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Hyperon binding energy in ${}^{6}_{\Lambda}$He and ${}^{7}_{\Lambda}$He

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Abstract. The three-body approach based on the configuration space Faddeev equations for systems of non-identical particles is proposed to describe light hypernuclei (A=6,7, S=-1) with α particle clustering. We focus on the model $(\alpha+\Lambda)+n+n$ for ${}^{7}_{\Lambda}$He hypernucleus for which the first experimental data have been recently reported. New evaluation for hyperon binding energy in ${}^{7}_{\Lambda}$He is done by using a relation between energies of the spin doublet (1^-,2^-) of ${}^{6}_{\Lambda}$He and the ${}^{7}_{\Lambda}$He ground state. Energies of low-lying levels of ${}^{6}_{\Lambda}$He hypernucleus are calculated within the cluster $\alpha+\Lambda+n$ model.

1 Introduction

P-shell hypernuclei, like the ${}^{7}_{\Lambda}$He and the ${}^{6}_{\Lambda}$He are being considered as appropriate systems to study the ΛN interaction and in particular their spin dependence [1]. The ${}^{7}_{\Lambda}$He hypernucleus is of some interest for studying the CSB effect of ΛN interaction [2]. The first reliable experimental ${}^{7}_{\Lambda}$He data obtained at the Jlab experiments (E01-011, E05-115) have been reported in Refs. [2, 3]. Theoretical investigations for ${}^{7}_{\Lambda}$He within the frameworks of three and four-body cluster models were undertaken in Refs. [4–7]. Up to now, the experimental data and predictions for the ground state energy are in disagreement. The experimental values for hyperon energy B_Λ are larger than the theoretical ones [8].

The ${}^{6}_{\Lambda}$He hypernucleus has been theoretically studied in Ref. [6, 9] within the cluster $\alpha+\Lambda+n$ model. Experimental data for ${}^{6}_{\Lambda}$He are restricted by the value of 1^- ground state energy. The recent theoretical result [6] for the energy is within experimental errors, but slightly overebound the system. The spin doublet (1^-, 2^-) of ${}^{6}_{\Lambda}$He is interesting due to study of spin dependence of the ΛN interaction [9].

In the present work, three-body cluster approach is applied for studying the ${}^{7}_{\Lambda}$He and the ${}^{6}_{\Lambda}$He nucleus. The configuration space Faddeev equations [10] are used for calculation of low-lying level energies of these hypernuclei. The system $\alpha+\Lambda+n+n$ can be consider as the four-body model of ${}^{7}_{\Lambda}$He hypernucleus that includes the bound subsystems $\alpha+\Lambda$ as ${}^{5}_{\Lambda}$He nucleus, $\alpha+\Lambda+n$ as ${}^{6}_{\Lambda}$He nucleus and $\alpha+n+n$ as ^{6}He nucleus. Thus, the consideration will be completed if all subsystems have been taken into account. It is clear that some correlations can be found between energies of the systems.

2 Theoretical assumptions

${}^{7}_{\Lambda}$He hypernucleus is modeled as the cluster system $(\alpha+\Lambda)+n+n$ [5, 11]. Here, the $\alpha+\Lambda$ pair is considered as ${}^{5}_{\Lambda}$He hypernucleus in the ground state with the hyperon energy of 3.12 MeV. The

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folding procedure is applied to construct the ${}^{5}_{\Lambda}$He-n potential [5]. In the $\alpha + \Lambda + n$ system, the Λn interaction is averaged by coordinates of a hyperon in ${}^{5}_{\Lambda}$He. One can assume that the two-body ${}^{5}_{\Lambda}$He-n model is a good approximation for ${}^{6}_{\Lambda}$He hypernicleus. However, an adjustment for the coefficients of spin-dependent part of effective ${}^{5}_{\Lambda}$He-n potential in p-wave is required. The adjustment can be done by reproducing the binding energy of the 1^- and the excitation energy of the 2^- states of ${}^{6}_{\Lambda}$He nucleus.

The s- and p-wave components of the folding potential are written in the form $V^{l=0,t(s)}_{\text{Core}-n} = \gamma V^{l=0}_{\alpha n} + V^{f,t(s)}_{\Lambda n}$, $V^{l=1,t(s)}_{\text{Core}-n} = V^{l=1}_{\alpha n} + \eta^{t(s)} V^{f,t(s)}_{\Lambda n}$. Here are three free parameters γ, η^t, η^s and Core means ${}^{5}_{\Lambda}$He. In this expression, the upper indicies s and t mean singlet and triplet spin states of the pair Core + n. The coefficient γ is adjusted using the experimental value for the ${}^{6}_{\Lambda}$He bound state energy within the $\alpha + n + n$ model [11]. We choose the spin-dependent coefficient $\eta^{t(s)}$ to reproduce the experimental and theoretical data for the 2^- and 1^- states of the ${}^{6}_{\Lambda}$He hypernucleus, respectively. $V^{f,t(s)}_{\Lambda N}$ is the triplet(singlet) component of the folding ΛN potential: $V^{f,t(s)}_{\Lambda N}(|\vec{r}|) = \int d\vec{\xi}\rho(|\vec{\xi}|) V^{s(t)}_{\Lambda N}(|\vec{r} - \vec{\xi}|)$, where $\rho(|\vec{r}|)$ is the Λ-particle density function for the ${}^{5}_{\Lambda}$He hypernucleus ($\int d\vec{r}\rho(|\vec{r}|) = 1$). $V^{s(t)}_{\Lambda N}$ is the s-wave component of ΛN potentials. $V^{l}_{\alpha N}$ is the partial component (s or p-wave) of the αN potential (spin-orbit coupling is not taken into account). Finally, we present the effective Core $-$ n potential in the form averaged over spins: $V^{l,\text{eff.}}_{\text{Core}-n} = \frac{3}{4} V^{l,t}_{\text{Core}-n} + \frac{1}{4} V^{l,s}_{\text{Core}-n}$. Because the experimental value for the 2^- state is still unknown, such three-body model encounters a difficulty. We have obtained the energy of 2^- as a result of our calculations for ${}^{6}_{\Lambda}$He within the cluster $\alpha + \Lambda + n$ model.

Numerical modeling for Core + n + n and $\alpha + \Lambda + n$ systems is based on the differential Faddeev equations. The set of potentials for the models includes the simulated NSC97f potential [12] for ΛN interaction, the TH $\alpha\Lambda$ potential taken from [13] and αn potential proposed in Ref. [14]. For nucleon-nucleon interaction we used the Malfliet-Tjon potential (MT-I-III) corrected by Friar et al. [15]. Calculations with this semi-realistic potential reproduce well the experimental data for three- and four-nucleon systems.

3 Numerical Results

The ${}^{7}_{\Lambda}$He binding energy is evaluated by calculating the correlation between the ${}^{7}_{\Lambda}$He binding energy and the excitation energy of the 2^- state of ${}^{6}_{\Lambda}$He nucleus. The strengths of the singlet and triplet components of the ${}^{5}_{\Lambda}$He-n potential consider as variable parameters of the model. These parameters were chosen to reproduce the ${}^{6}_{\Lambda}$He(2^-) excitation energy obtained by direct three-body $\alpha + \Lambda + n$ calculation. The calculated value for the binding energy of the ${}^{6}_{\Lambda}$He(1^-) is -0.65 MeV. The energy is measured from the ${}^{5}_{\Lambda}$He+n threshold. The energy of the ${}^{6}_{\Lambda}$He(2^-) resonance is about 0.11 MeV. Because the experimental value for the ${}^{6}_{\Lambda}$He ground state was reported to be E_Λ=-0.17± 0.10 MeV, our cluster model underestimates the binding energy of the hypernucleus. It is not surprising that the similar cluster system $\alpha + n + n$ is also weakly bound when calculation is performed with pair potentials only. To reproduce the ground state energy of ^{6}He, a three-body potential is used. One can assume that a weak three-body force is also needed to reproduce the experimental ${}^{6}_{\Lambda}$He data. The correlation between the Λ-hyperon energy B_Λ in ${}^{7}_{\Lambda}$He and the excitation energy E_x of the 2^- state of ${}^{6}_{\Lambda}$He hypernucleus is presented in figure 1a). The 1^- and 2^- energies are defined by the adjusting $\eta^{t(s)}$ parameters of the ${}^{5}_{\Lambda}$He$-n$ folding potential. The correlation has linear dependence. We calculate its dependence for the different values $E_{g.s.}$ of the ${}^{6}_{\Lambda}$He(1^-) ground state energy. The obtained value is 0.18 MeV for the 2^- excitation energy of ${}^{6}_{\Lambda}$He within three-body model. This result is presented by the vertical dashed line. The corresponding ${}^{6}_{\Lambda}$He(1^-) binding energy $E_{g.s.}$ is -0.16 MeV as is following from figure 1. The correlated value of the ${}^{7}_{\Lambda}$He hyperon binding energy is 5.69 MeV. The result is close to recently reported experimental value of 5.68±0.03±0.25 MeV [3]. This value is noted by the horizontal line Jlab(E01-111) with the experimental error given by the square left bracket.

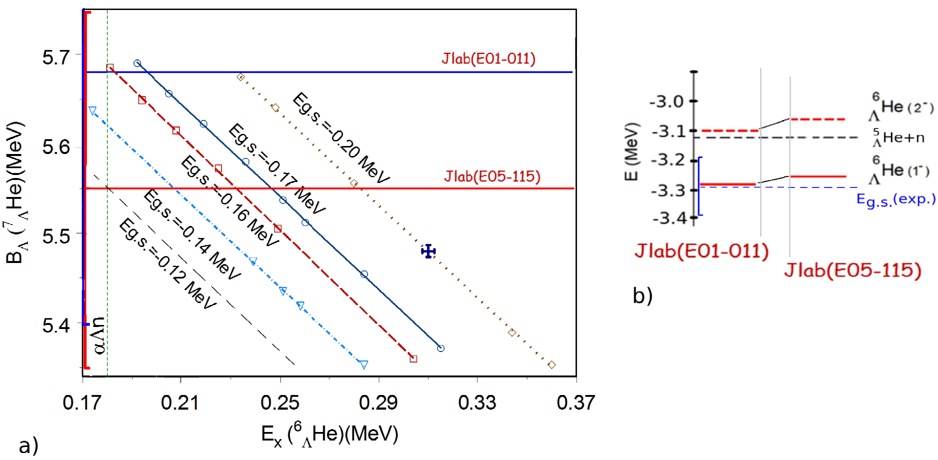

Figure 1. a) The correlation between the binding energy B_Λ of Λ-hyperon in ${}^{7}_{\Lambda}$He and the excitation energy E_x of the 2^- state of ${}^{6}_{\Lambda}$He hypernucleus for different values $E_{g.s.}$ of the ${}^{6}_{\Lambda}$He(1^-) ground state binding energy. The energies of 1^- and 2^- states are defined by the $\eta^{t(s)}$ parameters of the ${}^{5}_{\Lambda}$He$-n$ folding potential. The three-body $\alpha\Lambda n$ result for ${}^{6}_{\Lambda}$He is presented by vertical dashed line. The experimental data are shown by horizontal line with the experimental error given by the square bracket. Result of calculations [6] is shown by a cross. b) The 1^- ground state energy of ${}^{6}_{\Lambda}$He obtained by the correlation of 1a) for two experimental data (Jlab E01-011 and E05-115) for ${}^{7}_{\Lambda}$He. The ${}^{6}_{\Lambda}$He data are shown by horizontal line with the experimental error given by the square bracket. The level 2^- is presented for the excitation energy E_x of 0.18 MeV.

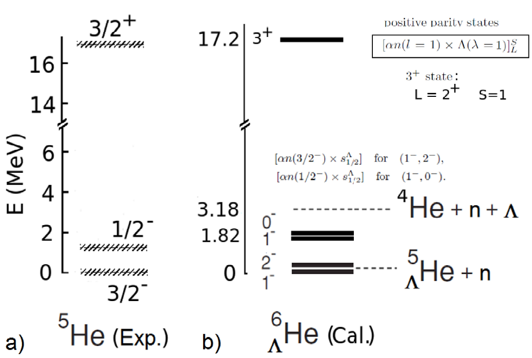

Figure 2. Excitation energies of low-lying levels of a) ^{5}He (experimental data) and b) ${}^{6}_{\Lambda}$He (calculated). The total spin J^π of each level is shown. Description of the spin-orbital configurations for the ${}^{6}_{\Lambda}$He levels is given from Ref.[9]. Λ + Core and $\alpha + \Lambda + n$ thresholds are shown by horizontal dashed line.

Result of calculations [6] is shown by a cross. Note that this result lies on the correlation dependence with $E_{g.s.}$ about -0.2 MeV. The difference between our calculation and the calculation [6] may be related with different sets of pair potential used. The result of our evaluation based on the E05-115 experimental data (5.55±0.10±0.11 MeV [2]) predicting the value of -0.12 MeV for the ${}^{6}_{\Lambda}$He(1^-) binding energy disagrees with existing experimental ${}^{6}_{\Lambda}$He(1^-) data. This situation is illustrated in figure 1b). To estimate the energies and widths of low-lying resonance states, we used the method of analytical continuation in the coupling constant [16]. A variant of this method with an additional non-physical three-body potential is employed. The strength parameter of this potential is considered as a variational parameter (coupling constant) for the analytical continuation of the bound state energy into the complex plane [14]. The structure of the ${}^{6}_{\Lambda}$He spectrum is studied within the three-body

$\alpha + \Lambda + n$ model. We present results of the Faddeev calculations for several low-lying levels in figure 2. Comparison with ^{5}He experimental data shows that direct relation can be establish between the spectra of ^{5}He and $^6_\Lambda$He. The spectrum of $^6_\Lambda$He is more complicated due to existence additional spin variable which forms the spin-doublets with small spacings reflecting the weak spin dependence of ΛN interaction. Description of the spin-orbital configurations for the $^6_\Lambda$He spectrum has been done in Ref. [9]. However, the excitation energies of the levels in [9] are quite different from ones shown in figure 2 due to strong ΛN potential used in [9] which overestimates the $^6_\Lambda$He binding energy. It has been previously shown in Ref. [11] that the low-lying states of the $^7_\Lambda$He nucleus can be qualitatively classified by analogy for corresponding states of the ^{6}He nucleus, within the ^{6}He(J$^\pi$) + $\Lambda(s)$ form. This analogy is generally supported in Ref. [7] where the calculations for the low-lying $^7_\Lambda$He states have been performed by using the four-body cluster model. The recent $^7_\Lambda$He experimental data [2] show the ground state and the excited state peaks of ^{7}Li$(e, eK^+)^7_\Lambda$He reaction (Jlab E05-115). The second peak is assumed to be related with the first 2^+ level of ^{6}He. According to the analysis in Ref. [7], the second 2^+ level of ^{6}He has to generate third excitation peak corresponding to energy $3/2_2$ $(5/2_2)$ level of the $^7_\Lambda$He.

4 Conclusion

In the presented work we focused on the correlation between energies of the spin doublet $(1^-,2^-)$ of $^6_\Lambda$He and the $^7_\Lambda$He ground state. The correlation allowed us to evaluate both $^6_\Lambda$He and $^7_\Lambda$He hyperon energies and to show relation to the existing experimental data. Another established correlation is between spectra of core nucleus and hypernucleus like ^{5}He and $^6_\Lambda$He, ^{6}He and $^7_\Lambda$He. These correlations are possible due to known "glue-like role" of hyperon in nuclear matter.

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