

Complete next-to-next-to-leading order calculation of $NN \rightarrow NN\pi$ in chiral effective field theory

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Abstract. We present the results of the pion production operator calculated up-to-and-including next-to-next-to-leading order (NNLO) in chiral effective field theory. We include explicit Delta degrees of freedom and demonstrate that they provide essential contribution required to understand neutral pion production data. Analysis of chiral loops at NNLO reveals new mechanisms which are important, but haven't been considered in phenomenological studies so far.

1 Introduction

Understanding of near-threshold pion production is of significant importance since it allows a direct test of chiral effective field theory (EFT), probes nucleon-nucleon dynamics at intermediate energies and provides access to isospin violation in few-nucleon processes (see refs. [1, 2] for review articles). Pion production contributes to many few-nucleon processes as a building block (see examples in figure 1) and can be investigated directly in the inelastic nucleon-nucleon reactions, such as $pp \rightarrow d\pi^+$ and $pp \rightarrow pp\pi^0$.

It has been known since years that neutral pion production in $pp \rightarrow pp\pi^0$ is the most challenging process since the experimental cross-section in this channel is suppressed by more than an order of magnitude, as compared to the charged channels near threshold. For example, at the energy $T_{\text{lab}} = 293.5$ MeV, which is just few MeV above the threshold, the total cross section of charged pion production is $\sigma_{\text{tot}}(pp \rightarrow d\pi^+) \simeq 42 \pm 5 \mu\text{b}$, while the cross section of neutral pion production is only $\sigma_{\text{tot}}(pp \rightarrow pp\pi^0) = 2.8 \pm 0.2 \mu\text{b}$ [3]. This experimental evidence¹ is fully in line with the chiral suppression of the leading production operators in the neutral channel and suggests the important role of higher order effects, especially chiral loops, which are discussed in this Contribution.

Our goal is to make a complete NNLO chiral EFT calculation of pion production operators and identify mechanisms needed to understand neutral pion production.

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¹See Refs. [4–6] for the latest measurements of $pp \rightarrow pp\pi^0$ and Ref. [7] for $pp \rightarrow d\pi^+$. Further data can be found e.g. in Ref. [2].

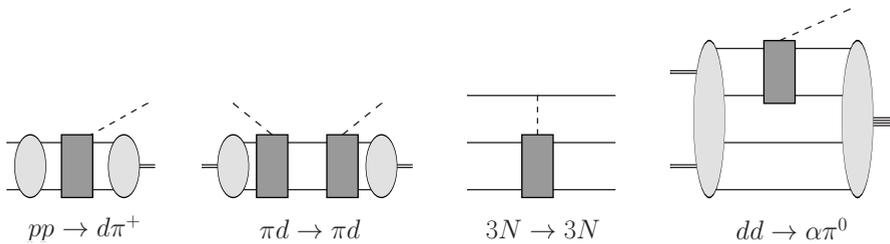


Figure 1. Examples of two-, three- and four-nucleon processes, where pion production operators play important role. Pion production operator is denoted by gray rectangle; single, double and quadruple solid lines correspond to nucleon, deuteron and the alpha particle respectively; dashed lines denote pion; grey ovals represent various few-nucleon interactions.

2 Method

Pion-nucleon dynamics at low energies is significantly constrained by the chiral symmetry of QCD. This allows the chiral EFT to be used to make systematic, QCD-based calculations with controlled uncertainty. The calculation is based on the chiral Lagrangian which includes pions and nucleons as explicit degrees of freedom. The perturbative expansion is performed in terms of the parameter $\chi = p/\Lambda_\chi$, where p corresponds to a typical dynamical scale due to small momenta and small masses in the problem, and $\Lambda_\chi \simeq 1$ GeV is the chiral symmetry breaking scale.

A typical procedure for the calculation of the pion production amplitudes consists of two steps.

1. *A calculation of the irreducible pion production operator* using perturbative chiral EFT and momentum counting scheme (MCS) [8] — a counting scheme which takes into account the initial momentum required to produce a pion.
2. *A calculation of the full amplitude* by making convolution of the irreducible pion production operator with nucleon-nucleon wave functions in the initial and final state. Nucleon-nucleon (NN) interactions are calculated by solving a nonperturbative problem using phenomenological or chiral NN potential [9].

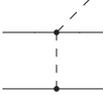
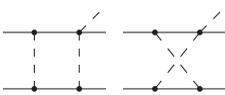
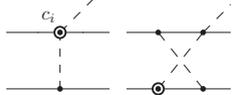
Delta resonance can be potentially important in the pion production process since the delta-nucleon mass difference is comparable to the momentum required to produce the pion. Thus, the explicit inclusion of the delta is important, for it insures that the low-energy constants are of more natural size and thus improves the convergence of the chiral expansion.

3 Pion production up to NNLO

In this section we present operators contributing to the $pp \rightarrow d\pi^+$ and $pp \rightarrow pp\pi^0$ channels up to NNLO MCS. Qualitatively, the contributions of production operators to these two channels are illustrated in table 1 for different chiral orders.

At leading order (LO) there are only several tree-level operators. Rescattering operator (top diagram in LO row) gives the largest contribution to the $pp \rightarrow d\pi^+$ amplitude, however due to isospin structure, its contribution to the $pp \rightarrow pp\pi^0$ is zero. Other LO contributions are dynamically suppressed for both channels. The suppression of the LO terms for neutral pion production qualitatively

Table 1. Diagrams contributing to the pion production operator up to NNLO MCS and a qualitative estimation of their contributions to different pion production channels. The first row shows the MCS order, the second row shows the diagrams which start to contribute at that order, and the last two rows indicate the impact of the diagrams on different pion production channels. Solid (dashed) lines denote nucleons (pions). Solid (circled) dots denote the leading (subleading) vertices from chiral Lagrangian.

	LO	NLO	NNLO
exemplary diagrams			
contrib. to $pp \rightarrow d\pi^+$	big [10]	zero [10]	of natural size, small correction to LO [11, 12]
contrib. to $pp \rightarrow pp\pi^0$	negligible [8, 13]	zero [14]	of natural size, but important contribution [11, 12]

explains why the total cross section for this channel is much smaller than for the charged one. However, the explicit calculations [8, 13] show that the theoretical cross section at LO is much smaller than required by data. Thus, the higher order terms should be considered.

At next-to-leading order (NLO), the one-loop diagrams start to contribute. However the sum of all irreducible NLO loop diagrams vanishes exactly [10, 11, 14]. The cancellation of NLO loops is not unexpected since the loops are divergent individually, however there is no contact term at this order which can absorb the divergency. The vanishing contribution from NLO is the reason why NNLO operators are that important, especially in the neutral channel.

At NNLO, there exist other tree-level and one-loop diagrams which also include subleading vertices. There are also two contact terms which renormalize divergent loops. The first step to study the subleading loop contributions was taken in Ref. [15], however the calculation was not complete.

4 Results and discussion

In Refs. [11, 12], we performed the complete NNLO chiral EFT calculation of the threshold pion production operators. The effect of the delta isobar on the pion production was also considered up to the same order. We have found that most of the NNLO loop-contributions cancel out, however, unlike the NLO case, the cancellation is not complete and there is a finite remainder. The residual non-vanishing contributions of pion-nucleon and delta loops appear to be of a similar size.

The interesting observation is that for $pp \rightarrow d\pi^+$, the contribution of loops with delta diminishes significantly the contributions from pion-nucleon loops. Thus, the total NNLO loop contribution will be quite small. Meanwhile, for the $pp \rightarrow pp\pi^0$ channel, the contribution of delta loops is much smaller than that of pion-nucleon loops, and the total loop-contribution will be important (see table 1).

Both of these observations are in line with the experimental data and the present state of the theory, because for the charged pion production the NLO calculation already describes data quite well, while for neutral pion production the theory underestimates the data significantly.

To conclude, we found that two-pion-exchange operators play important role especially for the neutral pion production. This might question phenomenological models (see Ref. [1] for details), where the agreement with data was achieved mainly using short-range mechanisms.

To make a quantitative statement about the importance of various mechanisms for the total cross section in $NN \rightarrow NN\pi$ one has to calculate the convolution integrals with the NN wave functions. This work is currently in process.

The methods developed in Refs. [11, 12] can be helpful in the study of charge symmetry breaking in $pn \rightarrow d\pi^0$ and $dd \rightarrow \alpha\pi^0$.

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