

# Compound nucleus formation probability $P_{CN}$ defined within the dynamical cluster-decay model

Sahila Chopra<sup>1,a</sup>, Arshdeep Kaur<sup>1,b</sup>, and Raj K. Gupta<sup>1,c</sup>

<sup>1</sup>Department of Physics, Panjab University, Chandigarh-160014, India

**Abstract.** With in the dynamical cluster-decay model (DCM), the compound nucleus fusion/ formation probability  $P_{CN}$  is defined for the first time, and its variation with CN excitation energy  $E^*$  and fissility parameter  $\chi$  is studied. In DCM, the (total) fusion cross section  $\sigma_{fusion}$  is sum of the compound nucleus (CN) and non-compound nucleus (nCN) decay processes, each calculated as the dynamical fragmentation process. The CN cross section  $\sigma_{CN}$  is constituted of the evaporation residues (ER) and fusion-fission (ff), including the intermediate mass fragments (IMFs), each calculated for all contributing decay fragments ( $A_1, A_2$ ) in terms of their formation and barrier penetration probabilities  $P_0$  and  $P$ . The nCN cross section  $\sigma_{nCN}$  is determined as the quasi-fission (qf) process where  $P_0=1$  and  $P$  is calculated for the entrance channel nuclei. The calculations are presented for six different target-projectile combinations of CN mass  $A \sim 100$  to superheavy, at various different center-of-mass energies with effects of deformations and orientations of nuclei included in it. Interesting results are that the  $P_{CN}=1$  for complete fusion, but  $P_{CN} < 1$  or  $\ll 1$  due to the nCN contribution, depending strongly on both  $E^*$  and  $\chi$ .

## 1 Introduction

The compound nucleus formation/ fusion probability  $P_{CN}$  is the least understood, but an important quantity in the study of heavy ion reactions. N. Bohr [1] in his compound nucleus (CN) model assumed  $P_{CN}=1$  for complete fusion, and treated the CN decay statistically. However, non-compound nucleus (nCN) decays are also observed, which means  $P_{CN} < 1$ , and hence Bohr's CN-model needs an extension. Some attempts (see, e.g., [2–4]) were made to use  $P_{CN}$  in determining the evaporation residue cross section  $\sigma_{ER}$ , given as the product of the capture cross section  $\sigma_{capture}$ , the CN formation probability  $P_{CN}$  and the survival probability  $W_{sur}$ , each term treated and calculated separately [3]. In this contribution, we introduce for the first time the definition of  $P_{CN}$  in to the dynamical cluster-decay model (DCM) of Gupta and collaborators [5–13], applied to reactions having non-zero nCN contribution.

Heavy-ion fusion reactions have received great attention in recent years, and this is an important and exciting field of research for nuclear physics. The advent of heavy ion reactions has facilitated to investigate the production and reaction mechanism of new heavy and super-heavy nuclei via fusion reactions. Heavy-ion reactions at below barrier energies give rise to highly excited compound nuclear systems that carry large angular momentum  $\ell$ , and hence decay by emitting multiple light particles (LPs:  $A \leq 4, Z \leq 2$ , like n, p,  $\alpha$ ) or their heavier counter-

parts, and  $\gamma$ -rays, called the evaporation residue (ER), and fusion-fission (ff) consisting of near-symmetric and symmetric fission fragments (nSF and SF), including also the intermediate mass fragments (IMFs) of masses  $5 \leq A \leq 20, 2 < Z < 10$ . In addition, many a times one of the several non-compound nucleus (nCN) decay process, like the quasi-fission (qf), deep-inelastic collisions, incomplete fusion or pre-equilibrium decay also contribute to the overall cross section. The cross section for such a CN decay is called the CN decay/ production cross-section, or simply the (total) fusion cross-section  $\sigma_{fusion}$ , given as

$$\begin{aligned}\sigma_{fusion} &= \sigma_{CN} + \sigma_{nCN} \\ &= \sigma_{ER} + \sigma_{ff} + \sigma_{nCN}\end{aligned}\quad (1)$$

In a fission-less decay, the contribution of  $\sigma_{IMFs}$ , that forms a part of  $\sigma_{ff}$ , is in general small, of the order of 5 to 10% of  $\sigma_{ER}$ , i.e.,  $\sigma_{IMFs} \approx 5 - 10\% \sigma_{ER}$ . Note that all these components of fusion cross section  $\sigma_{fusion}$  are individually measurable quantities. In case, the nCN component  $\sigma_{nCN}$  were not measured, it can be estimated empirically from the calculated and measured quantities, as

$$\sigma_{nCN} = \sigma_{fusion}^{Expt.} - \sigma_{fusion}^{Cal.}\quad (2)$$

It may be pointed out that different mass regions of compound nuclei constitute different combinations of these processes (ER, IMFs, ff and nCN) or a single one of them as the dominant mode. In the language of coupled channel calculations, in literature [2]  $\sigma_{fusion}$  is referred to as  $\sigma_{capture}$ , if calculated as "barrier crossing". However, we shall see that in DCM there is an additional fragment pre-formation factor  $P_0$ .

<sup>a</sup>e-mail: chopra.sahila@gmail.com

<sup>b</sup>e-mail: arshdeep.pu@gmail.com

<sup>c</sup>e-mail: rajkgupta.chd@gmail.com

In this paper, within the dynamical cluster-decay model (DCM), we attempt to define the CN formation probability  $P_{CN}$  and understand the role of  $\sigma_{nCN}$  component in  $\sigma_{fusion}$ . A brief description of the DCM is given in Sect. 2. Our calculations for the formation probability  $P_{CN}$ , using DCM, are given in Sect. 3. A summary and conclusions of our work are presented in Sect. 4.

## 2 The Dynamical cluster-decay model (DCM)

The DCM, based on the quantum mechanical fragmentation theory (QMFT), is worked out in terms of the collective coordinates of mass [and charge] asymmetries  $\eta = (A_1 - A_2)/(A_1 + A_2)$  [and  $\eta_Z = (Z_1 - Z_2)/(Z_1 + Z_2)$ ], and relative separation  $R$ , with multipole deformations  $\beta_{\lambda i}$  ( $\lambda=2,3,4$ ;  $i=1,2$ ), and orientations  $\theta_i$ . In terms of these coordinates, for each fragmentation ( $A_1, A_2$ ), we define the compound nucleus decay/formation cross section for  $\ell$  partial waves as

$$\sigma = \frac{\pi}{k^2} \sum_{\ell=0}^{\ell_{max}} (2\ell + 1) P_0 P; \quad k = \sqrt{\frac{2\mu E_{c.m.}}{\hbar^2}} \quad (3)$$

with  $P_0$  as the preformation probability, referring to  $\eta$ -motion, and  $P$ , the penetrability, to  $R$ -motion.  $\mu = mA_1A_2/(A_1 + A_2)$  is the reduced mass and  $\ell_{max}$ , the maximum angular momentum, is defined for LPs evaporation residue cross-section  $\sigma_{ER} \rightarrow 0$ .

The penetrability  $P$  is the WKB integral,

$$P = \exp\left(-\frac{2}{\hbar} \int_{R_a}^{R_b} \{2\mu[V(R, T) - Q_{eff}]\}^{1/2} dR\right), \quad (4)$$

with first and second turning points,  $R_a$  and  $R_b$ , satisfying

$$\begin{aligned} R_a(T) &= R_1(\alpha_1, T) + R_2(\alpha_2, T) + \Delta R(\eta, T), \\ &= R_i(\alpha, \eta, T) + \Delta R(\eta, T), \end{aligned} \quad (5)$$

and

$$V(R_a) = V(R_b) = Q_{eff}. \quad (6)$$

The choice of parameter  $R_a$  (equivalently,  $\Delta R$ ) in Eq. (5), for a best fit to the data, allows us to relate in a simple way the  $V(R_a, \ell)$  to the top of the barrier  $V_B(\ell)$  for each  $\ell$ .

In Eq. (3),  $P_0$ , the preformation probability referring to  $\eta$  motion, contains the structure information of the compound nucleus, and is the solution of the stationary Schrödinger equation in  $\eta$ , at a fixed  $R=R_a$ ,

$$\left\{ -\frac{\hbar^2}{2\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} \frac{1}{\sqrt{B_{\eta\eta}}} \frac{\partial}{\partial \eta} + V(R, \eta, T) \right\} \psi^\nu(\eta) = E^\nu \psi^\nu(\eta), \quad (7)$$

with

$$P_0(A_i) = |\psi(\eta(A_i))|^2 \sqrt{B_{\eta\eta}} \frac{2}{A}, \quad (8)$$

Here, the mass fragmentation potential  $V(\eta, T)$ , at fixed  $R = R_a$ , is the sum of liquid drop energy  $V_{LDM}$ , shell corrections, Coulomb, nuclear proximity and angular momentum dependent potentials  $V_C, V_P, V_\ell$  for deformed,

oriented co-planer nuclei, all T-dependent. The mass parameters  $B_{\eta\eta}$ , defining the kinetic energy term in Hamiltonian, are the smooth classical hydrodynamical masses.

The same equation (3) is used for  $\sigma_{nCN}$ , calculated as the quasi-fission (qf) process, since incoming nuclei keep their identity, and hence  $P_0=1$ , and then  $P$  is calculated for *incoming channel*.

Then, calculating  $\sigma$  for both the CN (=ER+ff) and nCN (=qf) processes, we define  $P_{CN}$  as the ratio of CN formation cross-section  $\sigma_{CN}$  and the total fusion cross-section  $\sigma_{fusion}$  which includes the non-compound nucleus component  $\sigma_{nCN}$ ,

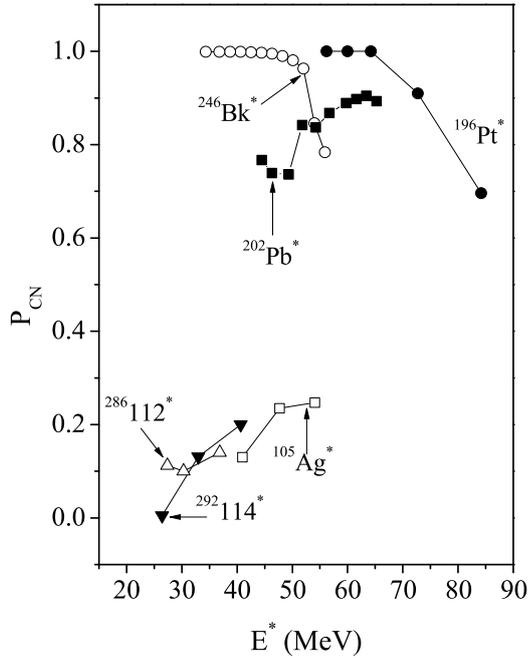
$$P_{CN} = \frac{\sigma_{CN}}{\sigma_{fusion}} = 1 - \frac{\sigma_{nCN}}{\sigma_{fusion}} \quad (9)$$

Apparently,  $P_{CN}$  gives the content of CN formation in the total fusion cross section.

## 3 Calculations and results

In this section, we present our calculations for the compound nucleus formation probability  $P_{CN}$  calculated by using the DCM. The non-compound nucleus component  $\sigma_{nCN}$  is calculated as the quasi-fission (qf) process,  $\sigma_{qf}$ . We consider a set of "hot" fusion reactions with different entrance channels leading to different compound nuclei, at various center of mass energies. Specifically, we have considered the possible decay processes of the six DCM studied compound nuclei, formed in  $^{12}\text{C}+^{93}\text{Nb} \rightarrow ^{105}\text{Ag}^*$  [13],  $^{11}\text{B}+^{238}\text{U} \rightarrow ^{246}\text{Bk}^*$  [5],  $^{64}\text{Ni}+^{132}\text{Sn} \rightarrow ^{196}\text{Pt}^*$  [11], and the  $^{48}\text{Ca}$ -based reactions  $^{48}\text{Ca}+^{154}\text{Sm} \rightarrow ^{202}\text{Pb}^*$  [7],  $^{48}\text{Ca}+^{238}\text{U} \rightarrow ^{286}112^*$  [10], and  $^{48}\text{Ca}+^{244}\text{Pu} \rightarrow ^{292}114^*$  [8] at different center-of-mass energies  $E_{c.m.}$ . The possible role of deformed configurations is also included in these works on DCM, except for the  $^{48}\text{Ca}+^{154}\text{Sm}$  reaction where only spherical nuclei are considered. The deformation effects in DCM are taken up to hexadecapole deformations ( $\beta_{2i}, \beta_{3i}, \beta_{4i}$ ) with compact orientations  $\theta_{ci}, i=1, 2$  [14], or up to only quadrupole deformation ( $\beta_{2i}$ ) with optimum orientations  $\theta_i^{opt}$  [15], of the "hot" fusion process, for the case of co-planer (azimuthal angle  $\Phi=0^0$ ) nuclei. For prolate-deformed nuclei, in a compact hot fusion process, the barrier is highest and interaction radius smallest in a belly-to-belly configuration (for  $\Phi=0^0$ ) or equator-cross (for  $\Phi \neq 0^0$ ). If one of the nucleus is spherical, the configuration is termed equatorial compact. The presence of large positive hexadecapole deformation ( $\beta_4 \gg 0$ ), together with  $\beta_2 > 0$ , result in a not-belly-to-belly compact or not-equatorial compact configuration, which differ from belly-to-belly or equatorial compact by as much as  $20^0$ . The same is true for oblate-deformed nuclei [16].

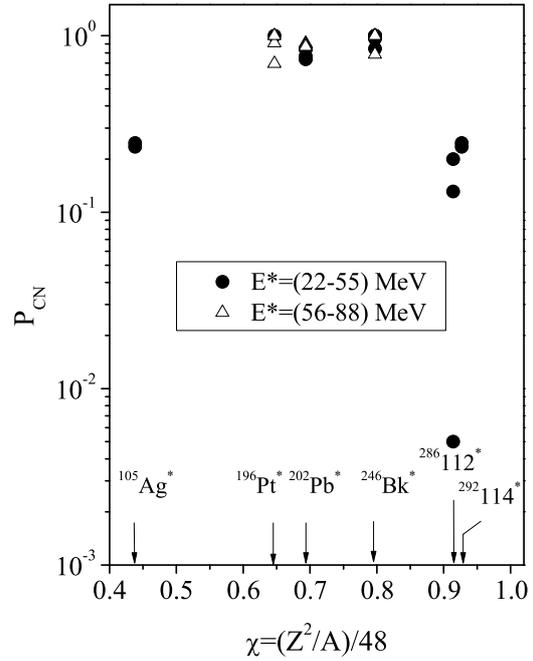
Knowing that the compound nucleus fusion/formation probability  $P_{CN}$  is not yet investigated in detail, it is of interest to see its variation with center-of-mass energy  $E_{c.m.}$ , CN charge  $Z_{CN}$ , its excitation energy  $E^*$ , the fissility parameter  $\chi = (Z^2/A)/48$ , the reaction entrance channels in terms of quantities such as the product  $Z_1Z_2$ , etc. We present our results for  $P_{CN}$  as a function of  $E^*$ , and the fissility  $\chi$ .



**Figure 1.** Variation of  $P_{CN}$  with compound nucleus excitation energy  $E^*$ . The reactions used for the compound systems formed are given in the text.

Fig. 1 shows the variation of  $P_{CN}$  with CN excitation energy  $E^*$  for the chosen six compound systems. We notice for the first time, a few interesting results in this figure: (i) The compound systems form two groups, one consisting of  $^{246}\text{Bk}^*$  and  $^{196}\text{Pt}^*$  showing  $P_{CN}=1$  for lower excitation energies, which decreases continuously as the excitation energy  $E^*$  increases. This means that the content of CN decreases, or equivalently, the nCN content increases with increase of  $E^*$ . (ii) The other group, consisting of  $^{105}\text{Ag}^*$ ,  $^{202}\text{Pb}^*$ ,  $^{286}112^*$  and  $^{292}114^*$ , on the other hand, show a reverse behaviour of increasing  $P_{CN}$  with increase of  $E^*$ . Interestingly,  $^{105}\text{Ag}^*$  shows a behaviour similar to superheavy systems  $^{286}112^*$  and  $^{292}114^*$ , all having  $P_{CN} \ll 1$ . This happens because  $^{105}\text{Ag}^*$  is shown [13] to contain a large nCN component and the two superheavy systems are known to decay dominantly via the qf process. For  $^{202}\text{Pb}^*$ , however, though  $P_{CN} < 1$  at lower  $E^*$  values, but approaches  $P_{CN}=1$  at higher  $E^*$ . Apparently, it will be interesting to extend the cases of  $P_{CN} \ll 1$  to higher energies and those of  $P_{CN} \approx 1$  to both lower and higher energies.

Fig. 2 shows the variation of  $P_{CN}$  with the fissility parameter  $\chi (=Z^2/A)/48$  for all the six CN  $^{105}\text{Ag}^*$ ,  $^{196}\text{Pt}^*$ ,  $^{202}\text{Pb}^*$ ,  $^{246}\text{Bk}^*$ ,  $^{286}112^*$  and  $^{292}114^*$  marked in the body of the figure, at various excitation energies  $E^*$  (see, [5, 7, 8, 10, 11, 13] for details), forming two energy groups of ranges  $E^*=22-55$  and  $56-88$  MeV. We notice in Fig. 2 that for the high energy range ( $E^*=56-88$  MeV; open triangles),  $P_{CN} \approx 1$  for all the systems with  $\chi$  lying between 0.6 and 0.8. On the other hand, for the low energy range ( $E^*=22-55$  MeV, filled circles),  $P_{CN}$  varies from  $\sim 0.25$  to almost zero (0.005) via nearly unity as  $\chi$  increases from



**Figure 2.** Variation of  $P_{CN}$  with fissility parameter  $\chi$ . The reactions used for the marked compound systems are given in the text.

0.45-0.9. Thus, for CN having  $\chi=0.6-0.8$ , the  $P_{CN}=1$  but for the superheavy systems, like  $^{286}112$  and  $^{292}114$  with higher  $\chi (=0.914$  and  $0.927)$ , the  $P_{CN} \ll 1$ , indicating the presence of large nCN effects. Similar to the cases of superheavy systems,  $P_{CN} \ll 1$  for very low (almost half)  $\chi (=0.44)$  case of CN  $^{105}\text{Ag}^*$ . Interestingly, a similar behaviour is observed in other published works [2], except for the case of  $P_{CN} \ll 1$  for low  $\chi$  compound systems.

## 4 Summary and Conclusions

Concluding, the compound nucleus (CN) fusion/formation probability  $P_{CN}$  is defined for the first time on the basis of the dynamical cluster-decay model (DCM) where the fusion cross section  $\sigma_{fusion}$  is calculated as the dynamical fragmentation process. The fusion cross section  $\sigma_{fusion}$ , taken as the sum of CN formation cross section  $\sigma_{CN}$  and the possible non-compound nucleus (nCN) contribution  $\sigma_{nCN}$ , is calculated for each contributing fragmentation ( $A_1, A_2$ ) in terms of its formation and barrier penetration probabilities  $P_0$  and  $P$ . The compound nucleus decay cross section  $\sigma_{CN}$  is the sum of cross sections due to the evaporation residues (ER) and fusion-fission (ff) processes, where ER is made up of light particles  $A_2 \leq 4$  (plus the complementary heavy fragments) and the ff are the near-symmetric and symmetric ( $A_1 = A_2 = A/2$ ) fragments, including the IMF ( $5 \leq A_2 \leq 20, 2 < Z_2 < 10$ ). On the other hand, the non-compound nucleus decay cross section  $\sigma_{nCN}$  is determined as the quasi-fission (qf) process where the incoming nuclei do not lose their identity, and hence  $P_0=1$  with  $P$  calculated for incoming channel.

The DCM is applied to some six "hot" fusion reactions at various incident energies, covering the mass region from  $A \sim 100$  to superheavy nuclei. The  $P_{CN}$  is calculated and its variation with CN excitation energy  $E^*$  and fissility parameter  $\chi$  studied. The interesting result is that for some compound systems  $P_{CN}=1$  at lower  $E^*$  values but decreases (nCN component increases) as  $E^*$  increases, whereas for other compound systems the variation of  $P_{CN}$  with  $E^*$  is reversed, i.e.,  $P_{CN} \ll 1$  at lower  $E^*$  values but it increases as  $E^*$  increases. Variation of  $P_{CN}$  with  $\chi$  is also interesting in that it is almost unity for systems with  $\chi=0.6-0.8$ , but is  $\ll 1$  for systems with very high or very low  $\chi$  values.

It will be interesting to extend these calculations to more and more reactions, and see if the above noted trends are kept the same or some new trends are added. Some authors [4] have even given a mathematical formulations of such trends based on a few "cold" fusion reactions only, hoping for their universality. Further studies are needed with in the DCM formulation.

## Acknowledgement

Work supported partially by the Department of Science & Technology (DST), Govt. of India. A part of the computation work was performed on the CAS-FIST-PURSE High Performance Computing Cluster (HPCC) at the Physics Department, Panjab University, Chandigarh (India).

## References

- [1] N. Bohr, Nature (London) **137**, 344 (1936).
- [2] R. Yanez, W. Loveland, J. S. Barrett, L. Yao, B. B. Back, S. Zhu, and T. L. Khoo, Phys. Rev. C **88**, 014606 (2013).
- [3] V. I. Zagrebaev, Y. Aritomo, M. G. Itkis, Yu. Ts. Oganessian, and M. Ohta, Phys. Rev. C **65** (2001) 014607.
- [4] V. I. Zagrebaev and W. Greiner, Phys. Rev. C **78**, 034610 (2008).
- [5] B. B. Singh, M. K. Sharma, and R. K. Gupta, Phys. Rev. C **77**, 054613 (2008).
- [6] S. K. Arun, R. Kumar, and R. K. Gupta, J. Phys. G: Nucl. Part. Phys. **36**, 085105 (2009).
- [7] S. Kanwar, M. K. Sharma, B. B. Singh, R. K. Gupta, and W. Greiner, Int. J Mod. Phys. E **18**, 1453 (2009).
- [8] R. K. Gupta, Niyti, M. Manhas, S. Hofmann, and W. Greiner, Int. J Mod. Phys. E **18**, 601 (2009).
- [9] R. K. Gupta, Lecture Notes in Physics 818 *Clusters in Nuclei*, ed C. Beck, Vol.I, (Springer Verlag, Berlin Heidelberg 2010), p223; and earlier references there in it.
- [10] Niyti, R. K. Gupta, and W. Griner, J. Phys. G: Nucl. Part. Phys. **37**, 115103 (2010).
- [11] M. K. Sharma, S. Kanwar, G. Sawhney, R. K. Gupta, and W. Greiner, J. Phys. G: Nucl. Part. Phys. **38**, 055104 (2011).
- [12] M. Bansal, S. Chopra, R. K. Gupta, R. Kumar, and M. K. Sharma, Phys. Rev. C **86**, 034604 (2012).
- [13] S. Chopra, M. Bansal, M. K. Sharma, and R. K. Gupta, Phys. Rev. C **88**, 014615 (2013).
- [14] R. K. Gupta, M. Manhas, and W. Greiner, Phys. Rev. C **73**, 054307 (2006).
- [15] R. K. Gupta, M. Balasubramaniam, R. Kumar, N. Singh, M. Manhas, and W. Greiner, J. Phys. G: Nucl. Part. Phys. **31**, 631 (2005).
- [16] M. Bansal and R. K. Gupta, Romanian J. Phys. **57**, 18 (2012).