

Spectroscopy of higher bottomonia

J. Ferretti^{1,a}

¹*Dipartimento di Fisica and INFN, Università di Roma "Sapienza", Piazzale A. Moro 5, I-00185 Roma, Italy*

Abstract. In this contribution, we discuss our recent unquenched quark model results for the spectrum of $b\bar{b}$ mesons with self energy corrections, due to the coupling to the meson-meson continuum. Our unquenched quark model predictions for the masses of the recently discovered $\chi_b(3P)$ states are compared to those of a re-fit of Godfrey and Isgur's relativized quark model to the most recent experimental data. The possible importance of continuum effects in the $\chi_b(3P)$ states is discussed. Finally, we show our quark model results for the radiative decays of the $\chi_b(3P)$ system and the open-bottom decays of $b\bar{b}$ mesons.

1 Introduction. An unquenched quark model for mesons

In the UQM for hadrons [1–7], the effects of $q\bar{q}$ sea pairs are introduced explicitly into the quark model (QM) via a QCD-inspired 3P_0 pair-creation mechanism [8]. The $q\bar{q}$ pairs have vacuum quantum numbers and their effect on hadron observables can be added as a perturbation. This approach is a generalization of the unitarized quark model by Törnqvist and Zenczykowski [9] and was also motivated by the work of Geiger and Isgur [10].

In the UQM for mesons [4–7], the meson wave function,

$$|\psi_A\rangle = \mathcal{N} \left[|A\rangle + \sum_{BCJ} \int d\vec{q} |BC\vec{q}\ell J\rangle \frac{\langle BC\vec{q}\ell J | T^\dagger | A\rangle}{E_a - E_b - E_c} \right], \quad (1)$$

is written as the superposition of a zeroth order quark-antiquark configuration and a sum over the possible higher Fock components, due to the creation of 3P_0 $q\bar{q}$ pairs. Above threshold, this coupling to the continuum leads to open-flavor strong decays; below threshold, it leads to virtual $q\bar{q} - q\bar{q}$ components in the meson wave function and a shift of the physical meson mass with respect to its bare mass, namely a self-energy correction. Something similar was also done in the baryon sector [1–3], with the intention to extend the range of applicability of the baryon QM in its various forms [11–20].

In Ref. [4], the UQM formalism was used to calculate the mass shifts of $1S$, $2S$ and $1P$ bottomonium states due to the coupling to the meson-meson continuum. This was just a preliminary study, in which the bare energies were taken as free parameters and their sum with the self-energies fitted to the physical masses of the mesons of interest. There, we also calculated the probability to find

the wave function of $1S$, $2S$ and $1P$ bottomonium states in the continuum component. The results vary between 2% and 6% and their smallness is due to the distance between these low-lying states and the first open-bottom decay thresholds.

In Ref. [5], the UQM was used to calculate the charmonium spectrum ($1S$, $2S$, $1P$, $2P$ and $1D$ states) with self-energy corrections. Unlike Ref. [4], the bare energies were calculated explicitly within a potential model [21], thus extending the predictive power of the UQM. Our results were used to discuss the nature of the $X(3872)$ [22], a meson characterized by $J^{PC} = 1^{++}$ quantum numbers [23], a very narrow width (less than 1.2 MeV [24]), and a mass 50 – 100 MeV lower than QM predictions. According to our study, the $X(3872)$ can be interpreted as a $c\bar{c}$ core plus higher Fock components due to the coupling to the meson-meson continuum. The problem of the mass difference between QM predictions [21, 25, 26] and the experimental data for the $X(3872)$ was solved thanks to a downward mass shift, due to self-energy corrections. See also Ref. [6].

In Ref. [7], this calculation was extended to bottomonia and used to provide predictions for $1S$, $2S$, $3S$, $1P$, $2P$, $3P$ and $1D$ resonances; the open-bottom decays of $b\bar{b}$ states were calculated too within the 3P_0 pair-creation model [8]. Analogously to what happens for the $X(3872)$ in the $c\bar{c}$ sector, threshold effects may play an important role in the $\chi_b(3P)$ system, discovered by ATLAS in 2012 [27, 28], because $\chi_b(3P)$ mesons are very close to the first open-bottom decay thresholds. Thus, to distinguish between QM and UQM interpretations for these mesons, our UQM results for the mass barycenter of the $\chi_b(3P)$ system were compared to those of a re-fit of Godfrey and Isgur's model mass formula to the most recent experimental data. In Ref. [6], the radiative transitions of $\chi_b(3P)$ mesons were calculated too.

In this contribution, we briefly analyze the results of Refs. [6, 7] for the bottomonium spectrum, the open-

^ae-mail: ferretti@ge.infn.it

Table 1. Mass barycenters of $\chi_b(1P)$, $\chi_b(2P)$ and $\chi_b(3P)$ systems (column 1) and mass splittings between the members of the $\chi_b(1P)$, $\chi_b(2P)$ and $\chi_b(3P)$ multiplets (column 2 and 3) in the UQM. See Ref. [7]. The results are expressed in MeV. The notation $\Delta M_{21}(1P)$ stands for the mass difference between the $\chi_{b2}(1P)$ and $\chi_{b1}(1P)$ resonances, $\Delta M_{10}(1P)$ for the mass difference between the $\chi_{b1}(1P)$ and $\chi_{b0}(1P)$ resonances, and so on.

$M_{\chi_b(1P)}^{\text{th}}$	$\Delta M_{21}(1P)$	$\Delta M_{10}(1P)$
9876	21	30
$M_{\chi_b(2P)}^{\text{th}}$	$\Delta M_{21}(2P)$	$\Delta M_{10}(2P)$
10242	13	18
$M_{\chi_b(3P)}^{\text{th}}$	$\Delta M_{21}(3P)$	$\Delta M_{10}(3P)$
10551	-2	85

bottom decays and the radiative transitions of $\chi_b(3P)$ mesons. These results are then used to discuss the possible importance of continuum effects in $\chi_b(3P)$ mesons.

2 Spectroscopy of higher bottomonia

2.1 Bottomonium spectrum in the UQM

Here, we discuss our UQM results for the spectrum of $b\bar{b}$ mesons with self energy corrections of Refs. [4, 7]. In our calculation, we fitted

$$M_a = E_a + \Sigma(E_a) \quad (2)$$

to the experimental data [24]. The physical mass of a meson, M_a , results from the sum of a bare energy term, E_a , computed within Godfrey and Isgur's relativized QM [21], and a self energy correction,

$$\Sigma(E_a) = \sum_{BC} \int_0^\infty q^2 dq \frac{|\langle BC\vec{q}\ell J | T^\dagger | A \rangle|^2}{E_a - E_{bc}}, \quad (3)$$

evaluated in the unquenched quark model formalism [1, 4–7]. See Fig. 1. In Ref. [7], we also gave results obtained by re-fitting the relativized QM mass formula [21] to the most recent experimental data [24], without including continuum effects. The re-fit was necessary to compute the open-bottom strong decays, which required more precise values for the masses of the higher-lying decaying mesons, like $4S$ states.

It is interesting to notice that (see Refs. [6, 7]): 1) below 10.5 GeV, our UQM and relativized QM results are very similar; 2) above 10.5 GeV, an evident discrepancy emerges in the case of $\chi_b(3P)$ resonances [27, 28], due to the vicinity of these states to the first open-bottom decay thresholds. This means that, far from thresholds, continuum effects can be (at least partially) re-absorbed into a redefinition of the model parameters, as also demonstrated in Ref. [29]; close to thresholds, this is not possible anymore and continuum effects can play an important role. For example, this also happens in the case of the $X(3872)$ [5, 6, 30–32] and $D_{sJ}^*(2317)$ [33] mesons.

Table 2. Mass barycenters of $\chi_b(1P)$, $\chi_b(2P)$ and $\chi_b(3P)$ systems (column 1) and mass splittings between the members of the $\chi_b(1P)$, $\chi_b(2P)$ and $\chi_b(3P)$ multiplets (column 2 and 3), from our re-fit of Godfrey and Isgur's mass formula of Ref. [7]. The results are expressed in MeV.

$M_{\chi_b(1P)}$	$\Delta M_{21}(1P)$	$\Delta M_{10}(1P)$
9894	21	30
$M_{\chi_b(2P)}$	$\Delta M_{21}(2P)$	$\Delta M_{10}(2P)$
10241	21	14
$M_{\chi_b(3P)}$	$\Delta M_{21}(1P)$	$\Delta M_{10}(1P)$
10510	17	13

Table 3. Experimental data [24] for the mass barycenters of $\chi_c(1P)$ and $\chi_c(2P)$ systems (column 1) and mass splittings between the members of the $\chi_c(1P)$ and $\chi_c(2P)$ multiplets (column 2 and 3). The results are expressed in MeV.

$M_{\chi_c(1P)}$	$\Delta M_{21}(1P)$	$\Delta M_{10}(1P)$
3494	46	96
$M_{\chi_c(2P)}$	$\Delta M_{21}(2P)$	$\Delta M_{10}(2P)$
3906	56	-47

2.2 Mass barycenter of the $\chi_b(3P)$ system

Our UQM results were also used to evaluate the mass barycenter of the $\chi_b(3P)$ system. See Table 1. Another estimation was obtained by re-fitting the relativized QM mass formula [21] to the most recent experimental data [24, 34]. See Table 2. The two sets of theoretical results can be then compared to the experimental data: $M_{\chi_b(3P)} = 10.534 \pm 0.009$ GeV [24], $M_{\chi_b(3P)} = 10.530 \pm 0.005(\text{stat.}) \pm 0.009(\text{syst.})$ GeV [27], and $M_{\chi_b(3P)} = 10.551 \pm 0.014(\text{stat.}) \pm 0.017(\text{syst.})$ GeV [28]. LHCb has also recently provided a measure for the mass of the $\chi_{b1}(3P)$ resonance [35]: $M_{\chi_{b1}(3P)} = 10515.7^{+2.2}_{-3.9}(\text{stat.})^{+1.5}_{-2.1}(\text{syst.})$ MeV. See also Ref. [36], where the authors provided predictions for the splittings between the members of the $\chi_b(3P)$ multiplet using several potential models.

It is worthwhile noting that: 1) our UQM and relativized QM predictions of Tables 1 and 2 are similar in the case of the mass barycenter, which is the average between the three meson masses of the multiplet, but differ for the splittings between the members of the multiplet; 2) the mass splitting scheme, with $M_{\chi_{b1}(1P,2P)} - M_{\chi_{b0}(1P,2P)} \approx 15 - 30$ MeV and $M_{\chi_{b2}(1P,2P)} - M_{\chi_{b1}(1P,2P)} \approx 15 - 20$ MeV, in the UQM is broken for $\chi_b(3P)$ mesons. Something similar also happens for $\chi_c(2P)$ mesons. See Table 3. Therefore, we think that new and more precise experimental data will be necessary to confirm or discard our hypothesis of $\chi_b(3P)$ mesons as made up of a $b\bar{b}$ core plus non-negligible continuum components, due to threshold effects.

3 Open bottom strong decays in the 3P_0 pair-creation model

Here, we discuss our results for the open-bottom strong decays of Ref. [7]. The transitions we considered are of

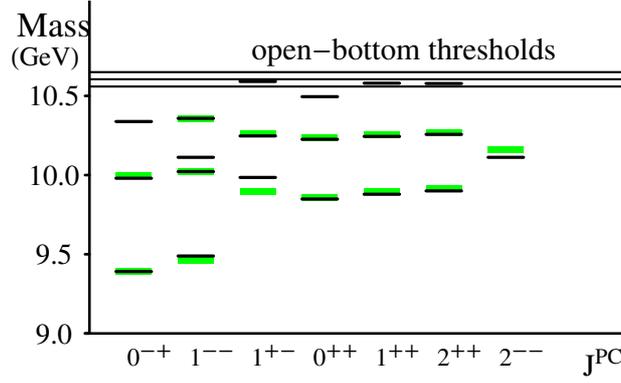


Figure 1. Spectrum of $b\bar{b}$ mesons in the UQM. Picture from Ref. [7]; APS copyright. The calculated masses of $b\bar{b}$ states are shown as black lines, the experimental data [24] as green boxes.

Table 4. Open-bottom decay widths (in MeV) for higher bottomonium states, as from Ref. [7]; APS copyright. Column 2 shows the values of the masses of the decaying $b\bar{b}$ states: when available, we used the experimental values from PDG [24] (\dagger), otherwise the theoretical predictions of the relativized QM [21], whose mass formula we re-fitted to the most recent experimental data [7]. Columns 3-8 show the decay width contributions from various BC channels, like $B\bar{B}$, $B\bar{B}^*$ and so on. The values of the 3P_0 model parameters were fitted to experimental data for the strong decays of $b\bar{b}$ resonances [7]. The symbol – in the table means that a certain decay is forbidden by selection rules or that the decay cannot take place because it is below the threshold.

Meson	Mass [MeV]	J^{PC}	$B\bar{B}$	$B\bar{B}^* / \bar{B}B^*$	$B^*\bar{B}^*$	$B_s\bar{B}_s$	$B_s\bar{B}_s^* / \bar{B}_sB_s^*$	$B_s^*\bar{B}_s^*$
$\Upsilon(10580)$ or $\Upsilon(4^3S_1)$	10.595 $10579.4 \pm 1.2^\dagger$	1^{--}	20	–	–	–	–	–
$\chi_{b2}(2^3F_2)$	10585	2^{++}	34	–	–	–	–	–
$\Upsilon(3^3D_1)$	10661	1^{--}	23	4	15	–	–	–
$\Upsilon_2(3^3D_2)$	10667	2^{--}	–	37	30	–	–	–
$\Upsilon_2(3^1D_2)$	10668	2^{+-}	–	55	57	–	–	–
$\Upsilon_3(3^3D_3)$	10673	3^{--}	15	56	113	–	–	–
$\chi_{b0}(4^3P_0)$	10726	0^{++}	26	–	24	–	–	–
$\Upsilon_3(2^3G_3)$	10727	3^{--}	3	43	39	–	–	–
$\chi_{b1}(4^3P_1)$	10740	1^{++}	–	20	1	–	–	–
$h_b(4^1P_1)$	10744	1^{+-}	–	33	5	–	–	–
$\chi_{b2}(4^3P_2)$	10751	2^{++}	10	28	5	1	–	–
$\chi_{b2}(3^3F_2)$	10800	2^{++}	5	26	53	2	2	–
$\Upsilon_3(3^1F_3)$	10803	3^{+-}	–	28	46	–	3	–
$\Upsilon(10860)$ or $\Upsilon(5^3S_1)$	$10876 \pm 11^\dagger$	1^{--}	1	21	45	0	3	1
$\Upsilon_2(4^3D_2)$	10876	2^{--}	–	28	36	–	4	4
$\Upsilon_2(4^1D_2)$	10877	2^{+-}	–	22	37	–	4	3
$\Upsilon_3(4^3D_3)$	10881	3^{--}	1	4	49	0	1	2
$\Upsilon_3(3^3G_3)$	10926	3^{--}	7	0	13	2	0	5
$\Upsilon(11020)$ or $\Upsilon(6^3S_1)$	$11019 \pm 8^\dagger$	1^{--}	0	8	26	0	0	2

the type

$$\begin{array}{c} A \\ b\bar{b} \end{array} \rightarrow \begin{array}{c} B \\ b\bar{q} \end{array} + \begin{array}{c} C \\ q\bar{b} \end{array}, \quad (4)$$

where q is a light quark (u , d or s). The decay widths were computed as [4–8, 37]:

$$\Gamma_{A \rightarrow BC} = \Phi_{A \rightarrow BC}(q_0) \sum_{\ell, J} |\langle BC \vec{q}_0 \ell J | T^\dagger | A \rangle|^2. \quad (5a)$$

Here, T^\dagger is the pair-creation operator of the 3P_0 model, where we substituted the constant pair-creation strength, γ_0 , with an effective one, γ_0^{eff} , to suppress unphysical heavy quark pair-creation [4–7, 38], and introduced a Gaussian quark form factor, because the pair of created

quarks has an effective size [4–7, 10, 29, 39]. $\Phi_{A \rightarrow BC}(q_0)$ is the standard relativistic phase space factor [4–8, 37],

$$\Phi_{A \rightarrow BC} = 2\pi q_0 \frac{E_b(q_0)E_c(q_0)}{M_a}, \quad (5b)$$

which depends on the relative momentum q_0 between B and C and on the energies of the two intermediate state mesons, $E_b = \sqrt{M_b^2 + q_0^2}$ and $E_c = \sqrt{M_c^2 + q_0^2}$. The values of the masses M_a , M_b and M_c , used in the calculation, were taken from the PDG [24] and Ref. [34], while, in the case of still unobserved states, we used the predictions from our re-fit of Godfrey and Isgur's model mass formula of Ref. [7].

Table 5. $E1$ radiative transitions of $\chi_b(3P)$ states, calculated with Eq. (6) [6]. In the calculation, we used our QM and UQM predictions for the masses of the $\chi_b(3P)$'s [7] and the wave functions of the relativized QM [21]. Table from Ref. [6]; APS copyright.

Transition	$E_\gamma(\text{QM})$ (MeV)	$\Gamma_{b\bar{b}}(\text{QM})$ (KeV)	$E_\gamma(\text{UQM})$ (MeV)	$\Gamma_{b\bar{b}}(\text{UQM})$ (KeV)
$\chi_{b0}(3^3P_0) \rightarrow \Upsilon(1^3S_1)\gamma$	983	0.6	984	0.2
$\chi_{b0}(3^3P_0) \rightarrow \Upsilon(2^3S_1)\gamma$	460	1.2	461	0.7
$\chi_{b0}(3^3P_0) \rightarrow \Upsilon(3^3S_1)\gamma$	138	6.1	139	6.4
$\chi_{b0}(3^3P_0) \rightarrow \Upsilon(1^3D_1)\gamma$	344	0.2	376	0.2
$\chi_{b0}(3^3P_0) \rightarrow \Upsilon(2^3D_1)\gamma$	69	0.9	–	–
$\chi_{b1}(3^3P_1) \rightarrow \Upsilon(1^3S_1)\gamma$	971	2.1	1061	1.7
$\chi_{b1}(3^3P_1) \rightarrow \Upsilon(2^3S_1)\gamma$	477	2.5	542	2.7
$\chi_{b1}(3^3P_1) \rightarrow \Upsilon(3^3S_1)\gamma$	155	7.4	223	22.8
$\chi_{b1}(3^3P_1) \rightarrow \Upsilon(1^3D_1)\gamma$	361	0	458	0
$\chi_{b1}(3^3P_1) \rightarrow \Upsilon(2^3D_1)\gamma$	86	0.4	–	–
$\chi_{b1}(3^3P_1) \rightarrow \Upsilon_2(1^3D_2)\gamma$	341	0	408	0.1
$\chi_{b1}(3^3P_1) \rightarrow \Upsilon_2(2^3D_2)\gamma$	79	1.0	–	–
$\chi_{b2}(3^3P_2) \rightarrow \Upsilon(1^3S_1)\gamma$	1010	3.9	1059	3.6
$\chi_{b2}(3^3P_2) \rightarrow \Upsilon(2^3S_1)\gamma$	489	3.8	540	4.0
$\chi_{b2}(3^3P_2) \rightarrow \Upsilon(3^3S_1)\gamma$	168	8.2	221	19.6
$\chi_{b2}(3^3P_2) \rightarrow \Upsilon(1^3D_1)\gamma$	373	0	456	0
$\chi_{b2}(3^3P_2) \rightarrow \Upsilon(2^3D_1)\gamma$	99	0	–	–
$\chi_{b2}(3^3P_2) \rightarrow \Upsilon_2(1^3D_2)\gamma$	354	0	406	0
$\chi_{b2}(3^3P_2) \rightarrow \Upsilon_2(2^3D_2)\gamma$	92	0.3	–	–
$\chi_{b2}(3^3P_2) \rightarrow \Upsilon_3(1^3D_3)\gamma$	357	0	441	0
$\chi_{b2}(3^3P_2) \rightarrow \Upsilon_3(2^3D_3)\gamma$	86	1.4	–	–

4 QM calculation of $\chi_b(3P)$'s $E1$ radiative transitions

In this section, we show our QM results for the radiative transitions of $\chi_b(3P)$ states from Ref. [6]. The $E1$ radiative transitions were calculated according to [25]:

$$\Gamma_{E1} = \frac{4}{3} C_{fi} \delta_{SS'} e_b^2 \alpha \left| \langle \psi_f | r | \psi_i \rangle \right|^2 E_\gamma^3 \frac{E_f^{(b\bar{b})}}{M_i^{(b\bar{b})}}. \quad (6a)$$

Here, $e_b = -\frac{1}{3}$ is the charge of the b -quark (in units of e), α is the fine structure constant, E_γ is the final photon energy, $E_f^{(b\bar{b})}$ is the total energy of the final $b\bar{b}$ state, $M_i^{(b\bar{b})}$ is the mass of the initial $b\bar{b}$ state, $\langle \psi_f | r | \psi_i \rangle$ is a radial matrix element, which involves the initial and final radial wave functions, and the angular matrix element C_{fi} is given by

$$C_{fi} = \max(L, L')(2J' + 1) \begin{Bmatrix} L' & J' & S \\ J & L & 1 \end{Bmatrix}^2. \quad (6b)$$

We calculated the matrix elements of Eqs. (6) assuming, for the initial and final states, the wave functions of Godfrey and Isgur's relativized QM [21]. We gave results considering, for the masses of the decaying $\chi_b(3P)$ resonances, predictions from the UQM and our re-fit of the relativized QM mass formula of Ref. [7]. This was done because the phase space factor of the decay strongly depends on the masses of the initial and final state mesons and our UQM and relativized QM predictions for the masses of $\chi_b(3P)$ states are quite different.

Finally, our results are reported in Table 5.

5 Conclusion

In this contribution, we discussed the results of an UQM calculation of the bottomonium spectrum with self-energy corrections [7]. In the UQM [1–7], the effects of $q\bar{q}$ sea pairs are introduced explicitly into the quark model (QM) via a QCD-inspired 3P_0 pair-creation mechanism [8]. The $q\bar{q}$ pairs have vacuum quantum numbers and their effect on hadron observables can be added as a perturbation. Our UQM results are similar to those of the relativized QM [21] for the majority of states we considered, but differ substantially in the case $\chi_b(3P)$ mesons. This happens because $\chi_b(3P)$ states are very close to the first open-bottom decay thresholds, thus meson-meson continuum effects can play an important role. A comparison between UQM and relativized QM results for the mass barycenter of $\chi_b(3P)$ states was done but, up to now, the present experimental data do not permit to conclude which of the two interpretations is preferable. Finally, we discussed our results for the open-bottom strong decay widths of $b\bar{b}$ states within the 3P_0 pair-creation model [7] and for the radiative transitions of $\chi_b(3P)$ states in the QM formalism [6].

In Refs. [40], the QM was extended to include the quark-antiquark-gluon higher Fock components to study the spectroscopy of hybrid mesons. In Ref. [41] can be found a classification of possible tetraquark states.

References

- [1] E. Santopinto and R. Bijker, Few Body Syst. **44**, 95 (2008); R. Bijker and E. Santopinto, Phys. Rev. C **80**, 065210 (2009).
- [2] E. Santopinto and R. Bijker, Phys. Rev. C **82**, 062202 (2010).

- [3] E. Santopinto, R. Bijker and J. Ferretti, *Few Body Syst.* **50**, 199 (2011); R. Bijker, J. Ferretti and E. Santopinto, *Phys. Rev. C* **85**, 035204 (2012).
- [4] J. Ferretti, G. Galatá, E. Santopinto and A. Vassallo, *Phys. Rev. C* **86**, 015204 (2012).
- [5] J. Ferretti, G. Galatá and E. Santopinto, *Phys. Rev. C* **88**, 015207 (2013).
- [6] J. Ferretti, G. Galatá and E. Santopinto, *Phys. Rev. D* **90**, 054010 (2014).
- [7] J. Ferretti and E. Santopinto, *Phys. Rev. D* **90**, 094022 (2014).
- [8] L. Micu, *Nucl. Phys. B* **10**, 521 (1969); A. Le Yaouanc, L. Oliver, O. Pene and J. -C. Raynal, *Phys. Rev. D* **8**, 2223 (1973); **9**, 1415 (1974); *Phys. Lett. B* **71**, 397 (1977); **72**, 57 (1977).
- [9] N. A. Törnqvist and P. Zenczykowski, *Phys. Rev. D* **29**, 2139 (1984); *Z. Phys. C* **30**, 83 (1986); P. Zenczykowski, *Annals Phys.* **169**, 453 (1986).
- [10] P. Geiger and N. Isgur, *Phys. Rev. Lett.* **67**, 1066 (1991); *Phys. Rev. D* **44**, 799 (1991); **47**, 5050 (1993); **55**, 299 (1997).
- [11] N. Isgur and G. Karl, *Phys. Rev. D* **18**, 4187 (1978); *Phys. Rev. D* **19**, 2653 (1979) [Erratum-ibid. *D* **23**, 817 (1981)]; *Phys. Rev. D* **20**, 1191 (1979).
- [12] S. Capstick and N. Isgur, *Phys. Rev. D* **34**, 2809 (1986).
- [13] R. Bijker, F. Iachello and A. Leviatan, *Annals Phys.* **236**, 69 (1994).
- [14] M. Ferraris, M. M. Giannini, M. Pizzo, E. Santopinto and L. Tiator, *Phys. Lett. B* **364**, 231 (1995); E. Santopinto, F. Iachello and M. M. Giannini, *Eur. Phys. J. A* **1**, 307 (1998); R. Bijker, F. Iachello and E. Santopinto, *J. Phys. A* **31**, 9041 (1998); M. De Sanctis, M. M. Giannini, E. Santopinto and A. Vassallo, *Nucl. Phys. A* **755**, 294 (2005); *Phys. Rev. C* **76**, 062201 (2007); E. Santopinto and M. M. Giannini, *Phys. Rev. C* **86**, 065202 (2012).
- [15] L. Y. Glozman, D. O. Riska, *Phys. Rept.* **268**, 263 (1996); L. Y. Glozman, W. Plessas, K. Varga, R. F. Wagenbrunn, *Phys. Rev. D* **58**, 094030 (1998).
- [16] U. Loring, B. C. Metsch and H. R. Petry, *Eur. Phys. J. A* **10**, 395 (2001).
- [17] R. Bijker, M. M. Giannini and E. Santopinto, *Phys. Lett. B* **595**, 260 (2004).
- [18] E. Santopinto, *Phys. Rev. C* **72**, 022201 (2005); J. Ferretti, A. Vassallo and E. Santopinto, *Phys. Rev. C* **83**, 065204 (2011); M. De Sanctis, J. Ferretti, E. Santopinto and A. Vassallo, *Phys. Rev. C* **84**, 055201 (2011); arXiv:1410.0590; E. Santopinto and J. Ferretti, arXiv:1412.7571.
- [19] G. Galata and E. Santopinto, *Phys. Rev. C* **86**, 045202 (2012).
- [20] I. G. Aznauryan *et al.*, *Int. J. Mod. Phys. E* **22**, 1330015 (2013).
- [21] S. Godfrey and N. Isgur, *Phys. Rev. D* **32**, 189 (1985).
- [22] S. K. Choi *et al.* [Belle Collaboration], *Phys. Rev. Lett.* **91**, 262001 (2003).
- [23] R. Aaij *et al.* [LHCb Collaboration], *Phys. Rev. Lett.* **110**, 222001 (2013).
- [24] K. A. Olive *et al.* [Particle Data Group Collaboration], *Chin. Phys. C* **38**, 090001 (2014).
- [25] E. Eichten, K. Gottfried, T. Kinoshita, K. D. Lane and T. -M. Yan, *Phys. Rev. D* **17**, 3090 (1978); **21**, 203 (1980).
- [26] T. Barnes, S. Godfrey and E. S. Swanson, *Phys. Rev. D* **72**, 054026 (2005).
- [27] G. Aad *et al.* [ATLAS Collaboration], *Phys. Rev. Lett.* **108**, 152001 (2012).
- [28] V. M. Abazov *et al.* [D0 Collaboration], *Phys. Rev. D* **86**, 031103 (2012).
- [29] P. Geiger and N. Isgur, *Phys. Rev. D* **41**, 1595 (1990).
- [30] M. R. Pennington and D. J. Wilson, *Phys. Rev. D* **76**, 077502 (2007).
- [31] B. -Q. Li, C. Meng and K. -T. Chao, *Phys. Rev. D* **80**, 014012 (2009).
- [32] I. V. Danilkin and Y. A. Simonov, *Phys. Rev. Lett.* **105**, 102002 (2010).
- [33] D. S. Hwang and D. -W. Kim, *Phys. Lett. B* **601** (2004) 137.
- [34] R. Mizuk *et al.* [Belle Collaboration], *Phys. Rev. Lett.* **109**, 232002 (2012).
- [35] R. Aaij *et al.* [LHCb Collaboration], *JHEP* **1410**, 88 (2014).
- [36] C. O. Dib and N. Neill, *Phys. Rev. D* **86**, 094011 (2012).
- [37] E. S. Ackleh, T. Barnes and E. S. Swanson, *Phys. Rev. D* **54**, 6811 (1996); T. Barnes, F. E. Close, P. R. Page and E. S. Swanson, *Phys. Rev. D* **55**, 4157 (1997); T. Barnes, S. Godfrey and E. S. Swanson, *Phys. Rev. D* **72**, 054026 (2005).
- [38] Y. S. Kalashnikova, *Phys. Rev. D* **72** (2005) 034010.
- [39] B. Silvestre-Brac and C. Gignoux, *Phys. Rev. D* **43**, 3699 (1991).
- [40] P. Guo, A. P. Szczepaniak, G. Galata, A. Vassallo and E. Santopinto, *Phys. Rev. D* **77**, 056005 (2008); **78**, 056003 (2008).
- [41] E. Santopinto and G. Galata, *Phys. Rev. C* **75**, 045206 (2007).