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Polarizability of pseudoscalar mesons from the lattice calculations

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Abstract. We explore the energy dependence of neutral and charge pions of the background constant abelian magnetic field in SU(3) lattice gauge theory without dynamical quarks. The energy of neutral pseudoscalar meson diminishes with the field, while the energy of charged one increases according with the theoretical expectations. Based on these data we estimate the magnetic polarizabilities of π^0 and $\pi^{\pm}$ mesons for various quark masses.

1 Introduction

Quantum Chromodynamics in a strong magnetic field presents an interesting topic for research. Magnetic fields of hadronic scale could exist in the Early Universe [1], in cosmic objects like magnetars and can be created in terrestrial laboratories (RHIC, LHC, FAIR, NICA) [2]. Background magnetic field also makes possible to calculate the magnetic polarizabilities of hadrons. To get the dipole magnetic polarizability and hyperpolarizability we measure the energy of a meson as a function of the uniform abelian field. The magnetic polarizabilities are important physical characteristics and describe the distibution of quark currents inside the hadron in the external field. It characterizes its internal structure. The magnetic field is also interesting quantity in connection with COMPASS [3] and JLab experiments. Comparison with the predictions of the experiments and chiral perturbation theory [4, 5] presents interest for fundamental science.

2 Details of calculations

We mention the details of calculations only breafly, because it is presented in our previous works [6, 7]. For the generation of $SU(3)$ gauge configurations we use the tadpole improved Lüscher-Weisz action. We solve Dirac equation numerically

$$D\psi_k = i\lambda_k\psi_k, \quad D = \gamma^{\mu}(\partial_{\mu} - iA_{\mu}) \tag{1}$$

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and find eigenfunctions ψ_k and eigenvalues λ_k for a test quark in the external gauge field A_μ. We find eigenstates of Dirac operator to calculate the correlators. From the correlators we obtain ground state energies. For the calculation of the fermion spectrum we use the Neuberger overlap operator [8]. Fermion fields obey periodical boundary conditions in space and antiperiodic boundary conditions in time. The background gauge field A_μ is a superposition of non-abelian $SU(3)$ gluon field and $U(1)$ abelian uniform magnetic field. We introduce the magnetic field in the symmetric gauge

$$A_{\mu ij} \to A_{\mu ij} + A^B_\mu \delta_{ij}, \;\; A^B_\mu(x) = \frac{B}{2}(x_1\delta_{\mu,2} - x_2\delta_{\mu,1}). \tag{2}$$

Magnetic field is directed along z axis and its value is quantized

$$qB = \frac{2\pi k}{(aL)^2}, \;\; k \in \mathbb{Z}, \tag{3}$$

where $q = -1/3\,e$ is the elementary quark charge.

We consider two types of quarks (u and d) which are degenerate in mass. Our simulations have been carried out on symmetrical lattices with the lattice volumes 16^4, 18^4, 20^4, lattice spacings 0.095 fm, 0.115 fm, 0.125 fm and various bare quark masses in the interval $[0.007, 0.06]$. We use 200-250 statistical independent gauge field configurations for the each lattice data set.

We calculate the following observables in the coordinate space and background gauge field A

$$\langle \psi^\dagger(x) O_1 \psi(x) \psi^\dagger(y) O_2 \psi(y) \rangle_A, \tag{4}$$

where $O_1, O_2 = \gamma_5, \gamma_\mu$ are Dirac gamma matrices, $\mu, \nu = 1, .., 4$ are Lorentz indices, $x = (\mathbf{n}a, n_t a)$ and $y = (\mathbf{n}'a, n'_t a)$ are the lattice coordinates. The spatial lattice coordinate $\mathbf{n}, \mathbf{n}' \in \Lambda_3 = \{(n_1, n_2, n_3) | n_i = 0, 1, ..., N-1\}$, n_t, n'_t are the numbers of lattice sites in the time direction. For the observables (4) the following equation is fulfilled

$$\langle \bar\psi O_1 \psi \bar\psi O_2 \psi \rangle_A = -\mathrm{Tr}\,[O_1 D^{-1}(x,y) O_2 D^{-1}(y,x)] + \mathrm{Tr}\,[O_1 D^{-1}(x,x)] \mathrm{Tr}\,[O_2 D^{-1}(y,y)]. \tag{5}$$

where $D^{-1}(x,y)$ is the Dirac propagator.

We perform Fourier transformation of (5) numerically and choose $\langle \mathbf{p} \rangle = 0$ as we are interested in the meson ground state energy. To obtain the masses we expand the correlation function to the exponential series

$$\tilde{C}(n_t) = \langle \psi^\dagger(\mathbf{0}, n_t) O_1 \psi(\mathbf{0}, n_t) \psi^\dagger(\mathbf{0}, 0) O_2 \psi(\mathbf{0}, 0) \rangle_A = \sum_k \langle 0|O_1|k\rangle \langle k|O_2^\dagger|0\rangle e^{-n_t a E_k}, \tag{6}$$

For a large n_t the main contribution to the correlator (6) comes from the ground state and due to the periodic boundary conditions has the following form

$$\tilde{C}_{fit}(n_t) = A_0 e^{-n_t a E_0} + A_0 e^{-(N_T - n_t) a E_0} = 2A_0 e^{-N_T a E_0/2} \cosh((\frac{N_T}{2} - n_t) a E_0), \tag{7}$$

where A_0 is a constant, E_0 is the ground state energy, a is the lattice spacing.

3 Results

3.1 Magnetic dipole polarizability of π^0

To calculate the magnetic polarizability of mesons we consider sufficiently large magnetic fields. The magnetic field of hadronic scale allows to measure the response of the internal structure to this field.

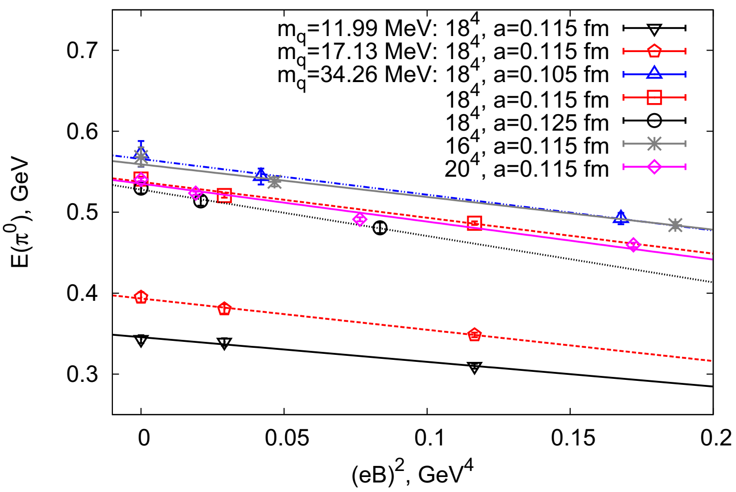

Figure 1. The energy of π^0 ground state as a function of the magnetic field value squared for various lattice volumes, spacings and the quark masses. Points correspond to the lattice data, lines are the lattice data fits obtained using the function (8).

For the neutral pion this is manifested in the deviation of the particle energy from the constant value, for the charged particle we observe the deviation of pion energy from Landau level.

We calculate the energy of neutral pion from the correlation function $C^{PS\,PS} = (\langle\bar{\psi}_d(\vec{0},n_t)\gamma_5\psi_d(\vec{0},n_t)\bar{\psi}_d(\vec{0},0)\gamma_5\psi_d(\vec{0},0)\rangle + \langle\bar{\psi}_u(\vec{0},n_t)\gamma_5\psi_u(\vec{0},n_t)\bar{\psi}_u(\vec{0},0)\gamma_5\psi_u(\vec{0},0)\rangle)/\sqrt{2}$. In Fig.1 we show the energy of π^0 ground state versus the field value squared for the lattice volumes 16^4, 18^4, 20^4, lattice spacings 0.095 fm, 0.105 fm, 0.115 fm, 0.125 fm and quark masses 11.99 MeV, 17.13 MeV, 34 MeV.

The each set of lattice data is fitted at $(eB)^2 \in [0, 0.3\ \text{GeV}^4]$ using the function

$$E = E(B = 0) - 2\pi\beta_m(eB)^2, \tag{8}$$

where $E(B = 0)$ and β_m are the fit parameters. At the considered range of fields the energy dependence is linear on the magnetic field and we calculate the value of β_m from its slope.

The magnetic polarizability is shown in Fig.2 for the lattice volume 18^4 as a function of the quark mass, the β_m values are also depicted for the lattice volumes 16^4, 20^4 and quark mass 34.26 MeV.

The value of β_m diminishes with the quark mass and extrapolation to the chiral limit gives the number $(3.2\pm0.4)\cdot10^{-4}\ \text{fm}^3$, $\chi^2/d.o.f = 0.715$ for the lattice volume 18^4 and lattice spacing 0.115 fm.

The values of magnetic polarizability for various quarks masses are a bit higher than assessments obtained in our previous work [6], where for the neutral particles we considered very simple model with only one type of quarks d. Correlation functions for d and u quarks behave differently in the external magnetic field because u and d quarks possess different absolute value of electric charges, in this work we take this into account. Further improvement of the accuracy in β_m determination might be carried out due to studing the effects of finite lattice volume and lattice spacing, consideration of weaker magnetic fields.

The value of magnetic dipole polarizability obtained within the framework of chiral perturbation theory is equal to $\beta_m = (1.5 \pm 0.3) \cdot 10^{-4}\ \text{fm}^3$ [4].

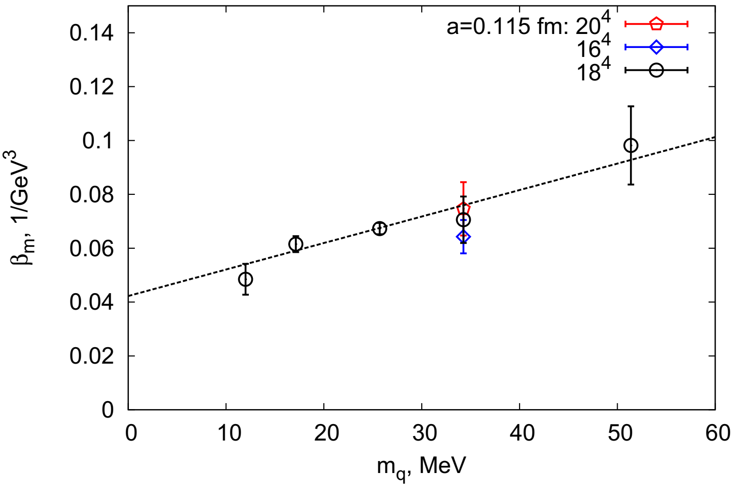

Figure 2. The dipole magnetic polarizability of neutral pion for the lattice volumes 16^4, 18^4, 20^4 and lattice spacing 0.115 fm depending on the quark mass.

3.2 Magnetic hyperpolarizability of π^0

At larger magnetic fields the terms with higher degrees on the magnetic field give contribution to the π^0 energy. In Fig.3 we show the energy of neutral pion for magnetic fields in the interval from 0 to 1.7 GeV^4 with the lattice data fits described by the formula

$$E = E(B = 0) - 2\pi\beta_m(eB)^2 - 2\pi\beta_m^h(eB)^4 - k(eB)^6, \tag{9}$$

where the mass of pion $E(B = 0)$, the dipole magnetic polarizability β_m, the magnetic hyperpolarizability β_m^h and k are fit parameters. The term proportional to B^3 is parity forbidden and π_0 can't decay to three (magnetic) photons.

The decay $\pi^0 \to 4\gamma_M$ is allowed, so the term $\sim (eB)^4$ appears in formula 9. The values β_m^h are negative for the all sets of data represented in Fig.3. For example for the lattice volume 18^4, quark mass 34.26 MeV we get the value $\beta_m^h = (-7 \pm 2) \cdot 10^{-7}$ fm^7 at lattice spacing $a = 0.095$ fm and $\beta_m^h = (-6.7 \pm 1.5) \cdot 10^{-7}$ fm^7 at $a = 0.115$ fm. For the lowest quark mass 17.13 MeV we obtain $\beta_m^h = (-8 \pm 2) \cdot 10^{-7}$ fm^7, $\chi^2/d.o.f. = 1.433$ using for fitting the formula (9).

3.3 Magnetic dipole polarizability of $\pi^\pm$

The energy levels of free charged (non-pointlike) particle in a constant abelian magnetic field parallel to z axis are described by the formula

$$E^2 = p_z^2 + (2n + 1)|qH| - gs_zqH + E^2(H = 0) - 2\pi\beta_mH^2, \quad H = eB. \tag{10}$$

We consider the particle momentum $p_z = 0$, ground state $n = 0$, meson charge $q = \pm 1$, spin $s_z = 0$ and $g = 0$. $E^2(H = 0)$ is the pion energy at zero magnetic field (the mass). So in our case

$$E^2 = |qH| + E^2(H = 0) - 2\pi\beta_mH^2, \quad H = eB. \tag{11}$$

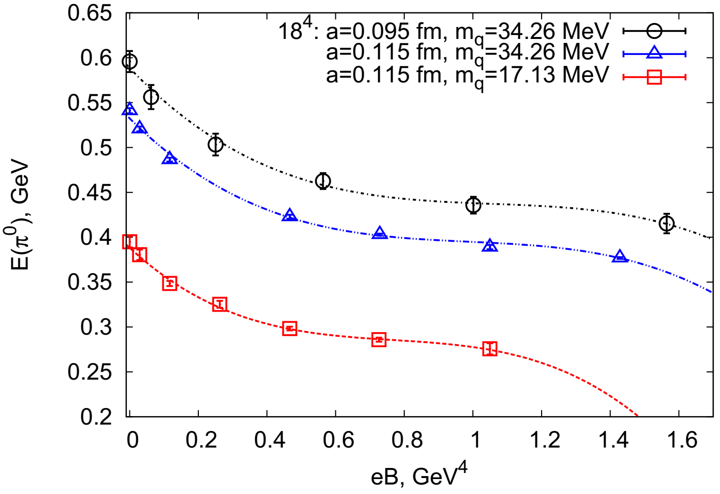

Figure 3. The energy of π^0 ground state as a function of the field value for the lattice volumes 16^4, 18^4, lattice spacings $a = 0.095$ fm, 0.105 fm, 0.115 fm and the quark mass equal to 34.26 MeV. Curves correspond to the fits of lattice data by the function (9).

We calculate the energy of charged pion from the correlation function $C^{PS\,PS} = \langle \bar{\psi}_u(\vec{0}, n_t)\gamma_5\psi_u(\vec{0}, n_t)\bar{\psi}_d(\vec{0}, 0)\gamma_5\psi_d(\vec{0}, 0)\rangle$. Its energy is depicted in Fig.4 for the lattice volumes 18^4, 20^4, lattice spacings 0.095 fm, 0.115 fm, 0.125 fm and quark masses 17.13 MeV, 34.26 MeV. In Fig.4 we also show the lattice data fits obtained using the formula (11).

The value of magnetic dipole polarizability has positive value for the all sets of data considered and its absolute value increases with the quark mass. For example fitting the lattice data at $eB \in [0; 2.5]$ for the volume 18^4, spacing 0.115 fm gives the $\beta_m = (2.8 \pm 0.2) \cdot 10^{-4}$ fm^3, $\chi^2/d.o.f = 4.48$ for the quark mass 17.13 MeV and the value $\beta_m = (1.3 \pm 0.2) \cdot 10^{-4}$ fm^3, $\chi^2/d.o.f = 3.68$ for the quark mass 34.26 MeV. Chiral perturbation theory predicts the value of magnetic dipole polarizability equal $\beta_m = -2.76 \cdot 10^{-4}$ fm^3 [5].

The values of magnetic polarizability obtained for the same lattice quark mass but for different lattice volumes and lattice spacings agree between each other within the errors. The magnetic polarizability depends strongly on the quark mass and chiral extrapolation will be done in the following work.

4 Conclusions

We calculate the ground state energies of neutral and charged pions depending on the value of the external abelian magnetic field for various lattice volumes, spacings and quark mass. From energy dependence we calculate the magnetic dipole polarizability for and magnetic hyperpolarizability of π^0. In the chiral limit the magnetic dipole polarizability is equal to $(3.2 \pm 0.4) \cdot 10^{-4}$ fm^3 for the lattice volume 18^4 and lattice spacing 0.115 fm, that is close to the prediction od ChPT. The further improvement in accuracy might be done in the following work. We didn't observe strong dependence of the β_m value from the lattice volume and lattice spacing. However it strongly depends on the lattice quark mass.

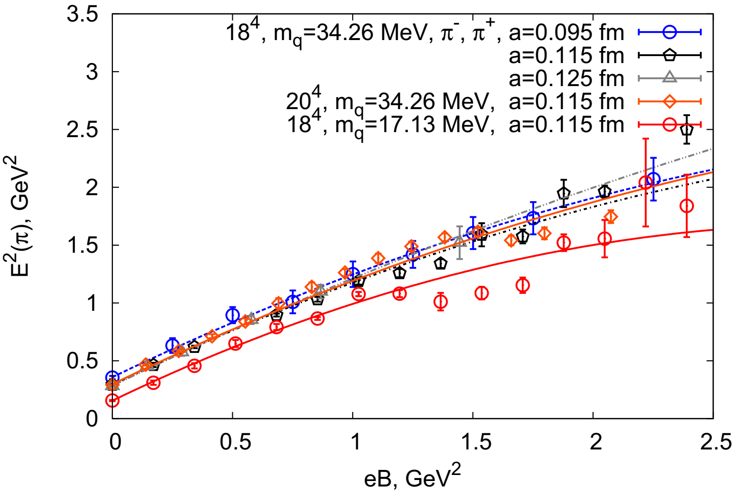

Figure 4. The energy squared of π^{+-} ground state versus the field value squared for the lattice volumes 18^4, 20^4, various lattice spacings and the quark masses 17.13 MeV and 34.26 MeV. Fits of lattice data are performed using the function (11)

We also find the contribution of hyperpolarizability β_m^h to the neutral pion energy. This is very tiny effect and we try to essess its value: for the all sets $\beta_m^h < 0$, for the lowest quark mass $\beta_m^h = (-8 \pm 2) \cdot 10^{-7}$ fm^7.

We calculate the magnetic dipole polarizability of charged pion for several lattice spacings, two lattice volumes and two bare lattice quark mass 34.26 MeV and 17.13 MeV. For the $m_q = 34.26$ MeV and $m_q = 17.13$ MeV we obtain $\beta_m = (1.3 \pm 0.2)10^{-4}$ fm^3 and $\beta_m = (2.8 \pm 0.2)10^{-4}$ fm^3 respectively. The dipole magnetic polarizability depends on the lattice quark mass and chiral extrapolation will be done soon. The sign of $\beta_m(\pi^\pm)$ doesn't agree with the prediction of ChPT [5].

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