

Hawking-Unruh hadronization and strangeness production in high energy collisions

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Abstract. The interpretation of quark (q)- antiquark (\bar{q}) pairs production and the sequential string breaking as tunneling through the event horizon of colour confinement leads to a thermal hadronic spectrum with a universal Unruh temperature, $T \simeq 165$ Mev, related to the quark acceleration, a , by $T = a/2\pi$. The resulting temperature depends on the quark mass and then on the content of the produced hadrons, causing a deviation from full equilibrium and hence a suppression of strange particle production in elementary collisions. In nucleus-nucleus collisions, where the quark density is much bigger, one has to introduce an average temperature (acceleration) which dilutes the quark mass effect and the strangeness suppression almost disappears.

1 Introduction

Hadron production in high energy collisions shows remarkably universal thermal features. In e^+e^- annihilation [1–3], in pp , $p\bar{p}$ [4] and more general hh interactions [3], as well as in the collisions of heavy nuclei [5], over an energy range from around 10 GeV up to the TeV range, the relative abundances of the produced hadrons appear to be those of an ideal hadronic resonance gas at a quite universal temperature $T_H \approx 160 - 170$ Mev (see fig.1) [6]. There is, however, one important non-equilibrium effect observed: the production of strange hadrons in elementary collisions is suppressed relative to an overall equilibrium. This is usually taken into account phenomenologically by introducing an overall strangeness suppression factor $\gamma_s < 1$ [7], which reduces the predicted abundances by γ_s, γ_s^2 and γ_s^3 for hadrons containing one, two or three strange quarks (or antiquarks), respectively. In high energy heavy ion collisions, strangeness suppression becomes less and disappears at high energies [8].

There is a still ongoing debate about the interpretation of the observed thermal behavior [9]. Indeed, in high energy heavy ion collisions multiple parton scattering could lead to kinetic thermalization, but e^+e^- or elementary hadron interactions do not readily allow such a description. Moreover, the universality of the observed temperatures, suggests a common origin for all high energy collisions. It has been recently proposed [10] that thermal hadron production is the QCD counterpart of Hawking-Unruh (H-U) radiation [11, 12], emitted at the event horizon due to colour confinement. In the case of approximately massless quarks, the resulting universal hadronization temperature is

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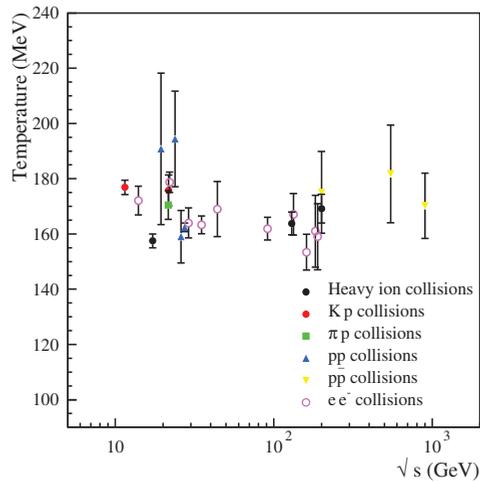


Figure 1. Hadronization temperature for different initial scattering configurations as a function of \sqrt{s} .

determined by the string tension σ , with $T \approx \sqrt{\sigma/2\pi} \approx 165$ MeV [10]. Moreover in ref.[13] it has been shown that strangeness suppression in elementary collisions naturally occurs in this framework without requiring an ad-hoc suppression factor due to the non-negligible strange quark mass, which modifies the emission temperature for such quarks.

In this contribution we briefly review the Hawking-Unruh hadronization approach and show that when the quark density is much bigger than in elementary scattering, as in relativistic heavy ion collisions, the effect of strange quark mass is washed out by the average acceleration due to the large number of light quarks and the strangeness suppression disappears.

2 Hawking-Unruh Hadronization

In this section we will recall the essentials of the statistical hadronization model and of the Hawking-Unruh analysis of the string breaking mechanism. For a detailed descriptions see ref. [3, 10, 13].

2.1 Statistical hadronization model

The statistical hadronization model assumes that hadronization in high energy collisions is a universal process proceeding through the formation of multiple colourless massive clusters (or fireballs) of finite spacial extension. These clusters are taken to decay into hadrons according to a purely statistical law and, for Lorentz-invariant quantities such as multiplicities, one can introduce the simplifying assumption that the distribution of masses and charges among clusters is again purely statistical [3], so that, as far as the calculation of multiplicities is concerned, the set of clusters becomes equivalent, on average, to a large cluster (*equivalent global cluster*) whose volume is the sum of proper cluster volumes and whose charge is the sum of cluster charges (and thus the conserved charge of the initial colliding system).

To obtain a simple expression for our further discussion, we neglect for the moment the conservation of the various discrete Abelian charges (electric charge, baryon number, strangeness, heavy

flavour) which has to be taken into account *exactly* in elementary collisions and we consider for the moment a grand-canonical picture. We also assume Boltzmann distributions for all hadrons. The multiplicity of a given scalar hadronic species j then becomes

$$\langle n_j \rangle^{\text{primary}} = \frac{VTm_j^2}{2\pi^2} \gamma_s^{n_j} K_2\left(\frac{m_j}{T}\right) \quad (1)$$

with m_j denoting its mass and n_j the number of strange quarks/antiquarks it contains. Here primary indicates that it gives the number at the hadronisation point, prior to all subsequent resonance decay. The Hankel function $K_2(x)$, with $K(x) \sim \exp\{-x\}$ for large x , gives the Boltzmann factor, while V denotes the overall equivalent cluster volume. In other words, in an analysis of 4π data of elementary collisions, V is the sum of the all cluster volumes at all different rapidities. It thus scales with the overall multiplicity and hence increases with collision energy. A fit of production data based on the statistical hadronisation model in elementary collisions thus involves three parameters: the hadronisation temperature T , the strangeness suppression factor γ_s , and the equivalent global cluster volume V . For heavy ion collisions there is a further parameter: the bariochemical potential, μ_B .

As previously discussed, at high energy the temperature turns out to be independent on the initial configuration and this result calls for a universal mechanism underlying the hadronization. In the next paragraph we recall the interpretation of the string breaking as QCD H-U radiation.

2.2 String breaking and event horizon

Let us outline the thermal hadron production process through H-U radiation for the specific case of e^+e^- annihilation. The separating primary $q\bar{q}$ pair excites a further pair $q_1\bar{q}_1$ from the vacuum, and this pair is in turn pulled apart by the primary constituents. In the process, the \bar{q}_1 shields the q from its original partner \bar{q} , with a new $q\bar{q}_1$ string formed. When it is stretched to reach the pair production threshold, a further pair is formed, and so on [15, 16]. Such a pair production mechanism is a special case of H-U radiation [17], emitted as hadron \bar{q}_1q_2 when the quark q_1 tunnels through its event horizon to become \bar{q}_2 .

The corresponding hadron radiation has a thermal spectrum with temperature given by the Unruh form $T_H = a/2\pi$, where a is the acceleration suffered by the quark \bar{q}_1 due to the force of the string attaching it to the primary quark Q . This is equivalent to that suffered by quark q_2 due to the effective force of the primary antiquark \bar{Q} . Hence we have

$$a_q = \frac{\sigma}{w_q} = \frac{\sigma}{\sqrt{m_q^2 + k_q^2}}, \quad (2)$$

where $w_q = \sqrt{m_q^2 + k_q^2}$ is the effective mass of the produced quark, with m_q for the bare quark mass and k_q the quark momentum inside the hadronic system $q_1\bar{q}_1$ or $q_2\bar{q}_2$. Since the string breaks [10] when it reaches a separation distance

$$x_q \simeq \frac{2}{\sigma} \sqrt{m_q^2 + (\pi\sigma/2)}, \quad (3)$$

the uncertainty relation gives us with $k_q \simeq 1/x_q$

$$w_q = \sqrt{m_q^2 + [\sigma^2/(4m_q^2 + 2\pi\sigma)]} \quad (4)$$

for the effective mass of the quark. The resulting quark-mass dependent Unruh temperature is thus given by

$$T(qq) \simeq \frac{\sigma}{2\pi w_q}. \quad (5)$$

Note that here it is assumed that the quark masses for q_1 and q_2 are equal. For $m_q \simeq 0$, eq. (5) reduces to $T(00) \simeq \sqrt{\sigma/2\pi}$, as obtained in [10].

If the produced hadron $\bar{q}_1 q_2$ consists of quarks of different masses, the resulting temperature has to be calculated as an average of the different accelerations involved. For one massless quark ($m_q \simeq 0$) and one of strange quark mass m_s , the average acceleration becomes

$$\bar{a}_{0s} = \frac{w_0 a_0 + w_s a_s}{w_0 + w_s} = \frac{2\sigma}{w_0 + w_s}. \quad (6)$$

From this the Unruh temperature of a strange meson is given by $T(0s) \simeq \sigma/\pi(w_0 + w_s)$ with $w_0 \simeq \sqrt{1/2\pi\sigma}$ and w_s given by eq.(4) with $m_q = m_s$. Similarly, we obtain $T(ss) \simeq \sigma/2\pi w_s$ for the temperature of a meson consisting of a strange quark-antiquark pair (ϕ).

The scheme is readily generalized to baryons. The production pattern is illustrated in Fig. ?? and leads to an average of the accelerations of the quarks involved. We thus have $T(000) = T(0) \simeq \sigma/2\pi w_0$ for nucleons, $T(00s) \simeq 3\sigma/2\pi(2w_0 + w_s)$ for Λ and Σ production, $T(0ss) \simeq 3\sigma/2\pi(w_0 + 2w_s)$ for Ξ production, and $T(sss) = T(ss) \simeq \sigma/2\pi w_s$ for that of Ω 's.

We thus obtain a resonance gas picture with five different hadronization temperatures, as specified by the strangeness content of the hadron in question: $T(00) = T(000)$, $T(0s)$, $T(ss) = T(sss)$, $T(00s)$ and $T(0ss)$. However we are not increasing the number of free parameters of the model since all the previous temperatures are completely determined by the string tension and the strange quark mass. Apart from possible variations of the quantities of σ and m_s , the description is thus parameter-free. As illustration, we show in table 1 the temperatures obtained for $\sigma = 0.2 \text{ GeV}^2$ and three different strange quark masses. It is seen that in all cases, the temperature for a hadron carrying non-zero strangeness is lower than that of non-strange hadrons and, as discussed in the next section, this leads to an overall strangeness suppression in elementary collisions, in good agreement with data [13], without the introduction of the ad-hoc parameter γ_s .

Table 1. Hadronization temperatures for hadrons of different strangeness content, for $m_s = 0.075, 0.100, 0.125$ Gev and $\sigma = 0.2 \text{ GeV}^2$ according to eq.(9).

| T | $m_s = 0.075$ | $m_s = 0.100$ | $m_s = 0.125$ |
|----------|---------------|---------------|---------------|
| $T(00)$ | 0.178 | 0.178 | 0.178 |
| $T(0s)$ | 0.172 | 0.167 | 0.162 |
| $T(ss)$ | 0.166 | 0.157 | 0.148 |
| $T(000)$ | 0.178 | 0.178 | 0.178 |
| $T(00s)$ | 0.174 | 0.171 | 0.167 |
| $T(0ss)$ | 0.170 | 0.164 | 0.157 |
| $T(sss)$ | 0.166 | 0.157 | 0.148 |

3 Strangeness Production

3.1 Elementary collisions

The different temperatures for hadrons carrying non-zero strangeness have been taken into account in a full statistical hadronization code [13] and the results are in quantitative agreement with the strangeness suppression observed in elementary collisions. However the result that a lower hadronization temperature for strange particles produces the same effect of γ_s can be easily understood in a simplified model where there are only two species: scalar and electrically neutral mesons, "pions" with mass m_π , and "kaons" with mass m_k and strangeness $s = 1$. According to the statistical model with the γ_s suppression factor, the ratio N_k/N_π is obtained by eq.(1) and is given by

$$\frac{N_k}{N_\pi} \Big|_{\gamma_s}^{stat} = \frac{m_k^2}{m_\pi^2} \gamma_s \frac{K_2(m_k/T)}{K_2(m_\pi/T)} \quad (7)$$

because there is thermal equilibrium at temperature T . On the other hand, in the H-U based statistical model there is no γ_s , but $T_k = T(0s) \neq T_\pi = T(00) = T$ and therefore

$$\frac{N_k}{N_\pi} \Big|_{H-U}^{stat} = \frac{m_k^2}{m_\pi^2} \frac{T_k}{T_\pi} \frac{K_2(m_k/T_k)}{K_2(m_\pi/T_\pi)}. \quad (8)$$

From previous eqs.(7-8), it is immediately clear that the difference in the hadronization temperatures, $T_k \neq T_\pi$, corresponds to a γ_s parameter given by

$$\gamma_s = \frac{T_k}{T_\pi} \frac{K_2(m_k/T_k)}{K_2(m_k/T_\pi)}. \quad (9)$$

In other terms, it is the mass dependence of the hadronization temperatures which reproduces the strangeness suppression. For $\sigma = 0.2 \text{ GeV}^2$, $m_s = 0.1 \text{ GeV}$, $T_\pi = 178 \text{ MeV}$ and $T_k = 167 \text{ MeV}$ (see table I), the crude evaluation by eq.(9) gives $\gamma_s \simeq 0.73$

The complete analysis, with the exact conservation of quantum numbers, has been carried out in ref. [13] and the Unruh-Hawking hadronization approach is in good agreement with data for different values of \sqrt{s} for (constant) values of the string tension and of the strange quark mass consistent with lattice results. A similar phenomenological study for proton-proton collisions is in progress.

3.2 Heavy ion collisions

The hadron production in high energy collisions occurs in a number of causally disconnected regions of finite space-time size [18]. As a result, globally conserved quantum numbers (charge, strangeness, baryon number) must be conserved locally in spatially restricted correlation clusters. This provides a dynamical basis for understanding the suppression of strangeness production in elementary interactions (pp , e^+e^-) due to a small strangeness correlation volume [19–22].

In the H-U approach in elementary collisions there is a small number of partons in a causally connected region and the hadron production comes from the sequential breaking of independent $q\bar{q}$ strings with the consequent species-dependent temperatures which reproduce the strangeness suppression. In contrast, the space-time superposition of many collisions in heavy ion interactions largely removes these causality constraints [18], resulting in an ideal hadronic resonance gas in full equilibrium.

The effect of a large number of causally connected quarks and antiquarks in the H-U scheme can be implemented by defining the average temperature of the system and determining the hadron multiplicities by the statistical model with this "equilibrium" temperature. More precisely, the average

temperature depends on the numbers of light quarks, N_l , and of strange quarks, N_s , which, in turn, are counted by the number of strange and non-strange hadrons in the final state at that temperature. A detailed analysis requires again a full calculation in the statistical model, that will be done in a forthcoming paper, however the mechanism can be roughly illustrated in the world of "pions" and "kaons" previously discussed.

Let us consider a high density system of quarks and antiquarks in a causally connected region. Generalizing our formulas in sec. 2, the average acceleration is given by

$$\bar{a} = \frac{N_l w_0 a_0 + N_s w_s a_s}{N_l w_0 + N_s w_s} \quad (10)$$

By assuming $N_l \gg N_s$, after a simple algebra, the average temperature, $\bar{T} = \bar{a}/2\pi$, turns out to be

$$\bar{T} = T(00) \left[1 - \frac{N_s}{N_l} \frac{w_0 + w_s}{w_0} \left(1 - \frac{T(0s)}{T(00)} \right) \right] + O[(N_s/N_l)^2] \quad (11)$$

Now in our world of "pions" and "kaons" one has $N_l = 2N_\pi + N_k$ and $N_s = N_k$ and therefore

$$\bar{T} = T(00) \left[1 - \frac{N_k}{2N_\pi} \frac{w_0 + w_s}{w_0} \left(1 - \frac{T(0s)}{T(00)} \right) \right] + O[(N_k/N_\pi)^2]. \quad (12)$$

On the other hand, in the H-U based statistical calculation the ratio N_k/N_π depends on the equilibrium (average) temperature \bar{T} , that is

$$N_k/N_\pi = \frac{m_k^2 K_2(m_k/\bar{T})}{m_\pi^2 K_2(m_\pi/\bar{T})}, \quad (13)$$

and, therefore, one has to determine the temperature \bar{T} by self-consistency of eq.(12) with eq.(13). This condition implies the equation

$$2 \frac{[1 - \bar{T}/T(00)]w_0}{[1 - T(0s)/T(00)](w_s + w_0)} = \frac{m_k^2 K_2(m_k/\bar{T})}{m_\pi^2 K_2(m_\pi/\bar{T})}, \quad (14)$$

that can be solved numerically.

For $\sigma = 0.2 \text{ GeV}^2$, $m_s = 0.1$ and the temperatures in table I, the average temperature turns out $\bar{T} = 174 \text{ MeV}$ and one can evaluate the Wroblewski factor defined by

$$\lambda = \frac{2N_s}{N_l} \quad (15)$$

where N_s is the number of strange and anti-strange quarks in the hadrons in the final state and N_l is the number of light quarks and antiquarks in the final state minus their number in the initial configuration.

The experimental value of the Wroblewski factor in high energy collisions is rather independent on the energy and is about $\lambda \simeq 0.26$ in elementary collisions and $\lambda \simeq 0.5$ for nucleus-nucleus scattering. In our simplified model, for e^+e^- annihilation, with the species-dependent temperatures in table I, one gets $\lambda \simeq 0.26$.

To evaluate the Wroblewski factor in nucleus-nucleus collisions one has to consider the average "equilibrium" temperature \bar{T} and the number of light quarks in the initial configuration. The latter point requires a realistic calculation in the statistical model which includes all resonances and stable particles. However to show that one is on the right track, let us neglect the problem of the initial configuration and let us evaluate the effect of substituting in eq.(8) the species-dependent temperatures with the equilibrium temperature \bar{T} . With this simple modification one has an increasing of the Wroblewski factor, $\lambda = 0.33$.

In other terms, in the toy model, the change from a non-equilibrium condition, with species-dependent temperatures, to an equilibrated system with the average temperature \bar{T} is able to reproduce part of the observed growing of the number of strange quarks with respect to elementary interactions.

4 Comments and Conclusions

Strictly speaking, the Unruh effect has been derived for an infinite life-time of the accelerated system but the causally connected region, defined by the event horizon, for a finite time accelerated quark (anti-quark) trajectory is much smaller. The problem if an accelerated system with finite life-time and a bounded trajectory still observes a thermal spectrum has been discussed in details in ref. [23, 24]. It turns out that the spectrum is thermal but with a modified relation (with respect to the Unruh formula $T = a/2\pi$) between the acceleration and the temperature given by

$$\frac{1}{T} = \frac{2\pi}{a} - \frac{2\pi}{a^2 L} \quad (16)$$

where a is the acceleration and L is the linear size of the accelerated trajectory. In our case $a = \sqrt{2\pi\sigma}$, L is given by eq.(3) (with $m_q = 0$) and the correction to the Unruh formula is about 10%, which can be reabsorbed in a different value of σ , well compatible with phenomenological indications.

Therefore the Hawking-Unruh approach to the hadronization can explain the origin of the universal temperature and, essentially with no free parameters, describes the strangeness suppression in elementary collisions and the strangeness enhancement in heavy ion scatterings.

Moreover, it can be easily understood why in heavy ion collisions, for $\mu_B \approx 0$ the average energy per particle, $\langle E \rangle / \langle N \rangle$, is about 1.08 Gev [25–28]. Indeed, the energy of the pair produced by string breaking, i.e., of the newly formed hadron, is given by(see Sec. 2)

$$E_h = \sigma R = \sqrt{2\pi\sigma} \quad (17)$$

In the central rapidity region of high energy collisions, one has $\mu_B \approx 0$ so that E_h is in fact the average energy $\langle E \rangle$ per hadron, with an average number $\langle N \rangle$ of newly produced hadrons. Hence one obtains

$$\frac{\langle E \rangle}{\langle N \rangle} = \sqrt{2\pi\sigma} = 1.09 \pm 0.08 \text{ Gev} \quad (18)$$

for $\sigma = 0.19 \pm 0.03 \text{ Gev}^2$.

Finally, high energy particle physics, and in particular hadron production, is, in our opinion, the promising sector to find the analogue of the Hawking-Unruh radiation for two main reasons: color confinement and the huge acceleration that cannot be reached in any other dynamical systems.

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