_1}^{res} = 1.435(53)_{(-109)}^{(+109)}$ GeV compared to $m_{a_1}^{exp} = 1.230(40)$ GeV. The a_1 width $\Gamma_{a_1}(s) \equiv g^2 p/s$ is parametrized in terms of the coupling, which results in $g_{a_1\rho\pi} = 1.71(39)$ GeV compared to $g_{a_1\rho\pi}^{exp} = 1.35(30)$ GeV derived from experiment.

7 Conclusions

I have reviewed recent lattice results for conventional and exotic mesons. A number of precise results for states well below strong decay threshold are available for a number of years and there is impressive

agreement with experiment. I reported on the first rigorous simulations aimed at near-threshold states Z_c^+ , $X(3872)$, D_{s0}^* (2317) as well as light, strange and charmed resonances.

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