

Experimental study of shear bands formation in a granular material

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Abstract. We present an experimental investigation of the formation of shear bands in a granular sample submitted to a biaxial test. Our principal result is the direct observation of the bifurcation at the origin of the localization process in the material. At the bifurcation, the shear band is spatially extended: we observe a breaking of symmetry without any sudden localization of the deformation in a narrow band. Our work thus allows to clearly distinguish different phenomena: *bifurcation* which is a punctual event which occurs before the peak, *localization* which is a process that covers a range of deformation of several percents during which the peak occurs and finally *stationary shear bands* which are well-defined permanent structures that can be observed at the end of the localization process, after the peak.

1 Introduction

Localization of the deformation and shear band formation in granular materials is an ubiquitous phenomenon which is still very far from being understood [1–7]. Theoretically the phenomenon is described as a bifurcation [2, 8, 9]. Experimental works using full-field measurements have studied the distribution of the deformation in a granular materials showing that strain heterogeneities are observed before failure [3, 10–14] but, to the best of our knowledge, the bifurcation initiating the localization process has never been reported in a quantitative study. Here, we report an experimental study where we are able to characterize objectively the distribution of plastic strain in the sample. Using image analysis tools we are able to exhibit a symmetry breaking in the strain distribution coining the bifurcation that occurs in the system.

Our quantitative measurements also allow to propose some disambiguation about different notions. We will show in the following that we are able to distinguish between three different phenomena:

- *bifurcation*: we are able to evidence a symmetry breaking in the plastic strain distribution which allows us to identify the critical value of the strain at which a bifurcation takes place. This phenomenon can be qualified as sudden as it is punctual during the loading, but it does not correspond to the emergence of a failure plane as the strain field is still diffuse at this stage. This phenomenon occurs before the peak of the loading curve.
- *localization*: starting at the bifurcation, a process of focussing in the plastic strain field is observed from a diffuse, extended field towards a narrow band. We thus underline that localization is not a sudden, catastrophic event but qualifies an evolution in the spatial extension

of the strain distribution from very diffuse field to a strongly confined one.

- *stationary shear bands*: the narrowing of the spatial distribution of the plastic activity finally reaches a stationary state. At this stage well-defined, stationary shear bands are observed.

2 Experiment

The experimental setup is a biaxial test in a plane strain configuration that has been extensively described elsewhere [12].

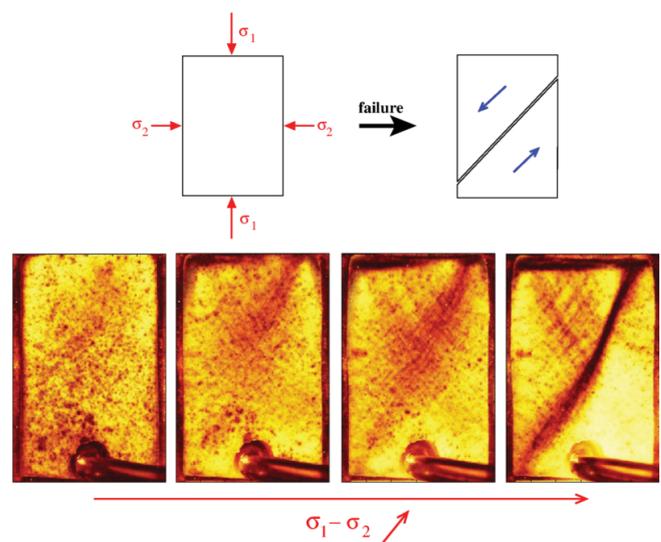


Figure 1. Top: Principle of a biaxial test. Bottom: Examples of deformation maps obtained at different stages of the loading process during the test. Colorcode: yellow, local strain $\approx 10^{-5}$, black, $\geq 10^{-3}$.

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A rectangular sample of size $85 \times 55 \times 25 \text{ mm}^3$ of granular material (glass beads, $d = 70 - 110 \mu\text{m}$, volume fraction ~ 0.60), submitted to a confining pressure of 30 kPa, is loaded by a quasi-static uniaxial compression. An original interferometric measurement method of micro-deformations based on diffusing-wave spectroscopy [15, 16] is used to obtain deformation maps of the sample (see bottom row of Fig. 1). This method gives access to deformation of the order of 10^{-5} all along the loading with a spatial resolution of a few beads diameter. Those maps are directly link to the local deformation between two successive images so that we have access to a spatially-resolved measurement of the incremental strain.

The observed behavior is very reproducible (see References [12, 17] and the movies in supplemental material of those references). Intermittent heterogeneities of the strain are observed from the very beginning of the loading [12–14]. This behavior corresponds to correlated plastic events in the amorphous material and have been analyzed and interpreted in references [13, 18]. The evolution of this pattern as the loading proceeds is complex. Finally, from an axial deformation of about 6.5%, well defined stationary shear bands are observed [12, 17] with an angle corresponding to the Mohr-Coulomb angle [19]. The angles measured directly on the maps are shown in insert Figure 3(a) and the Mohr-Coulomb values deduced independently from the loading curve are indicated on the graph. We see that we have a good correspondence between those angles for each of our experiments, showing that we indeed observe the Mohr-Coulomb angle.

Here we study more specifically the formation of the final shear bands by image analysis of the incremental deformation maps obtained during loading.

3 Image analysis method

We have developed different image analysis tools [17, 20, 21] in order to characterize quantitatively the process of formation of the shear band. We present here only the results obtained using projection analysis. Other methods allows to obtain complementary informations that confort the results presented here (see Ref. [17] for a full detailed study).

The image analysis consists in a projection of the values of the pixels on a line of variable inclination (see Figure 2 top). A projection at a given angle provides a profile of intensity. For an optimal angle of the line a maximum in the profile is detected (see Figure 2 bottom). This optimal profile allows to deduce both the inclination of the shear band (from the angle of the optimal line) and the width of the shear band from the width of the profile.

Those observables allow us to quantify the orientation and degree of localization of the strain distribution process during the loading and to characterize the bifurcation which precedes and initiates the localization process in the material. It also allows us to identify the axial strain from which stationary shear bands can be unambiguously identify.

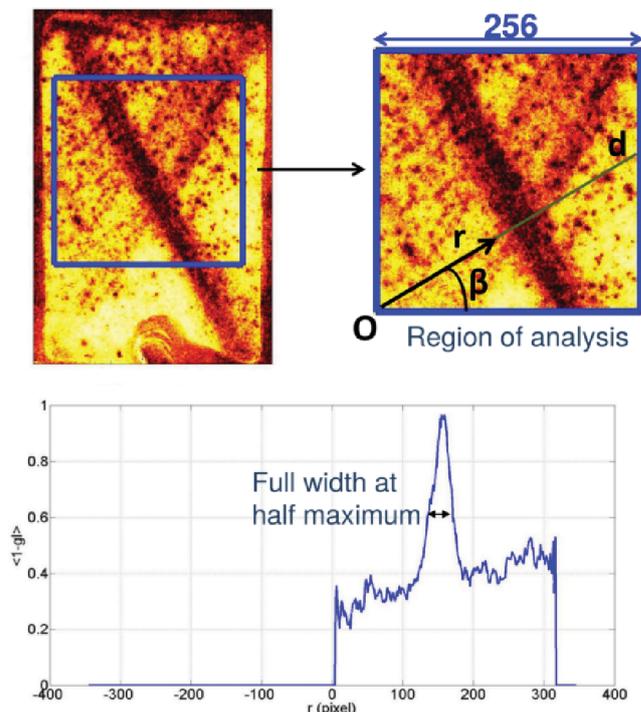


Figure 2. Principle of the image analysis: we define a region of interest for the analysis to avoid artefacts due to boundaries. We project the grey level of the pixels on a line and search for the inclination β leading to a maximum of intensity in the projected profile. We obtain from the inclination of the line the orientation of the strain field and from the width of the curve at half maximum the width of the distribution of the strain.

4 Results

Figure 3 gathers our observations for three different experiments done in the same conditions and corresponding each to a color. Fig. 3(a) shows the loading curves for each test and in inset are shown the final deformation maps also for each experiment. We observe on those maps the final shear bands. The angles measured directly on the maps are in agreement with the ones deduce from the loading curves using a Mohr-Coulomb failure criterium [19].

Figure 3(b) presents the angle of inclination of the strain field obtained from image analysis (see previous section and Fig. 2) as a function of the axial strain for each experiment. Two different zones can be identified: from 0 to roughly 4.5%, the angles are widely fluctuating and the error bars on their determination are huge. This means that during the first half of the loading no definite direction can be determined in the strain field. The plastic field distribution is thus isotropic. After an axial strain of 4.5% a definite and well-resolved angle is observed for each experiments. The angles observed are not identical from one experiment to the other but all the observed angles gather in two groups of values symmetrical with respect to the loading axis. This coins a symmetry breaking and thus a bifurcation: at $\epsilon \approx 4.5\%$ an orientation emerges spontaneously in the strain field. We observe that the chosen direction is well-defined and stays roughly constant until the

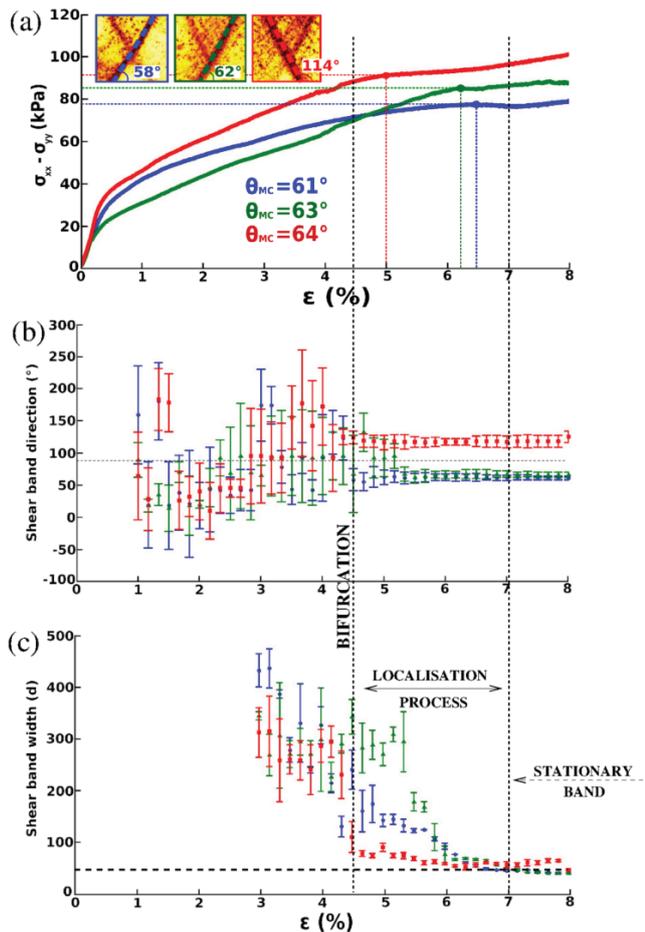


Figure 3. (a) Loading curves of the three different tests done in the same conditions. Insert: final deformation maps exhibiting stationary shear bands. The angles directly measured are in agreement with the Mohr-Coulomb angles θ_{MC} that can be deduced from the loading curves. (b) Principal orientation of the spatial distribution of the deformation field for the same three experiments. (c) Width of the distribution of the deformation field.

end of the test and thus corresponds to the Mohr-Coulomb angle. We underline that we do not observe at this stage a failure plane but only an overall orientation in a diffuse field. This direction is chosen spontaneously by the system between two conjugated symmetrical direction.

Figure 3(b) shows the evolution of the width of the projected profiles as a function of the axial strain for each test. Before the bifurcation this width has no real meaning as the image is approximatively isotropical. We thus measure width of the order of the size of the region of interest. When the bifurcation occurs, the width of the distribution becomes meaningful. We observe in Figure 3(b) that at the bifurcation the profiles are still very large, meaning that the strain field is very diffuse when the bifurcation occurs. From $\epsilon \approx 4.5\%$ to $\epsilon \approx 6.5\%$ we observe a narrowing of the spatial extension of the profiles until a stationary value is reached, identical for all our experiments. We thus observe that the phenomenon of localization is not a sudden event but on the contrary a progressive process that

starts with a bifurcation and ends when the width of the bands have reached a stationary value. Interestingly, such an evolution of the shear band width has been reported in very recent theoretical and numerical works on shear band formation [22].

Our experimental results lead us to distinguish the bifurcation phenomenon from the formation of the failure plane, i.e. the stabilization of shear bands on a constant width. Those two phenomena are not concomitant in a test but on the contrary occurs at two well-separated values of the axial strain from either side of the local maxima of the loading curve. Our analysis allows to clearly identify and distinguish them. A more thorough analysis of this localization process can be found in Ref. [17].

5 Conclusion

We have presented an experimental study of the shear band formation during a biaxial test of a granular sample. Using an unmatched resolution in deformation measurement as well as efficient image analysis tools, we provide a quantitative study of the localization process and most especially of the bifurcation that initiates this localization.

Our experiment clearly shows that it is necessary to distinguish the *bifurcation* which coins a symmetry breaking generating a general orientation in a diffuse, extended strain field, from the establishment of the *stationary shear bands*. Those two phenomena are not simultaneous but on the contrary are well separated during the loading and are respectively observed before and after the local maximum of the loading curve. Between those two events, a *localization* process takes place. We think that our experiment is the first to be able to characterize quantitatively all those phenomena and thus that it will certainly be a benchmark for future theoretical works. An very surprising result is the fact that the angle chosen at the bifurcation does not evolve strongly during the remaining of the loading. Consequently, the Mohr-Coulomb angle emerges in the system very soon during the loading and before a modeling of the system as two parts in frictional sliding has any relevancy.

To understand the micromechanical processes at the origin of the bifurcation, we are presently performing numerical simulations of a bidimensional biaxial test using discrete elements. Those studies should allow us to observe the influence of different parameters on the shear band formation and study the processes underlying the failure process [4–7].

Acknowledgements

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