

Non-perturbative pion dynamics for the X(3872)

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Abstract. We discuss the role of non-perturbative pion dynamics on the near-threshold resonant X(3872) charmonium state, which is assumed to be an S-wave $D\bar{D}^*$ bound system. We calculate the contribution to the width of the X(3872) from the $D\bar{D}\pi$ intermediate state treated non-perturbatively and compare it with different approximate approaches. Further, we explore the quark-mass dependence of the pole position of the X(3872) state. We find that the trajectory of the X(3872) depends strongly on the assumed quark-mass dependence of the short-range interactions which can be determined in lattice QCD calculations.

1 Introduction

After more than a decade after the discovery of the X(3872) by the Belle collaboration its nature still remains an open question, see Ref. [1] for a review. The resonance has the mass $M_X=(3871 \pm 0.17)$ MeV and thus resides very close to the neutral $D\bar{D}^*$ threshold

$$E_B = M_{D^0} + M_{\bar{D}^{*0}} - M_X = (0.12 \pm 0.26) \text{ MeV}. \quad (1)$$

It is therefore natural to assume that it has a large molecular admixture [2], see also Refs. [3, 4].

Recently, the quantum numbers of this state were determined by the LHCb Collaboration to be 1^{++} [5] which is consistent with its interpretation as an S-wave $D^0\bar{D}^{*0}/\bar{D}^0D^{*0}$ bound state, see e.g. [6, 7]. The small binding energy relative to the $D^0\bar{D}^{*0}$ threshold allows for an effective field theory (EFT) formulation of the problem in analogy to the deuteron¹. The pionless EFT framework based on pure contact $D\bar{D}^*$ interactions was first applied to the X(3872) in Ref. [8]. Due to the relevance of other dynamical scales, such a treatment is expected to be valid only in very narrow region around the threshold. In particular, the three-body neutral channel $\pi^0 D^0\bar{D}^0$ opens at the energy 7 MeV below the

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¹Implications of heavy quark and heavy flavour symmetries were utilised in Ref. [10] to predict partner states of the X(3872).

$D^0\bar{D}^{*0}$ threshold while the charged three-body channels $D^\pm D^\mp \pi^0$ and $D^+ \bar{D}^0 \pi^- / D^- D^0 \pi^+$ reside about 2 MeV above it. Furthermore, also the charged two-body channel $D^c \bar{D}^{*c}$ (with $c = \pm$) is located around 8 MeV above the neutral channel. To incorporate the long-range pion physics, the so-called X-EFT was developed in Ref.[9] based on the assumption that pions can be treated perturbatively. Recently, this framework was extended to include higher-order corrections and then used to predict the pion-mass dependence of the X-pole [11]. On the other hand, the perturbative treatment of pions is known to be not applicable in the deuteron channel [12] which shows certain similarities with the X(3872). The role of non-perturbative pions was investigated in many phenomenological studies, see e.g. Refs.[14–16], all of them however include one-pion-exchange (OPE) in the static limit, i.e. under the assumption that the D-mesons are infinitely heavy particles. Meanwhile, the pion in the $D\bar{D}^*$ potential can go on shell and thus the three-body $\pi D\bar{D}$ unitarity cuts should be taken into account².

In this Contribution we discuss effects induced by the non-perturbative pion dynamics on the X(3872) state within the EFT framework, see Refs.[17, 18] for more details. In particular, we test the validity of the static OPE approximation for the partial decay width $X(3872) \rightarrow D\bar{D}\pi$ and study the dependence of the X binding energy on the light quark masses which is a precondition to extract a valuable information about the $D\bar{D}^*$ interactions from upcoming and ongoing lattice simulations.

2 Formalism

We solve a system of coupled-channels Faddeev-type three-body equations for the $D\bar{D}\pi$ system in the $J^{PC} = 1^{++}$ channel

$$\begin{aligned} a_{00}^{m'}(\mathbf{p}, \mathbf{p}', E) &= \lambda_0 V_{00}^{m'}(\mathbf{p}, \mathbf{p}') - \sum_{i=0,c} \lambda_i \int \frac{d^3k}{\Delta_i(k)} V_{0i}^{nm}(\mathbf{p}, \mathbf{k}) a_{i0}^{m'}(\mathbf{k}, \mathbf{p}', E), \\ a_{c0}^{m'}(\mathbf{p}, \mathbf{p}', E) &= \lambda_c V_{c0}^{m'}(\mathbf{p}, \mathbf{p}') - \sum_{i=0,c} \lambda_i \int \frac{d^3k}{\Delta_i(k)} V_{ci}^{nm}(\mathbf{p}, \mathbf{k}) a_{i0}^{m'}(\mathbf{k}, \mathbf{p}', E), \end{aligned} \quad (2)$$

where λ_i stand for the known isospin coefficients and the OPE potential containing the three-body propagator at leading order reads³

$$V^{m'}(\mathbf{p}, \mathbf{p}') = -g^2 \frac{\mathbf{p}'_n \mathbf{p}_{n'}}{2m + m_\pi + \frac{p^2}{2m} + \frac{p'^2}{2m} + \frac{(\mathbf{p}+\mathbf{p}')^2}{2m_\pi} - M - i0}. \quad (3)$$

Here, the indices n, n' are contracted with the corresponding indices of the D^* polarisation vectors, m, m_* and m_π stand for the D, D^* and the pion mass, in order, and the energy E is defined relative to the neutral two-body threshold $M = m_{*0} + m_0 + E$. Furthermore, the strength of the potential g is extracted from the decay width $D^* \rightarrow D\pi$, see e.g. Ref. [18] for a more extended discussion of the input quantities. The OPE potential (3) connects the four D -meson channels defined as $|0\rangle = D^0\bar{D}^{*0}$, $|\bar{0}\rangle = \bar{D}^0 D^{*0}$, $|c\rangle = D^+ D^{*-}$, $|\bar{c}\rangle = D^- D^{*+}$, and the amplitude $a_0 = (a_{00} - a_{c0})/2$ contains the relevant information about the X-pole. Note that the same three-body cut is also taken into account in the $D\bar{D}^*$ propagators Δ_i due to dressing D^* by the self-energy (πD) loops.

It should be stressed that in full analogy to the NN problem [19, 20], the OPE does not fall off at large momenta and thus requires renormalisation. The $D\bar{D}^*$ potential in an S-wave (V^{SS}) is to be modified to include the contact interaction C_0

$$V^{SS}(p, p') \rightarrow C_0 + V^{SS}(p, p'). \quad (4)$$

²It is shown in Ref. [13] that the 3-body unitary cuts play the crucial role in the $D_\alpha \bar{D}_\beta$ system, if the D_β width is dominated by the S-wave $D_\beta \rightarrow D_\alpha \pi$ decay.

³In Ref. [21] the role of relativistic corrections in the non-perturbative approach including 3-body effects was addressed.

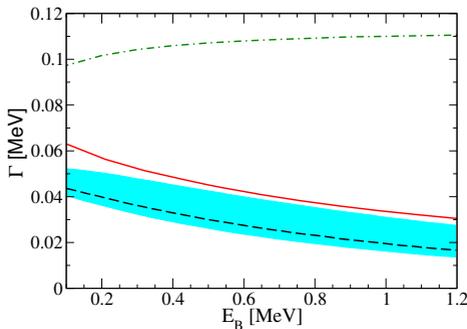


Figure 1. The X width as a function of the binding energy E_B for several different calculations: (i) solution of the problem with the non-perturbative OPE in the static limit — (green) dot-dashed line; (ii) solution of the full dynamical problem with non-perturbative OPE — (red) solid line; (iii) the X-EFT calculations: LO [6] – the dashed line and NLO [9] – the (blue) band.

For the sharp cut-off regularisation scheme used in our calculation $C_0(\Lambda)$ is adjusted to produce a bound state of the $X(3872)$ for any given cut-off Λ .

In order to analyse the light quark-mass dependence we allow all quantities such as the D and D^* -meson masses, the coupling constant and the pion decay constant to vary with m_π , i.e. we perform an expansion of all such quantities in terms of the parameter $\delta m_\pi/M$ [18], where the small scale is the difference of the running and physical pion masses $\delta m_\pi = m_\pi - m_\pi^{\text{ph}}$, while the large scale M is given by a typical hadronic scale ~ 1 GeV. In addition to OPE, also the contact term has to vary with m_π to ensure that the binding energy $E_B(m_\pi)$ is approximately Λ -independent for the running pion mass. Assuming that the leading correction to the the physical-limit quantity $C_0^{\text{ph}}(\Lambda)$ is analytic with the quark masses, we may write

$$C_0(\Lambda, m_\pi) = C_0^{\text{ph}} + \delta C_0 = C_0^{\text{ph}}(\Lambda) \left(1 + f(\Lambda) \frac{m_\pi^2 - m_\pi^{\text{ph}2}}{M^2} \right). \quad (5)$$

The leading Λ -dependence of the contact interaction is captured by $C_0^{\text{ph}}(\Lambda)$, while the dimensionless function $f(\Lambda)$ absorbs the extra Λ -dependence which appears for values of the pion mass away from the physical point. Therefore, we fix the Λ -dependence of the contact interaction requiring that both the binding energy E_B as well as its slope at the physical point, $(\partial E_B / \partial m_\pi)|_{m_\pi=m_\pi^{\text{ph}}}$, are Λ -independent.

3 Discussion and conclusions

First, we discuss the impact of non-perturbative pions on the decay width $X \rightarrow D\bar{D}\pi$ [17], as shown in Fig. 1. We find that the perturbative inclusion of pions is justified, while the static approximation with non-perturbative pions leads to a significant overestimation of this observable. Thus, we conclude that the appropriate treatment of the three-body dynamics is mandatory.

The pion mass dependence of the binding energy of the $X(3872)$ is illustrated in Fig. 2. The trajectory of the $X(3872)$ depends strongly on the assumed quark-mass dependence of the short-range interactions which can be parametrized by the slope $(\partial E_B / \partial m_\pi)|_{m_\pi=m_\pi^{\text{phys}}}$ and is, in principle, measurable in lattice QCD. This is demonstrated in Fig. 2 where the resulting pion mass dependence is shown for two different values of the slope of a natural size. The sizeable difference between the pion-full and pion-less approaches at higher values of the pion mass for positive values of the slope indicates the important role of pion dynamics in this scenario, see also Ref. [11] for an analogous study within the X-EFT⁴. These findings will be useful for chiral extrapolations of the future lattice-QCD

⁴Our results are in conflict with those of Ref. [22], the differences however can be traced back to conceptual problems of the approach used there, see Ref. [23] for a detailed discussion.

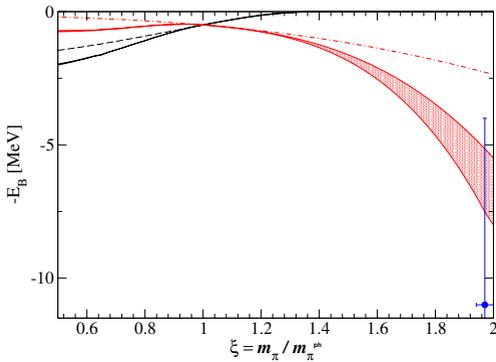


Figure 2. Pion mass dependence of the X(3872) binding energy. The red filled band corresponds to the positive slope $(\partial E_B / \partial m_\pi) \big|_{m_\pi = m_\pi^{\text{ph}}} = 0.7 \times 10^{-2}$ while the black filled band corresponds to the negative slope $(\partial E_B / \partial m_\pi) \big|_{m_\pi = m_\pi^{\text{ph}}} = -1.5 \times 10^{-2}$. In both cases, the ultraviolet cutoff in the integral equations is varied in the range $\Lambda \in [400, 700]$ MeV. The dashed and dash-dotted curves represent the corresponding results of the pionless approach. The blue point depicts the first lattice calculation of the X(3872) [24].

results for the X(3872) binding energy (see Ref. [24] for the first results) and will provide insights into its binding mechanism once the value of the slope parameter is determined.

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