

## The Falsification of Nuclear Forces

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**Abstract.** We review our work on the statistical uncertainty analysis of the NN force. This is based on the Granada-2013 database where a statistically meaningful partial wave analysis comprising a total of 6713 np and pp published scattering data from 1950 till 2013 below pion production threshold has been made. We stress the necessary conditions required for a correct and self-consistent statistical interpretation of the discrepancies between theory and experiment which enable a subsequent statistical error propagation and correlation analysis

### 1 Introduction

Error propagation and uncertainty quantification have recently become a central topic in nuclear physics [1–6]. In the particular field of phenomenological Nucleon-Nucleon (NN) interactions uncertainties can be classified as statistical or systematic<sup>1</sup>. Statistical uncertainties are the result of unavoidable random fluctuations during the experimental process. Most NN scattering measurements consist of counting events which corresponds to a Poisson distribution. If the number of events is large enough then the distribution can be safely approximated as a normal one. This allows to fix the parameters of a phenomenological potential via the usual chi square minimization process to reproduce the collection of NN scattering data. As a consequence the statistical uncertainty of the experimental data propagates into the fitting parameters in the form of a confidence region in which the parameters are allowed to vary and still give an accurate description of the data. Such confidence region can be easily determined by the parameters' covariance matrix if the assumption of normally distributed residuals is fulfilled. Systematic uncertainties are a consequence of our lack of knowledge of the actual form of the NN potential and the assumptions that have to be made in order to give a representation of the NN interaction. Some potentials are separable in momentum space while others are not, some are energy dependent and others range from fully local to different types non-localities in coordinate space. Even though most of the potentials are fitted to the same type of experimental NN scattering

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<sup>1</sup>Numerical uncertainties are also present but can be made small enough to be dominated by the other two.

data, their predictions of unmeasured scattering observables or nuclear structure properties can sometimes be incompatible. This residual incompatibilities beyond statistical consistency and equivalence is what we refer as systematic uncertainties. In this contribution we review our determination of the NN interaction statistical uncertainties along with the necessary conditions for such type of analysis.

## 2 Coarse graining and the delta-shell potential

Coarse graining embodies the Wilsonian renormalization [7] concept and represents a very reliable tool to simplify the description of pp and np scattering data while still retaining all the relevant information of the interaction up to a certain energy range set by the de Broglie wavelength of the most energetic particle considered. The  $V_{lowk}$  potentials in momentum space are a good example of an implementation of coarse graining by removing the high-momentum part of the interaction [8, 9]. In 1973 Aviles introduced the delta-shell (DS) potential in the context of NN interactions [10]. In [11] the DS potential was used to implement coarse graining in coordinate space via a local potential that samples the np interaction at certain concentration radii  $r_i$  by

$$V(r) = \sum_{i=1}^n \frac{\lambda_i}{2\mu} \delta(r - r_i), \quad (1)$$

where  $\mu$  is the reduced mass and  $\lambda_i$  are strength coefficients. After fitting the  $\lambda_i$  parameters to np phase-shifts the properties and form factors of the deuteron were calculated. A variational method with harmonic oscillator wave functions was used to calculate upper bounds to the binding energy of the double-closed shell nuclei  ${}^4\text{He}$ ,  ${}^{16}\text{O}$  and  ${}^{40}\text{Ca}$  [11].

## 3 Description of NN scattering data

In order to quantify the statistical uncertainties of the NN interaction a fit to experimental data becomes necessary. The usual first step to fit a phenomenological potential to reproduce scattering phase-shifts is not sufficient to get an accurate description of the actual experimental data. As was recently shown in [6], the local chiral effective potential of [12] fitted to phase-shifts yields a significantly large  $\chi^2/\text{d.o.f.}$  value when compared to experimental scattering data. However it is possible that small readjustments of the potential parameters have a significant impact in lowering the total  $\chi^2$ . Given the wide applicability of this type of interactions [13, 14] a full fledged fit to NN experimental scattering data would be of great interest.

Historically, a successful description of the complete database with  $\chi^2/\text{d.o.f.} \lesssim 1$  has never been possible. The potentials and PWA were gradually improved over time by explicitly including different physical effects like OPE in the long range part, charge symmetry breaking in the central channel and electromagnetic interactions among others. The first PWA with  $\chi^2/\text{d.o.f.} \lesssim 1$  was obtained in 1993 when the Nijmegen group introduced the  $3\sigma$  criterion to exclude over 1000 inconsistent data[15]. The  $3\sigma$  criterion deals with possible over and underestimations of the statistical uncertainties by excluding data sets with improbably high or improbably low values of  $\chi^2$  (for a clear description of this process see [16]). However, this method identifies only inconsistencies between individual data sets and a model trying to describe the complete database. To improve on this method, so that inconsistencies between each data set and the rest of the database can be found, the  $3\sigma$  criterion was applied iteratively to the *complete* database and the potential parameters were refitted to the *accepted* data sets until no more data are excluded or recovered [17]. The self-consistent data base obtained with this procedure contains 6713 experimental points and recovers 300 of initially discarded data with the usual  $3\sigma$

criterion [18]. Although the 300 extra data do not significantly change the potential parameters, their inclusion can only improve the estimate of statistical errors. A simultaneous fit to pp and np scattering data was made representing the short range part of the interaction with a DS potential and OPE for the long range part. The fit requires a total of 46 parameters and yields  $\chi^2/\text{d.o.f.} = 1.04$  to the self-consistent data base [17, 19].

## 4 Description of NN scattering errors

On a more fundamental level, any chi-squared distribution can be used to test goodness of fit, provided that the experimental data can be assumed to have a normal distribution [20, 21]. If the residuals defined as

$$R_i = \frac{O_i^{\text{exp}} - O_i^{\text{theor}}}{\Delta O_i^{\text{exp}}} \quad (2)$$

a theoretical model correctly describes the data if they follow the standard normal distribution  $N(0, 1)$ .<sup>2</sup> This self-consistency condition, which can only be checked *a posteriori* and entitles legitimate error propagation, has usually been overlooked in the NN literature. In [22] a few of these tests are reviewed along with a recently proposed Tail-Sensitive test [23] and it was found that the three potentials DS-OPE [17], DS- $\chi$ TPE [24] and Gauss-OPE [22] have standard normal residuals. The three more recent potentials DS-Born, Gauss- $\chi$ TPE and Gauss-Born also were found to have normally distributed residuals [25]. A simple and straightforward recipe to apply the Tail-Sensitive test to any set of empirical data with a sample size up to  $N = 9000$  was developed in [26]. Thus, this six new potentials are the first to qualify for error estimation in nuclear physics. A direct application of normality is the re-sampling of experimental data via Monte Carlo techniques, as is noted in [27], for a robust analysis of possible asymmetries on the potential parameters distribution. Most recently a Monte Carlo method was used to calculate a realistic statistical uncertainty of the Triton binding energy stemming from NN scattering data [28].

## 5 Conclusions

In this contribution we outline the two main requirements for a correct quantification of the NN statistical uncertainties; a correct description of the experimental data and the reproduction of experimental errors. Although the first one may seem obvious, some widely used interactions in nuclear structure and nuclear reaction calculations are fitted to phase-shifts instead of experimental data and as shown in [6] those two descriptions are not entirely equivalent. The second requirement can be easily reformulated into positively testing for the normality of residuals. Normality of residuals provides a criterion of “falsibility” to distinguish those NN interactions that can come in conflict with observation and those that cannot. Interactions that fail the criterion [5, 29] come in conflict with observation, even if they apparently give a good description of the data, because the chi-squared distribution cannot be used to test goodness of fit, and require more complex statistical techniques to analyze the data. Even though the description here is for statistical uncertainties, a certain glance into the systematic uncertainty can already be perceived by looking into the differences in phase-shifts and scattering amplitude predictions given by the different realistic potentials with  $\chi_{\text{min}}^2/\text{d.o.f.} \sim 1$ . In particular the six interactions fitted to the Granada self-consistent database, give similar statistical uncertainties but present inconsistent phase-shifts at low angular momentum and high energy. The discrepancies between different potentials, accounting for the systematic uncertainty, are usually an order of magnitude larger than the statistical error bars.

<sup>2</sup>More generally, if the sufficient goodness of fit condition  $\chi^2/\nu = 1 \pm \sqrt{2/\nu}$  is not fulfilled one can scale globally the errors  $\Delta O_i^{\text{exp}} \rightarrow \alpha \Delta O_i^{\text{exp}}$  by a common Birge factor  $\alpha$  provided the residuals follow a scaled normal distribution  $N(0, \alpha)$ .

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