

Long-range rapidity correlations between mean transverse momenta

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Abstract. The forward-backward correlation strength between event-mean transverse momenta of particles produced in separated rapidity intervals in high energy hadronic collisions is analyzed in the simple model with the string fusion on a transverse lattice. The coefficient of long-range rapidity p_T - p_T correlation is obtained using the negative binomial distribution for the number of strings, which is closer to reality than the poissonian one. The results demonstrate good agreement with the asymptotes of the correlation coefficient analytically calculated earlier at large string density. It is observed that at LHC energy the p_T - p_T correlation coefficient reveals the drop for most central collisions. The model calculations show that the physical reason for this decrease is the attenuation of color field fluctuations due to the string fusion at large string density, whereas at RHIC energy the string density is not enough to provide a decline of the correlation coefficient for most central collisions.

1 Introduction

As expected, the experimental studies of long-range rapidity correlations can give us the information about the initial stage of high energy hadronic interactions [1]. In [2] the study of the long-range forward-backward (FB) correlations between multiplicities in two separated rapidity windows to find a signature of the string fusion and percolation phenomenon [3–5] in ultrarelativistic heavy ion collisions has been proposed. Later it was realized [6–12] that the investigations of the FB correlations involving intensive observables, such e.g. as the event-mean transverse momentum, enable to obtain clearer signal about the initial stage of hadronic interaction, including the process of string fusion, compared to usual FB multiplicity correlations.

In the present paper we consider the correlation between mean-event transverse momenta of charged particles in separated rapidity intervals [9, 11, 12], as an example of the correlation between intensive observables:

$$p_{tF} = \frac{1}{n_F} \sum_{i=1}^{n_F} |\mathbf{p}_{tF}^i|, \quad p_{tB} = \frac{1}{n_B} \sum_{i=1}^{n_B} |\mathbf{p}_{tB}^i|. \quad (1)$$

We perform the MC calculations in the model, which allows to take into account the string fusion effects by introducing a lattice in transverse plane [7, 8]. We obtain the coefficient of long-range

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rapidity p_r - p_t correlation, b_{pp} , using the negative binomial distribution (NBD) for the fluctuation in number of strings, which is closer to reality than the poissonian one, which was exploited in the previous works [9, 11]. The results demonstrate good agreement with the asymptotes of the correlation coefficient, analytically calculated at large string density in [12]. We also discuss the impact of the obtained results on the b_{pp} correlation coefficient centrality dependence at RHIC and LHC energies.

2 Definitions

To quantify the FB correlation between the event-mean transverse momenta, (1), measured event-by-event in rapidity intervals, $\delta\eta_F$ and $\delta\eta_B$, separated by some rapidity gap, $\Delta\eta$, one usually introduces the FB correlation function (the regression) [9, 11, 12]:

$$\langle p_B \rangle_{p_F} = f(p_F), \quad (2)$$

which gives the dependence of the mean value of the variable p_B on the variable p_F . (From here we will omit the subscript t .) As usually we will characterize the strength of the FB correlation by the correlation coefficient b_{pp} [13, 14]:

$$b_{pp} = \frac{\langle p_F p_B \rangle - \langle p_F \rangle \langle p_B \rangle}{\langle p_F^2 \rangle - \langle p_F \rangle^2} = \frac{\text{cov}(p_F, p_B)}{D_{p_F}}. \quad (3)$$

Another possible option [8, 9, 12]:

$$b'_{pp} = \left. \frac{d\langle p_B \rangle_{p_F}}{dp_F} \right|_{p_F=\langle p_F \rangle}. \quad (4)$$

These two definitions coincide, when the correlation function (2) is linear (a linear regression), yet may differ in a general case (see discussion in [15]).

Due to the locality of strong interaction in rapidity the FB correlation naturally divides into two parts: the short- and the long-range ones. The short-range correlation takes place only between particles produced from a same source (string). It goes to zero for large enough rapidity gaps, $\Delta\eta \gtrsim 2$. The origin of the long-range correlations at large $\Delta\eta$ is event-by-event variance in the number and properties of particle sources (strings [16], cut pomerons [17] and so on), formed in high energy hadronic interaction. In present paper we will consider only this long-range part of the correlation, as we are interested in the study of the variations in number and properties of the strings, produced at the initial stage of the hadronic interaction.

Moreover for the considered correlation between event-mean transverse momenta, compared to the correlation between multiplicities, we expect smaller influence on results of the trivial fluctuations in the number of sources (strings) - the so-called ‘‘volume’’ fluctuations. Being intensive variables, the p_F and p_B do not depend directly on the number of strings. They are sensitive to changes in the characteristics of the sources, e.g. due to the string fusion processes.

3 The model

We will take into account the string fusion effects in simplified form by introducing a lattice in the transverse plane. This discrete version of string fusion model [5] was introduced in [7] and then was exploited for a description of various phenomena (correlations, anisotropic azimuthal flows, the ridge) in high energy hadronic collisions [8–12, 18–20]. In this approach one splits the impact parameter

plane into M cells with the area equal to the transverse area of a single string, σ_{str} , and supposes the fusion of all strings with the centers in a given cell. Then each string configuration can be completely defined by the set of integers:

$$C_\eta = \{\eta_1, \dots, \eta_M\}, \quad (5)$$

where η_i is a number of initial strings fused in a given i -th cell. This leads to the splitting of the transverse area into domains with different, fluctuating values of color field within them.

We suppose that the η_i in each cell fluctuates around the same mean value η with a scaled variance ω_η . Then in accordance with the string fusion prescription [5] the mean number of charged particles in forward and backward observation windows, produced from the fragmentation of fused strings in the i -th cell, is given by the expressions

$$\bar{n}_i^F = \mu_F \sqrt{\eta_i}, \quad \bar{n}_i^B = \mu_B \sqrt{\eta_i}, \quad (6)$$

where μ_F is a mean number of particles produced from decay of a single string in the forward window $\delta\eta_F$. It is clear that μ_F is proportional to the window width: $\mu_F = \mu_0 \delta\eta_F$ and the same for the backward window. Because we study only LRC, we also suppose the independent fragmentation of the i -th fused string in forward and backward windows. So the numbers of particles n_i^F and n_i^B fluctuates independently around the above mean values with some scaled variance ω_μ .

Finally, in accordance with the string fusion ideas [5] we suppose that the transverse momentum distribution of particles, produced from the η_i fused strings, has enlarged mean transverse momentum:

$$\bar{p}(\eta_i) = p_0 \sqrt[4]{\eta_i}, \quad (7)$$

where p_0 is a mean transverse momentum for particles produced from a decay of a single string.

4 Correlation between event-mean transverse momenta

In present work we are using the definition (3) for the MC calculation of the correlation coefficient b_{pp} . This approach requires the calculation only some averages, entering in (3). Whereas all our previous calculations of the correlation coefficients in string fusion model [7–12] were based on the approach in which we have calculated at first the FB correlation function (2) and only then the corresponding correlation coefficient by formula (4).

Basically to calculate b_{pp} by direct MC simulations in this model one has, along with the string configuration C_η , (5), to generate also the following configurations:

$$C_n^F = \{n_1^F, \dots, n_M^F\}, \quad C_n^B = \{n_1^B, \dots, n_M^B\}, \quad (8)$$

$$C_p^F = \{p_1^{1F}, \dots, p_1^{n_1^F F}; \dots; p_M^{1F}, \dots, p_M^{n_M^F F}\}, \quad (9)$$

and the same for C_p^B . Here n_i^F and n_i^B are the numbers of charged particles, produced in forward and backward window from the fragmentation of fused string in i -th cell and p_i^{jF} and p_i^{jB} are the transverse momenta of all n_i^F and n_i^B particles produced from i -th cell in forward and backward window.

Yet such a direct MC calculation of the correlation coefficient is not optimal. The approach, in which part of the computations is performed analytically, proves to be more practical. It allows considerably reduce the amount of MC simulations to get the same accuracy.

As it is shown in [21] in the model under consideration the ingredients, entering the numerator and denominator in the formula (3), can be presented, after averaging over the configurations C_p^F and C_p^B , as follows

$$\langle p_F \rangle = \sum_i \langle \langle n_i^F / n_F \rangle_{C_n^F} \bar{p}(\eta_i) \rangle_{C_\eta}, \quad \langle p_B \rangle = \sum_i \langle \langle n_i^B / n_B \rangle_{C_n^B} \bar{p}(\eta_i) \rangle_{C_\eta}, \quad (10)$$

$$\langle p_F p_B \rangle = \left\langle \left[\sum_i \langle n_i^F / n_F \rangle_{C_n^F} \bar{p}(\eta_i) \right] \left[\sum_k \langle n_k^B / n_B \rangle_{C_n^B} \bar{p}(\eta_k) \right] \right\rangle_{C_\eta}, \quad (11)$$

$$\langle p_F^2 \rangle = \left\langle \sum_i \langle n_i^F / n_F^2 \rangle_{C_n^F} d_{p_i}(\eta_i) + \sum_{i,k} \langle n_i^F n_k^F / n_F^2 \rangle_{C_n^F} \bar{p}(\eta_i) \bar{p}(\eta_k) \right\rangle_{C_\eta}, \quad (12)$$

where $n_F = \sum_{i=1}^M n_i^F$ and $n_B = \sum_{i=1}^M n_i^B$. The $d_{p_i}(\eta_i) \equiv \overline{p^2}(\eta_i) - \bar{p}^2(\eta_i)$ is the variance of transverse momentum for particles produced from a decay of the fused string in the i -th cell.

For popular transverse momentum distributions with one-dimensional parameter \tilde{p} , due to dimensional reasons, we have from (7) $\tilde{p} \sim \sqrt[4]{\eta_i}$. This leads to the following connection:

$$d_{p_i}(\eta_i) = \gamma \bar{p}^2(\eta_i), \quad (13)$$

where the dimensionless parameter γ depends only on the type of the distribution and is not changed by string fusion (see [11, 21]).

In paper [8] it was shown that for the poissonian distribution in n_i^B averaging of the n_i^B / n_B over configurations C_n^B reduces to the simple substitution:

$$n_i^B \rightarrow \bar{n}_i^B = \mu_B \bar{n}(\eta_i) = \mu_B \sqrt{\eta_i}. \quad (14)$$

In general case we can use this substitution for n_i^B and the similar one for n_i^F only as some approximation. The MC simulations show that this approximation works in some cases very well [22, 23]. In this approximation the formulae (10)-(12) are simplified to

$$\begin{aligned} \langle p_F \rangle &= \langle p_B \rangle = \left\langle \frac{\sum_i \bar{n}(\eta_i) \bar{p}(\eta_i)}{\sum_k \bar{n}(\eta_k)} \right\rangle_{C_\eta} & \langle p_F p_B \rangle &= \left\langle \left[\frac{\sum_i \bar{n}(\eta_i) \bar{p}(\eta_i)}{\sum_k \bar{n}(\eta_k)} \right]^2 \right\rangle_{C_\eta}, \\ \langle p_F^2 \rangle &= \gamma \left\langle \frac{\sum_i \bar{n}(\eta_i) \bar{p}^2(\eta_i)}{[\sum_k \bar{n}(\eta_k)]^2} \right\rangle_{C_\eta} + \langle p_F p_B \rangle. \end{aligned} \quad (15)$$

The remaining averaging over configurations C_η in (15) can be done easily by MC simulations.

Taking into account that the event-by-event fluctuation in the number of strings, $N = \sum_i \eta_i$, is non-poissonian in both pp and AA collisions (see e.g. [24]), in present work we have used the NBD as the distribution in the number of strings, which is closer to reality than the poissonian one.

In figure 1 we present the value of the correlation coefficient b_{pp} , (3), between event-mean transverse momenta as a function of η (the η/σ_{str} is a string density), calculated by MC simulations using the formulae (10)-(12) (the points) and by simplified MC simulations based on the formulae (15) (the solid curves). For comparison the results for both the NBD and the poisson distribution in a number of strings are presented (upper and lower curves). In the same figure the asymptotes of b_{pp} at large string density, analytically calculated by the formula

$$b_{pp} = \omega_\eta \mu_F / (\omega_\eta \mu_F + 16\gamma \sqrt{\eta}), \quad (16)$$

obtained in [12], are also presented by the dashed curves.

We see in figure 1, comparing the points and the solid curves, that the simplified MC procedure (15), based on the substitution (14), works very well both for the NBD and the poisson distribution of the number of strings. We see also that the value of the correlation coefficient calculated using the NBD with $\omega_\eta = 2$ is approximately two times higher then for the poisson distribution, as we could expect from the asymptotic formula (16). For both distributions we observe also the good agreement

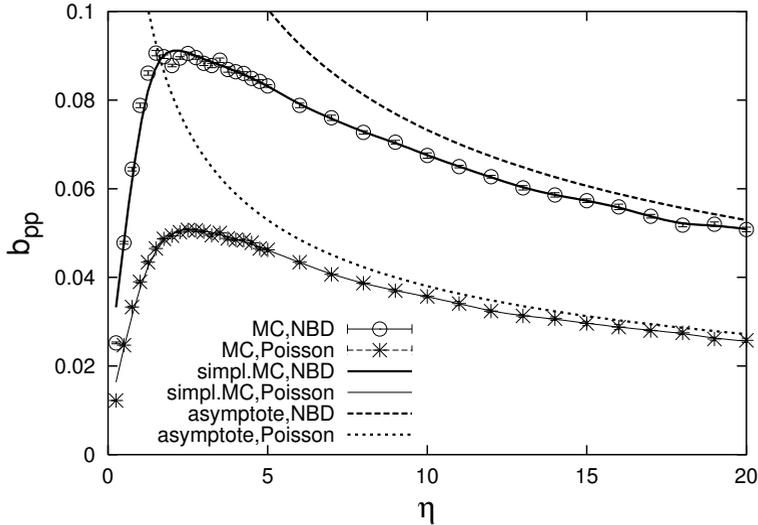


Figure 1. The coefficient b_{pp} , (3), of the long-range FB correlation between event-mean transverse momenta, arising due to string fusion processes, as a function of η (the η/σ_{str} is a string density), calculated for the NBD ($\omega_\eta = 2$) and the poisson distribution ($\omega_\eta = 1$) in a number of strings (upper and lower curves). The results are obtained by 100000 MC simulations, using (10)-(12), (the points) and by simplified MC simulations, using (15), (the solid curves). The dashed curves - the asymptotes of b_{pp} , (16), analytically calculated at large string density in [12]. The value of other parameters: $\mu_F = \mu_B = 1$ and $\omega_\mu = 1$ (the multiplicity and scaled variance in the observation window for a single string); $M = 64$ (the number of cells); $\gamma=0.5$, see the formula (13).

of the MC results with the analytical asymptotes, (16), found in [12] at large string density, yet this asymptote starts to work at larger string density η for the NBD, than for the poisson one.

Basing on the dependency, shown in figure 1, the general behavior of the p_t - p_t correlation coefficient, b_{pp} , as a function of the heavy ion collision energy and centrality can be qualitatively understood. We have to take into account that the maximum string density, achieved for 0-10% of the most central Au-Au collisions at energy 200 GeV/nucleon in RHIC, corresponds to the value $\eta=2.88\pm 0.09$ and the one for 0-5% of the most central Pb-Pb collisions at energy 2.76 TeV/nucleon at LHC reaches $\eta=10.56\pm 1.05$ [11, 25]. So, depending on the collision centrality, it varies approximately in the intervals (0, 3) for RHIC and (0, 11) for LHC.

In figure 1 we see that the increase of the string density η with centrality from 0 to 3 leads to the monotonic growth of the correlation coefficient, which at RHIC energies reaches the highest value in the most central collisions. Whereas at LHC energies the correlation coefficient reaches the highest value at some intermediate value of centrality, which corresponds to the string density $\eta \approx 3$. The further increase of η with centrality from 3 to 11 leads to the decrease of the correlation coefficient, in contrast with the situation at RHIC energies. So at LHC energies the p_t - p_t correlation coefficient reaches the highest value in semicentral collisions at some intermediate value of centrality.

Note that this decrease of the correlation coefficient for the most central collisions at LHC energies corresponds to the decrease of the asymptotic (16) with the growth of η . It is clear from the figure 1 that the RHIC data, corresponding to the η region (0, 3), can not be described by the asymptotic formula (16), which starts to work at $\eta \gtrsim 7 \div 8$. This asymptotic decrease with string density, is reached only for central heavy ion collisions at LHC energies.

5 Concluding remarks

The forward-backward correlation strength between event-mean transverse momenta of particles produced in separated rapidity intervals in high energy hadronic collisions is analyzed in the simple model with the string fusion on a transverse lattice. It is found that at LHC energy the p_t - p_t correlation coefficient must reveal the drop for most central collisions. The model calculations show that the physical reason for this decrease is the attenuation of color field fluctuations due to the string fusion at large string density, whereas at RHIC energy the string density is not enough to provide a decline of the correlation coefficient even for most central collisions. This qualitative conclusion is confirmed by the results [26, 27], obtained in more realistic dipole-based MC string fusion model [28, 29].

The developed methods can be applied to the analysis of the long-range rapidity correlations involving heavy flavor particles. Assuming the sensitivity of the heavy flavor particle yields to the string fusion processes it seems fruitful to study the correlation between the event-mean transverse momentum and the relative yield of strange and charmed particles.

The research was funded by the grant of the Russian Science Foundation (project 16-12-10176).

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