

Study of the $KK\bar{K}$ system in hyperspherical formalism

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Abstract. The three-body kaonic system $KK\bar{K}$ is studied in the framework of a single-channel potential nonrelativistic model using the method of hyperspherical harmonics in momentum representation. Calculations are performed with two sets of KK and $K\bar{K}$ potentials. A quasibound state for the $KK\bar{K}$ system with spin-parity 0^- and isospin $1/2$ is found below the three-kaon threshold.

Recently, there has been special interest in few-body systems constituted of two or more kaons. Particularly noteworthy is the possibility of formation of quasibound states in a $KK\bar{K}$ system. We study the $KK\bar{K}$ system using a nonrelativistic potential model within the framework of the hyperspherical harmonics (HH) method in momentum representation and consider the $KK\bar{K}$ system as three interacting kaons. Once the two-body interactions for the $K\bar{K}$ and KK subsystems are defined one can determine the wave function of the $KK\bar{K}$ system by solving the Schrödinger equation $H\Psi = E\Psi$ for the Hamiltonian $H = \hat{T} + V_{\bar{K}K}(r_{12}) + V_{KK}(r_{13}) + V_{\bar{K}K}(r_{23})$. Here \hat{T} is the operator of the kinetic energy and the potentials of the effective KK and $K\bar{K}$ interactions are $V_\alpha(r_{ij})$, where the index α indicates the type of interaction: $\alpha \in KK, K\bar{K}$. For the description of the effective kaon-kaon interactions we use the local potentials from Refs. [1] and [2] that can be written in one-range Gaussian form as $V_\alpha(r) = \sum_{l=0,1} U_\alpha^l \exp[-(r/b)^2] P_\alpha$, where b is the range parameter and P_α is the isospin projection

operator. The set of values of the potential depth for $K\bar{K}$ interaction and two optimized values for the range parameter (set A and set B) are given in Table 1. In Ref. [2] the $K\bar{K}$ interaction is derived under the assumption that $K\bar{K}$ forms the quasibound states $f_0(980)$ and $a_0(980)$ in the $I = 0$ and $I = 1$ channels, respectively, and reproduces the masses and widths of these resonances. Table 1 also presents sets of parameters (A1 and B1) and (A2 and B2) for the effective KK interaction that reproduce lattice QCD calculations [3] for the scattering lengths $a_{\bar{K}\bar{K}} = -0.14$ fm and $a_{\bar{K}K} = -0.10$ fm, respectively. Starting with the Hamiltonian for the $KK\bar{K}$ system one can write the integral Schrödinger equation in the momentum representation. Then, following Ref. [4], we expand the wave function Ψ of the $KK\bar{K}$ system in terms of the symmetrized HH in momentum space and substitute this expansion into the integral Schrödinger equation. As a result, we obtain a system of coupled integral equations for the hyperradial functions. The solution of this system of equations allows us to construct the wave function Ψ for the $KK\bar{K}$ system and determine the binding energy B . The convergences of binding energy calculations for the ground state of the $KK\bar{K}$ system as a function of the grand angular momentum for the sets of parameters A2 and B1 are shown in Table 2. A reasonable convergence for the ground state energy is reached for the grand angular momentum 10, and we limit our considerations to this

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Table 1. Sets of parameters for the $K\bar{K}$ and KK interactions.

Interaction	A ($b = 0.66$ fm)		B ($b = 0.47$ fm)	
	$U^{I=0}$, MeV	$U^{I=1}$, MeV	$U^{I=0}$, MeV	$U^{I=1}$, MeV
$K\bar{K}$	$-630 - 210i$	$-630 - 210i$	$-1155 - 283i$	$-1155 - 283i$
KK ($a_{\bar{K}K} = -0.14$ fm)	0	104	0	313
KK ($a_{\bar{K}K} = -0.10$ fm)	0	70	0	205

Table 2. The convergence of the ground state energies of the $KK\bar{K}$ system for sets of parameters A1 and B2.

	$\mu = 0$	$\mu = 2$	$\mu = 4$	$\mu = 6$	$\mu = 8$	$\mu = 10$
B , MeV, for A2	20.6	22.4	23.3	23.8	23.9	24.2
B , MeV, for B1	15.6	16.9	17.7	18.1	18.3	18.6

Table 3. Results of calculations and comparison with the Faddeev and variational calculations from Ref. [5].

	Faddeev [5]	Variational [5] with A1	A1	A2	B1	B2
Mass, MeV	1420	1467	1464.1	1463.8	1469.4	1468.2
Width, MeV	50	110	48.4	49.1	42.0	41.1
B , MeV		21	23.9	24.2	18.6	19.8
$\sqrt{\langle r^2 \rangle}$, fm		1.6	1.61	1.56	1.67	1.63

value. The width of the bound state is evaluated from the imaginary part of the $K\bar{K}$ interactions as $\Gamma = -2 \langle \Psi | \text{Im} (V_{\bar{K}K}(r_{12}) + V_{\bar{K}K}(r_{23})) | \Psi \rangle$. The results of our calculations for the binding energy and the width for the $KK\bar{K}$ system are presented in Table 3. To compare our results with those obtained in Ref. [5] we consider the same K meson mass $m_K = 496$ MeV. The total mass of the $KK\bar{K}$ system ranges from 1463.8 to 1469.4 MeV. The width falls into the 41–49 MeV range for all sets of the $K\bar{K}$ and KK interactions. The quasibound state for the $KK\bar{K}$ with spin-parity 0^- and total isospin $1/2$ is found to be below the three-kaon threshold. The comparison of our results with those obtained with the variational method [5] shows that, while the binding energies calculated within the HH and variational methods are close enough, the difference for the widths are more than 50%. A different scenario is observed for the HH and the Faddeev calculations in the momentum representation: the difference in masses is more than 40 MeV.

Thus, our calculations using the three body nonrelativistic potential model predict a quasibound state for the $KK\bar{K}$ system with a mass around 1460 MeV that can be associated with the $K(1460)$ resonance. Our results support the conclusion obtained through the variational calculations and a coupled-channel approach using the Faddeev equations that $K(1460)$ could be considered a dynamically generated resonance.

References

- [1] Y. Kanada En'yo, D. Jido, Phys. Rev. C **78**, 025212 (2008).
- [2] D. Jido, Y. Kanada-En'yo, Phys. Rev. C **78**, 035203 (2008).
- [3] S.R. Beane, et.al. Phys. Rev. D **77**, 094507 (2008).
- [4] R. Ya. Kezerashvili and Sh. M. Tsiklauri, EPJ Web Conf. **81**, 02022 (2014).
- [5] A. Martinez Torres, D. Jido, and Y. Kanada-En'yo, Phys. Rev. C **83**, 065205 (2011).