

# Proton-Deuteron Scattering and Test of Time-Reversal Invariance

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**Abstract.** The integrated proton-deuteron scattering cross section  $\tilde{\sigma}$  for transversely polarized protons ( $P_y^p$ ) and tensor polarized deuterons ( $P_{xz}$ ) constitutes a null test signal for time-reversal invariance violating but P-parity conserving effects. This cross section will be measured at COSY. Using the generalized optical theorem and Glauber theory we study the null-test observable  $\tilde{\sigma}$  for different types of T-odd P-even NN-interactions. The formalism includes full spin dependence of elementary pN-amplitudes and S- and D-components of the deuteron wave function.

## 1 Introduction

Time-invariance-violating (T-odd) P-parity conserving (P-even) (TVPC) interactions do not arise at the fundamental level within the standard model. This type of interaction can be generated by radiative corrections to the T-odd P-odd interaction discovered in the physics of kaons and B-mesons. However, in this case its intensity is too low to be observed in experiments at present [1]. Thus, observation of TVPC effects would be considered as an indication of physics beyond the standard model.

As was shown in Ref. [2], the total polarized cross section  $\tilde{\sigma}$  of the proton-deuteron scattering with vector polarization of the proton  $p_y^p$  and tensor polarization of the deuteron  $P_{xz}$  constitutes a null-test observable for TVPC effects. The dedicated experiment is planned at COSY [3] at proton beam energy 135 MeV. The first analysis of the TVPC null-test signal [4] was done within the nonmesonic deuteron breakup channel  $pd \rightarrow ppn$  estimated in the single scattering approximation. Recently we used the spin-dependent formalism [5] of the Glauber theory to calculate the cross section  $\tilde{\sigma}$  [6] and "null-combinations" of some differential spin observables of the  $pd$  elastic scattering [7] which deviate from zero if the TVPC effects occur. The formalism includes full spin dependence of elementary pN-amplitudes and S- and D-components of the deuteron wave function. This formalism allows one to explain existing data on the non-polarized differential cross section and spin observables of the elastic  $pd$  scattering at 135 MeV [8]. Here we consider some qualitative arguments concerning the  $\rho$ -meson contribution to  $\tilde{\sigma}$  and briefly explain the role of the deuteron D-wave.

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## 2 Elements of formalism

Time-reversal symmetry conserving and P-parity conserving (TCPC or T-even P-even) interactions lead to the following transition amplitude of the elastic  $pd$  scattering at 0 degree [9]

$$e_{\beta}^{\prime*} M(0)_{\alpha\beta}^{TCPC} e_{\alpha} = g_1[\mathbf{e} \mathbf{e}^{\prime*} - (\mathbf{m}\mathbf{e})(\mathbf{m}\mathbf{e}^{\prime*})] + g_2(\mathbf{m}\mathbf{e})(\mathbf{m}\mathbf{e}^{\prime*}) + ig_3\{\boldsymbol{\sigma}[\mathbf{e} \times \mathbf{e}^{\prime*}] - (\boldsymbol{\sigma}\mathbf{m})(\mathbf{m} \cdot [\mathbf{e} \times \mathbf{e}^{\prime*}])\} + ig_4(\boldsymbol{\sigma}\mathbf{m})(\mathbf{m} \cdot [\mathbf{e} \times \mathbf{e}^{\prime*}]), \quad (1)$$

where  $\mathbf{e}$  ( $\mathbf{e}'$ ) is the polarization vector of the initial (final) deuteron,  $\mathbf{m}$  is the unit vector along the beam momentum,  $\boldsymbol{\sigma}$  is the Pauli matrix,  $g_i$  ( $i = 1, \dots, 4$ ) are complex amplitudes. To the right-hand side of Eq.(1) one can add the TVPC (T-odd P-even) term in a very general form

$$e_{\beta}^{\prime*} M(0)_{\alpha\beta}^{TVPC} e_{\alpha} = \tilde{g}\{(\boldsymbol{\sigma} \cdot [\mathbf{m} \times \mathbf{e}]) (\mathbf{m} \cdot \mathbf{e}^{\prime*}) + (\boldsymbol{\sigma} \cdot [\mathbf{m} \times \mathbf{e}^{\prime*}]) (\mathbf{m} \cdot \mathbf{e})\}, \quad (2)$$

where  $\tilde{g}$  is the TVPC transition amplitude. The matrix elements of the operators (1), (2) are

$$\langle \mu' = \frac{1}{2}, \lambda' = 1 | M^{TCPC} | \mu = \frac{1}{2}, \lambda = 1 \rangle = g_1 + g_4, \quad (3)$$

$$\langle \mu' = \frac{1}{2}, \lambda' = -1 | M^{TCPC} | \mu = \frac{1}{2}, \lambda = -1 \rangle = g_1 - g_4, \quad (4)$$

$$\langle \mu' = \frac{1}{2}, \lambda' = 0 | M^{TCPC} | \mu = \frac{1}{2}, \lambda = 0 \rangle = g_2, \quad (5)$$

$$\langle \mu' = \frac{1}{2}, \lambda' = 0 | M^{TCPC} + M^{TVPC} | \mu = -\frac{1}{2}, \lambda = 1 \rangle = \sqrt{2}g_3 + i\sqrt{2}\tilde{g}, \quad (6)$$

$$\langle \mu' = \frac{1}{2}, \lambda' = -1 | M^{TCPC} + M^{TVPC} | \mu = -\frac{1}{2}, \lambda = 0 \rangle = \sqrt{2}g_3 - i\sqrt{2}\tilde{g}. \quad (7)$$

where  $\mu$  ( $\mu'$ ) and  $\lambda$  ( $\lambda'$ ) are spin projections of the initial (final) proton and deuteron on the beam direction, respectively. All diagonal matrix elements of the  $M^{TVPC}$  operator are zeros.

The total cross section of the  $pd$  scattering has the form [8]

$$\sigma_{tot} = \sigma_0 + \sigma_1 \mathbf{p}^p \cdot \mathbf{p}^d + \sigma_2 (\mathbf{p}^p \cdot \mathbf{m})(\mathbf{p}^d \cdot \mathbf{m}) + \sigma_3 P_{zz} + \tilde{\sigma} p_y^p P_{xz}^d, \quad (8)$$

where  $\mathbf{p}^p$  ( $\mathbf{p}^d$ ) is the vector polarization of the initial proton (deuteron) and  $P_{zz}$  and  $P_{xz}$  are the tensor polarizations of the deuteron. The OZ axis is directed along the proton beam momentum  $\mathbf{m}$ , OY  $\uparrow\uparrow \mathbf{p}^p$ , OX  $\uparrow\uparrow [\mathbf{p}^p \times \mathbf{m}]$ . In Eq. (8) the terms  $\sigma_i$  with  $i = 0, 1, 2, 3$  are non-zero only for T-even P-even interactions corresponding to Eq. (1) and the last term  $\tilde{\sigma}$  constitutes a null-test signal of T-invariance violation with P-parity conservation. Using the generalized optical theorem we find  $\tilde{\sigma} = -4\sqrt{\pi}Im\frac{2}{3}\tilde{g}$ .

Hadronic amplitudes of  $pN$  scattering are taken as [5]

$$M_N(\mathbf{p}, \mathbf{q}; \boldsymbol{\sigma}, \boldsymbol{\sigma}_N) = A_N + C_N \boldsymbol{\sigma} \hat{\mathbf{n}} + C'_N \boldsymbol{\sigma}_N \hat{\mathbf{n}} + \sum_{l=n,q,k} B_N^l(\mathbf{q})(\boldsymbol{\sigma} \hat{\mathbf{l}})(\boldsymbol{\sigma}_N \hat{\mathbf{l}}), \quad (9)$$

where  $\hat{\mathbf{q}}$ ,  $\hat{\mathbf{k}}$  and  $\hat{\mathbf{n}}$  are defined as unit vectors along the vectors  $\mathbf{q} = \mathbf{p} - \mathbf{p}'$ ,  $\mathbf{k} = \mathbf{p} + \mathbf{p}'$  and  $\mathbf{n} = [\mathbf{k} \times \mathbf{q}]$ , respectively;  $\mathbf{p}$  ( $\mathbf{p}'$ ) is the initial (final) proton momentum;  $\boldsymbol{\sigma}_N$  is the Pauli matrix acting on the spin state of the nucleon  $N$ . We consider the following terms of the TVPC NN interaction which were under discussion in Ref. [4]:

$$t_{pN} = h_N [(\boldsymbol{\sigma} \cdot \mathbf{k})(\boldsymbol{\sigma}_N \cdot \mathbf{q}) + (\boldsymbol{\sigma}_N \cdot \mathbf{k})(\boldsymbol{\sigma} \cdot \mathbf{q}) - \frac{2}{3}(\boldsymbol{\sigma}_N \cdot \boldsymbol{\sigma})(\mathbf{k} \cdot \mathbf{q})] / m_p^2 + g_N [\boldsymbol{\sigma} \times \boldsymbol{\sigma}_N] \cdot [\mathbf{q} \times \mathbf{k}] / m_p^2 + g'_N (\boldsymbol{\sigma} - \boldsymbol{\sigma}_N) \cdot i [\mathbf{q} \times \mathbf{k}] [\boldsymbol{\tau} \times \boldsymbol{\tau}_N]_z / m_p^2. \quad (10)$$

Here  $\boldsymbol{\sigma}$  ( $\boldsymbol{\sigma}_N$ ) is the Pauli matrix acting on the spin state of the proton (nucleon  $N = p, n$ ),  $\boldsymbol{\tau}$  ( $\boldsymbol{\tau}_N$ ) is the corresponding matrix acting on the isospin state;  $m_p$  is the proton mass. In the framework of the phenomenological meson exchange interaction the term  $g'$  corresponds to  $\rho$ -meson exchange, and  $h$ -term provides the axial meson  $h_1$  exchange.

## 2.1 $g'$ -term

The  $g'$  term contributes only to the charge exchange transitions, because the non-zero matrix elements of the isospin-operator connected with the  $g'$  term in Eq. (10) are the following

$$\langle n, p | [\boldsymbol{\tau} \times \boldsymbol{\tau}_N]_z | p, n \rangle = -i2, \quad \langle p, n | [\boldsymbol{\tau} \times \boldsymbol{\tau}_N]_z | n, p \rangle = i2. \quad (11)$$

The isospin matrix element of the C-odd isospin operator  $\mathcal{T}_z = [\boldsymbol{\tau} \times \boldsymbol{\tau}_N]_z$  in Eq. (11) changes the sign under replacement  $p \leftrightarrow n$ . In contrast, the similar matrix elements for T-even P-even (strong) NN-interaction are equal one to other. This difference is one cause for the vanishing of the amplitude  $\tilde{g}$  for the double scattering mechanism of the process  $pd \rightarrow pd$ . As was shown in Ref. [6], the  $g'$ -term gives zero contribution to  $\tilde{g}$  within the Glauber model. Below we discuss this observation briefly.

The TVPC charge-exchange Glauber operator of the double scattering has a form [6]

$$O_{TVPC}^c = -\frac{1}{2} [M_{np \rightarrow pn}(\mathbf{q}_2) t_{pn \rightarrow np}(\mathbf{q}_1) + t_{np \rightarrow pn}(\mathbf{q}_2) M_{pn \rightarrow np}(\mathbf{q}_1)], \quad (12)$$

where  $\mathbf{q}_1 = \mathbf{q}/2 + \mathbf{q}'$  is the transferred momentum in the first and  $\mathbf{q}_2 = \mathbf{q}/2 - \mathbf{q}'$  in the second collision and  $\mathbf{q}$  is the total transferred momentum. For the next step one has to calculate the matrix element of the operator (12) over the deuteron states  $\psi(\mathbf{r})$  with the factor  $\exp(i\mathbf{q}'\mathbf{r})$  and integrate over  $\mathbf{q}'$ . Under the sign of this integral the operator (12) is not changed after the substitution  $\mathbf{q}_1 \leftrightarrow \mathbf{q}_2$  [5, 6]. Therefore, one may add to the right side of Eq. (12) the term  $O_{TVPC}^c(1 \leftrightarrow 2)$  and divide the obtained sum by a factor of 2. In collinear kinematics ( $\mathbf{q} = 0$ ), this symmetry and linear dependence of  $g'$ -term on  $[\mathbf{q} \times \mathbf{k}]$  lead to cancellation of the spin-independent term  $A_N$  in the transition operator (12). The same is true for the  $B_N$  terms in Eq. (9). Thus, only  $C_N$  and  $C'_N$  terms may contribute as [6]

$$O_{TVPC}^c = \frac{g'}{\Pi} \left[ C_n(\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}_1)(\boldsymbol{\sigma}_n - \boldsymbol{\sigma}_p) \cdot \mathbf{n}_1 - C'_n(\boldsymbol{\sigma}_n \cdot \hat{\mathbf{n}}_1)(\boldsymbol{\sigma}_p \cdot \mathbf{n}_1) + C'_n \mathbf{n}_1 \hat{\mathbf{n}}_1 + \right. \\ \left. C_p(\boldsymbol{\sigma} \cdot \hat{\mathbf{n}}_1)(\boldsymbol{\sigma}_p - \boldsymbol{\sigma}_n) \cdot \mathbf{n}_1 - C'_p(\boldsymbol{\sigma}_p \cdot \hat{\mathbf{n}}_1)(\boldsymbol{\sigma}_n \cdot \mathbf{n}_1) + C'_p \mathbf{n}_1 \hat{\mathbf{n}}_1 \right], \quad (13)$$

where  $\mathbf{n}_1 = [\mathbf{k} \times \mathbf{q}']$ ,  $\hat{\mathbf{n}}_1 = \mathbf{n}_1/|\mathbf{n}_1|$  and  $\mathbf{q}' = \mathbf{q}_2 = -\mathbf{q}_1$ ;  $\Pi$  is a constant. In Eq. (13) the  $C'_N$ -terms do not contain the proton beam spin  $\boldsymbol{\sigma}$ . We can show that this is a consequence of Eqs. (11). According to Eqs. (6), (7), it means that the contribution of the  $C'_N \times g'$  term to the amplitude  $\tilde{g}$  is zero. Furthermore, due to Eqs. (11) the remaining terms with  $C_N$  in Eq. (13) contain the difference  $\boldsymbol{\sigma}_n - \boldsymbol{\sigma}_p$ , but not the sum. These terms can be rewritten as  $\mathbf{V}_p \boldsymbol{\sigma}_p + \mathbf{V}_n \boldsymbol{\sigma}_n = 2(\mathbf{V}_p + \mathbf{V}_n)(\boldsymbol{\sigma}_p + \boldsymbol{\sigma}_n) \equiv 0$  [6]. Thus, the contribution of the operator (13) to the amplitude  $\tilde{g}$  vanishes and this fact is directly connected with Eqs. (11).

Strong suppression of the contribution of the  $\rho$ -meson as compared to the axial  $h_1$  meson was found numerically in the Faddeev calculations [10] of the null-test signal for the  $nd$  scattering at 100 keV, but no explanation of this result was offered. We suppose that the cause for this suppression is the same spin-isospin structure of the scattering amplitude which leads to the vanishing  $\rho$ -meson contribution within the Glauber approach.

## 2.2 $h$ - and $g$ -terms

For the single scattering mechanism the amplitude  $\tilde{g}$  vanishes within the Glauber theory. Using Eqs. (3)-(7) for the double scattering mechanism with  $pN$ -amplitudes (9) and (10) we find for the  $h$ - and  $g$ -terms in Eq.(10) that all T-even P-even amplitudes of the  $pd$ -scattering are zeros:  $g_1 = g_2 = g_3 = g_4 = 0$ . Furthermore, we find for the TVPC amplitude

$$\tilde{g} = \frac{i}{4\pi m_p} \int_0^\infty dq q^2 [S_0^{(0)}(q) - 2\sqrt{2}S_2^{(1)}(q)] [C'_n(q)(g_p - h_p) + C'_p(q)(g_n - h_n)], \quad (14)$$

where  $S_0^{(0)} = \int_0^\infty dr u^2(r) j_0(qr)$  and  $S_2^{(1)}(q) = 2 \int_0^\infty dr u(r) w(r) j_2(qr)$  are the elastic form factors of the deuteron, and  $u(r)$  and  $w(r)$  are the S-wave and D-wave  $w(r)$  of the deuteron, respectively, [6]. We can show that the S-D wave interference, not considered in Ref.[6], considerably diminishes the null-test signal  $\tilde{\sigma}$  at the energies of the planned COSY experiment [3]  $\sim 100$  MeV as compared to the pure S-wave contribution and provides an enhancement at 700-800 MeV.

### 3 Summary

In contrast to Ref. [4] we show, using the optical theorem, that within the single scattering approximation the null-test observable  $\tilde{\sigma}$  is zero. Our result obtained within the Glauber theory is formulated by Eq. (14). Only the amplitude  $C'_N$  appears in Eq.(14) whereas other T-even P-even  $pN$  amplitudes, which were found in Ref. [4] to contribute to the TVPC null-test signal, are absent in Eq. (14). Furthermore, we find the deuteron D-wave gives a valuable contribution to the null-test signal for the case of the  $h$ - and  $g$ -type of interaction. The  $g'$ -term caused by the  $\rho$ - meson exchange in the TVPC NN-interaction makes a zero contribution to  $\tilde{\sigma}$  and this result is true in the case when both the S- and D-components of the deuteron wave function are taken into account. We discuss some symmetry arguments to clarify a cause for the vanishing contribution of the  $g'$ -term. The  $g'$ -optical potential [11] and the corresponding coupling constant of the  $\rho$ -meson to the nucleon  $\bar{g}_\rho$  is widely used as a measure of intensity of the TVPC effects [12, 13]. Since the  $g'$ -term gives zero contribution to  $\tilde{\sigma}$  within the Glauber theory, this parameter cannot be applied straightforwardly for the nucleon-deuteron scattering as a scale of the TVPC interactions at large enough energies. However, the  $g'$ -term can give contribution to the null-test signal  $\tilde{\sigma}$  if this interaction is included into the deuteron bound state [6]. One-pion exchange is excluded from the TVPC NN-interaction [12], however, two-pion exchange probably contributes similarly to the P-violating  $pp$ -interaction [14]. Finally, TVPC NN forces can contribute to the electromagnetic p-d interaction due to the toroidal quadrupole form factor of the deuteron [15].

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