EPJ Web of Conferences **140**, 02027 (2017)
Powders & Grains 2017
DOI: 10.1051/epjconf/201714002027

On the submerging of a spherical intruder into granular beds

Chuan-Yu Wu[1]*, *Ling* Zhang[1], and *Lan* Chen[2]

[1] *Department of Chemical and Process Engineering, University of Surrey, Guildford, GU2 7XH, UK*
[2] *School of Medical Instrument and Food Engineering, University of Shanghai for Science and Technology, Shanghai 200093, China*

Abstract. Granular materials are complex systems and their mechanical behaviours are determined by the material properties of individual particles, the interaction between particles and the surrounding media, which are still incompletely understood. Using an advanced discrete element method (DEM), we simulate the submerging process of a spherical projectile (an intruder) into granular materials of various properties with a zero penetration velocity (i.e. the intruder is touching the top surface of the granular bed and released from stationary) and examine its settling behaviour. By systematically changing the density and size of the intruder and the particle density (i.e. the density of the particles in the granular bed), we find that the intruder can sink deep into the granular bed even with a zero penetration velocity. Furthermore, we confirm that under certain conditions the granular bed can behave like a Newtonian liquid and the submerging intruder can reach a constant velocity, i.e. the terminal velocity, identical to the settling of a sphere in a liquid, as observed experimentally. A mathematical model is also developed to predict the maximum penetration depth of the intruder. The model predictions are compared with experimental data reported in the literature,good agreement was obtained, demonstrating the model can accurately predict the submerging behaviour of the intruder in the granular media.

1 Introduction

Penetration of a projectile into granular media is a ubiquitous process in nature, astrophysical, geophysical and technological applications, ranging from the penetration of crab's legs and human feet into beach sand, landing of planetary exploratory vehicles, asteroid impact on the moon and earth surface, to the high speed impact of military projectile to defensive materials. Two key questions associated with this process are how deep the projectile can penetrate and how large the impact crater is. Because of its technological relevance, penetration dynamics has been intensively explored over last three centuries. However, due to the complexity of the mechanical behaviour of granular materials and diversity of granular material properties, our understanding of the penetration processes is far from complete. For instance, majority of previous studies focused on the penetration of projectile at high impact speeds and explored the dependence of penetration depth and crater size on impact speed and materials properties. An interesting phenomenon reported in the literature published in the last decade is that a projectile can sink into granular materials even with zero initial impact velocity ([1], [2], [3], [4]). Recently, using a superlight granular material (expanded polystylene beads with a density of 14 kg/m3), Pacheo-Vazquez et al. ([4]) revealed that a projectile could penetrate indefinitely in the granular material with a terminal velocity, which is identical to the settling of a sphere in a Newtonian liquid. This indicates that during the penetration of a projectile into granular media, the granular material acts as a liquid, similar to conventional Newtonian liquids. In other words, for granular materials, there is a potential liquid phase.

Here, with the aim of investigating whether there is a liquid phase for granular materials through which a projectile can reach a terminal velocity and penetrate indefinitely, we employ an advanced numerical method, the discrete element method (see the Methods section), and analyse the penetration of a spherical projectile of different sizes into granular media with zero initial speed, i.e. the projectile is placed on the top surface of the granular bed and released from rest. In contrast to the experimental study of Pacheo-Vazquez et al. ([4]) who used one particular granular material but varied the mass of the projectile, we systematically change the density of the particles in the granular materials for a given projectile, while keeping the initial packing structure of the granular bed identical, and examine how deep the projectile can sink into the granular materials and the kinematics of the projectile during the penetration process.

2 Submerging kinematics

We modeled a 2D system with the discrete element method (DEM), as detailed in Goey et al. [5]. A granular bed was first prepared by sedimentation of a particle system, which was randomly generated, under the action of the

* e-mail: c.y.wu@surrey.ac.uk

DOI: 10.1051/epjconf/201714002027

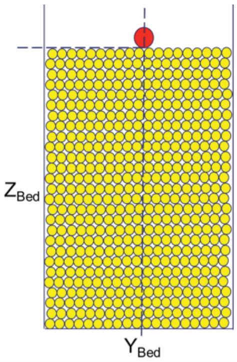

Figure 1. Illustration of the DEM model set up [5]

Table 1. Simulation parameters of granular beds and intruder

Parameter	Granular Bed	Intruder
Diameter	0.005 m	0.01 m
Density	14 to 490kg/m^3	$4,900 kg/m^3$
Young's modulus	3.37 GPa	210 GPa
Poisson's ratio	0.15	0.3

gravity. This bed was modelled with a random packing of 24,000 spheres, with a bed height of 0.8 m. This granular bed was contained in a rectangular container with rigid walls, as shown in Figure 1. The container was wide enough ($> 8d_p$, where d_p is the diameter of particles in the granular bed) to avoid any boundary effects from the side walls. During the sedimentation process, the motions of the spheres were traced until a final stable state was reached (when all the spheres in the bed had settled with a negligible velocity of $< 1x10^{-5} m/s$). A spherical intruder with a diameter of d_i was released with zero impact velocity, directly from the centre of the surface of granular beds. To examine the effect of the density ratio of the system (ranging from 1 to 350) on the subsiding behaviour, the density of the particles in the granular bed (ρ_p) was varied from $14 kg/m^3$ to $490 kg/m^3$, while keeping the density of the intruder constant at $\rho_i = 4900 kg/m^3$. The properties of the granular beds and intruder are presented in Table 1.

We first use a projectile of a diameter d_i that is twice as that of the particles, d_p, in the granular bed, i.e. the projectile/particle diameter ratio $\zeta = d_i/d_p = 2$. The density of the projectile is fixed at $\rho_i = 4900 kg/m^3$, but the density of the particles in the granular bed ρ_p is systematically varied to simulate various materials with a projectile/particle density ratio ($\psi = \rho_i/\rho_p$) in the range of 1 and 350, and to probe the effect of the density ratio ψ on the penetration dynamics. We observe that for $\psi > 1$ the projectile can sink into the granular bed to a certain depth before it reaches the bottom of the container and the penetration depth increases as the density ratio ψ increases (Fig. 2a). Once the projectile is released from stationary, it is accelerated under gravity to reach a maximum penetration velocity, thereafter it is decelerated to a halt at a finite penetration depth (Fig. 2b). These results are qualitatively in agreement with the experimental observations of Lohse et

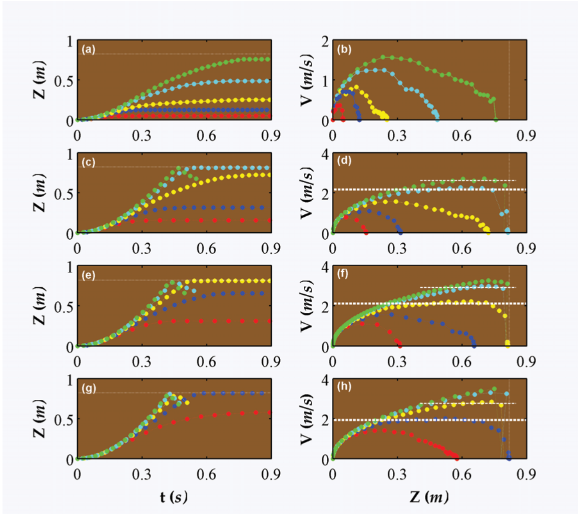

Figure 2. Kinematics of the intruder during submerging

al. [2], Hou et al. [3] and Pacheco-Vazquez & Ruiz-Suarez [4].

Using the same granular beds, we then systematically increase the size of the projectile to $\zeta = 3\ 5$ (see Figs 2c-. 2h). We notice that, for these larger projectiles, the penetration behaviour can be classified into three distinctive regimes depending on the density ratio ψ: 1) At low density ratios, the projectile is accelerated initially but decelerated to a halt before it reaches the bottom of the container as observed for the cases with a diameter ratio $\zeta = 2$. And the maximum penetration depth increases as the density ratio increases and as the diameter ratio increases (Figs . 2d, . 2f and . 2h). 2) At high density ratios, the projectile is accelerated all the way as it sinks until it hits the bottom of the container and bounces back. 3) At the intermediate density ratios, the projectile is initially accelerated until a certain velocity is reached, thereafter it travels at this velocity until it reaches to the bottom of the container, instead of being decelerated, which is similar to the experimental observation of Pacheco-Vazquez et al. [4].

The penetration behaviour in the third regime is interesting as it implies that the projectile reaches a constant velocity, identical to the terminal velocity of a sphere settling in a Newtonian fluid, can penetrate indefinitely in the granular bed if there was no bottom boundary. This also indicates that a liquid phase exists for the penetration of a projectile in granular materials, in which the projectile can penetrate indefinitely with a terminal velocity. On examining the penetration behaviour shown in Fig. 2, we find that the transition from frictional granular solids to the liquid phase depends on not only the density ratio ψ but also the diameter ratio ζ.

3 Penetration model

To understand the phase transition behaviour, it is necessary to analyse the dynamics of the projectile during its penetration. As the projectile sinks in the granular bed, it is primarily subjected to two forces: the gravitational force

DOI: 10.1051/epjconf/201714002027

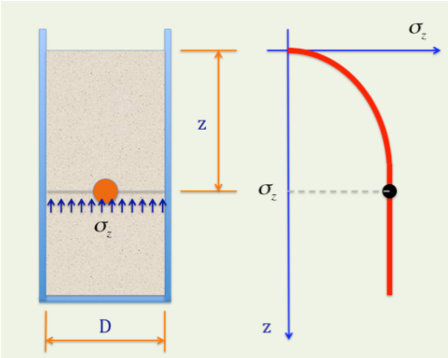

Figure 3. The resistance stress on the intruder

$m_i g$ (m_i – the mass of the projectile; and g – the gravitational acceleration) acting downwards and a upwards resistance force F_r induced by the contact with the particles in the granular bed. Based upon the Janson-Walker analysis [6] and assuming that the granular bed is homogeneous during submerging of intruder, the compressive stress σ_z in the vertical direction (Fig.3) at a distance z from the free surface of a packed granular bed is given as

$$\sigma_z = \frac{\rho_b g D}{4\mu_w K}\left[1 - e^{\left(\frac{-4\mu_w K}{D} z\right)}\right] \tag{1}$$

where $\rho_b = (1-\epsilon)\rho_p$ is the bulk density of the granular bed with ϵ being its porosity. D is the diameter of the container, μ_w is the wall friction coefficient and K is the stress transmission ratio. The resistance force F_r acting on the intruder can hence be approximated by

$$F_r = c_D A \sigma_z = \frac{C_D \pi D_i^2 (1-\epsilon)\rho_b g D}{4\mu_w K}\left[1 - e^{\left(\frac{-4\mu_w K}{D} z\right)}\right] \tag{2}$$

where A is the surface area of the projectile and c_D is a parameter related to the dynamic nature of the penetration process , in which the rearrangement of particles leads to the densification of the granular bed and intermittent strong force chain between the projectile and the granular bed. This will result in a much higher pressure than the static pressure given in Eq.1. Thus it is expected that c_D generally has a value greater than unity. Equations (1) and (2) show that σ_z and F_r increase from zero at $z = 0$ to a limiting value at a great depth. According to Newton's second law of motion,

$$m_i \frac{d^2 z}{dt^2} = m_i g - F_r \tag{3}$$

If the projectile penetrates to a finite depth Z_{max}, applying energy balance,

$$m_i g Z_{max} - \int_0^{Z_{max}} F_r d_z = 0 \tag{4}$$

Substituting Eq.(2) into (4), we obtain

$$m_i g Z_{max} - \int_0^{Z_{max}} \frac{C_D \pi d_i^2 (1-\epsilon)\rho_b g D}{4\mu_w K}\left[1 - e^{\left(\frac{-4\mu_w K}{D} z\right)}\right] d_z = 0 \tag{5}$$

Equation (6) can be integrated to obtain

$$m_i g Z_{max} - \frac{C_D \pi d_i^2 (1-\epsilon)\rho_b g D}{4\mu_w K} x \left[Z_{max} + \frac{D}{4\mu_w K} e^{\left(\frac{-4\mu_w K}{D} z\right)} - \frac{D}{4\mu_w K}\right] d_z = 0 \tag{6}$$

Re-arranging (6), we have

$$\frac{Z_{max}}{D} - \frac{6C_D(1-\epsilon)D\rho_p}{4\mu_w K d_i \rho_i}\left[\frac{Z_{max}}{D} + \frac{1}{4\mu_w K} e^{\left(\frac{-4\mu_w K}{D} z\right)} - \frac{1}{4\mu_w K}\right] = 0 \tag{7}$$

Introducing dimensionless parameters

$$\gamma = \frac{Z_{max}}{D},$$

$$\eta = \frac{D}{d_i},$$

$$\delta_b = 6(1-\epsilon),$$

$$\xi_b = 4\mu_w K,$$

Equation (7) can be rewritten as

$$\frac{\xi_b \gamma}{\xi_b \gamma + e^{-\xi_b \gamma} - 1} = C_D \frac{\delta_b}{\xi_b} \frac{\eta}{\psi} \tag{8}$$

For shallow penetration (say $\gamma << 1$), Equation (8) can be approximated as

$$\gamma = \frac{2}{C_D \delta_b} \frac{\psi}{\eta} \tag{9}$$

Equation (9) indicates that 1) the penetration depth is linearly proportional to the density ratio and the diameter of the intruder; 2) the higher the packing fraction of the bed, the smaller the penetration depth. Equation (9) can also be expressed as

$$Z_{max} = \frac{2d_i}{C_D \delta_b} \frac{\rho_i}{\rho_p} \tag{10}$$

Equation (10) implies that the maximum penetration depth is independent of the size of the container, but linearly proportional to the diameter of the intruder and the density ratio.

4 Model validation

To validate the model developed in the previous section, comparison between the model prediction and the experimental data for shallow and deep penetration reported in Lohse et al. ([3]), Hou et al. ([2]), Ambroso et al. ([1]), and Pacheco-Vazquez & Ruiz-Suarez ([4]) were presented in Figs 4-7. It is clear that the predictions using the developed model are in excellent agreement with the experimental data obtained for both shallow penetration and deep penetration.

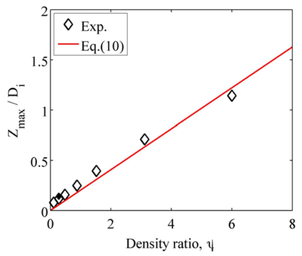

Figure 4. Maximum penetration depth: prediction vs. experiments (Ambrosso et al. [1])

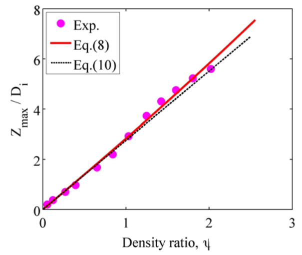

Figure 5. Maximum penetration depth: prediction vs. experiments (Lohse et al. [3])

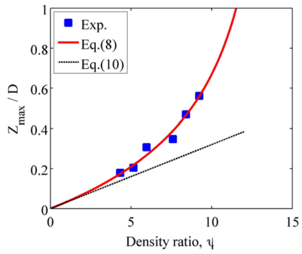

Figure 6. Maximum penetration depth: prediction vs. experiments (Hou et al. [2])

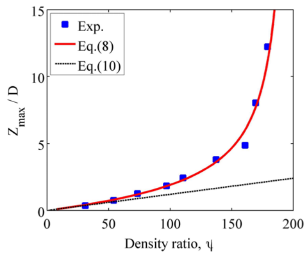

Figure 7. Maximum penetration depth: prediction vs. experiments (Pacheco-Vazquez et al. [4])

5 Conclusions

The submersing process of an intruder in granular materials was analysed numerically and analytically. An analytical model was developed and its accuracy was validated by comparison with experimental data reported in the literature. It was shown that the maximum penetration depth is a function of the diameter of the intruder, the packing density of the granular bed and the ratio of the intruder density to the density of particles in the granular bed.

Acknowledgements

CYW would like to acknowledge Chunlei Pei and Cher Hui Geoy for performing the DEM simulations discussed in Section 2.

References

[1] M.A. Ambroso, C. Santore, A. Abate and D. Durian, Phys. Rev. E **71**, 051305 (2005).

[2] M. Hou, Z. Peng, R. Liu, K. Lu and C. Chan, Phys. Rev. E **72**, 062301(2005).

[3] D. Lohse, R. Rauhe, R. Bergmann and D. van der Meer, Nature **432**, 689 (2004).

[4] F. Pacheco-Vazquez, G. Caballero-Robledo, J. Solano-Altamirano, E. Altshuler, A. Batista-Leyva and J. Ruiz-Suarez, Phys. Rev. Lett. **106**, 218001 (2011).

[5] C.H. Goey, C. Pei and C.Y. Wu, AIP Conference Proceedings **1542**, 437-440 (2013).

[6] J.P.K. Seville and C.Y. Wu, *Particle Technology and Engineering*, (Elsevier, London, 2016), 285.