

# Magnetic Properties of Liquid RE-Co (RE = Dy, Er) Alloys

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The magnetic susceptibilities  $\chi$  of liquid Dy-Co and Er-Co alloys have a large and negative temperature coefficient, which suggests that the Dy, Er, and Co ions in their liquid alloys are in the magnetic state. However, the temperature dependence of  $\chi$  in both systems becomes weak near the content of 70 at% Co. It is interesting that the compositional dependence of  $\chi$  for liquid Dy-Co and Er-Co alloys has a minimum at content of 80 at% Co, respectively. On the Co-rich side, the magnetic susceptibility of liquid Dy-Co and Er-Co alloys obeyed the Curie-Weiss law with regard to their temperature dependence of  $\chi$ . On the rare earth-rich side, the magnetic susceptibilities of liquid Dy-Co and Er-Co alloys also exhibited Curie behavior with a reasonable value for the effective number of Bohr magnetons. The compositional dependence of  $\chi_{4f}$  for liquid Dy-Co and Er-Co alloys was extracted by subtracting the corresponding data for liquid La-Co alloys from  $\chi$  for the liquid Dy-Co and Er-Co alloys, respectively.

## 1 Introduction

Busch and colleagues [1] reported the electronic properties of liquid transition metal (TM), rare earth metal (RE), and their alloys. Schlapbach [2] reported more detailed dependence of magnetic susceptibility on composition with a minimum for liquid Ce-TM (TM = Fe, Co, Ni) alloys and found a continuous transition of Ce ions into the nonmagnetic state. Moreover, Singer et al. [3] extended and explained the compositional dependence of  $\chi$  for liquid Ce-TM, Pr-TM, La-TM (TM = Fe, Co, Ni), and Sm-Fe alloys. In their paper [3], the compositional dependence of  $\chi$  for liquid La-TM alloys plays an important role in understanding the magnetic susceptibility of liquid RE-TM (RE = Ce, Pr, Sm) alloys. Actually, the compositional dependence of  $\chi_{4f}$  can be obtained by subtracting the corresponding data for liquid La-TM alloys from  $\chi$  for liquid RE-TM (RE = Ce, Pr, Sm) alloys.

We attempt to obtain the magnetic susceptibilities as the functions of temperature and composition for liquid Dy-Co and Er-Co alloys. The magnetic susceptibility of liquid Dy-Co and Er-Co alloys consists of the two large terms due to the 3d- and 4f-electron states.

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The contribution of these two electron states provides a convenient sectionalization in the each region on the Co-rich and RE-rich (RE = Dy, Er) sides. Therefore, these two terms can be separated to obtain the temperature dependence of  $\chi_{3d}$  and  $\chi_{4f}$  for liquid RE-Co (RE = Dy, Er) alloys. The  $P_{\text{eff}}$  values for the Dy, Er, and Co ions in liquid Dy-Co and Er-Co alloys can thus be estimated from the temperature dependence of  $\chi_{3d}$  and  $\chi_{4f}$ . We investigate the trend of  $P_{\text{eff}}$  for Dy and Er ions on the basis of the s-f and indirect d-f interactions and compare the trend of  $P_{\text{eff}}$  for Ce, Pr, Gd, Dy, and Er ions in liquid RE-Co alloys (RE = Ce, Pr, Gd, Dy, Er), respectively.

## 2 Experimental

Magnetic susceptibility measurements were made using the Faraday method with a torsion balance [4, 5]. The force applied to the sample in the cell was quantified by an automatic method of feedback current control. Mohr's salt ( $\chi = 1.26 \times 10^{-2} \text{ cm}^3/\text{mol}$  at room temperature) was employed as a standard sample. The field strength  $H$  of the electromagnet was 10 kOe with a 6.0 cm gap between pole pieces, and  $H \cdot dH/dx$  was  $6.25 \pm 0.12 \text{ (kOe)}^2/\text{cm}$ .

The measurement system can be evacuated in a vacuum of  $1.2 \times 10^{-2} \text{ Pa}$  and filled with a purified Ar gas at a pressure of 7 kPa. The RE-Co alloy sample was placed in a magnesia (MgO) crucible packed loosely with a MgO rod, and was completely melted and homogenized in a silicon carbide furnace under the above condition. The measurements of liquid Co and Er were carried out up to about 1570 °C. The measurements on the Co-rich side were carried out from the temperature between 1500 °C and 1550 °C to the melting point. The chemical composition of each sample was determined by accurately measuring the mass of each component before mixing. After this measurement, the decrease in weight of a 0.2 g sample was less than 4 mg. The temperature of the liquid alloy was determined using Pt-Pt13%Rh thermocouples placed close to the sample. The elemental purity was 99.9% for Dy, Er, and Co.

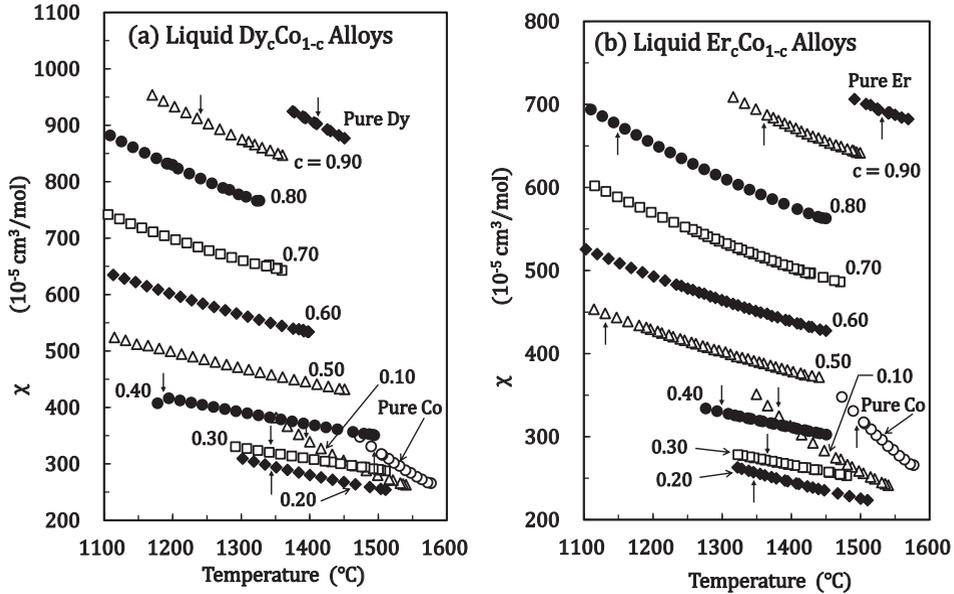
## 3 Results

As shown in Fig. 1(a), liquid Co has a large and negative temperature coefficient of magnetic susceptibility. On the Co-rich side, the magnetic susceptibilities  $\chi$  of liquid Dy-Co alloys decrease with increasing Dy content up to 20 at% Dy and these temperature coefficients of  $\chi$  become weak up to 30 at% Dy. With further increasing Dy content, the magnetic susceptibilities of liquid Dy-Co alloys increase with a large and negative temperature coefficient of  $\chi$ . This suggests that the Co and Dy ions in the liquid Dy-Co alloys are in magnetic states. The values of  $\chi$  for liquid Co and Dy are in good agreement with those obtained in the previous works [5, 6].

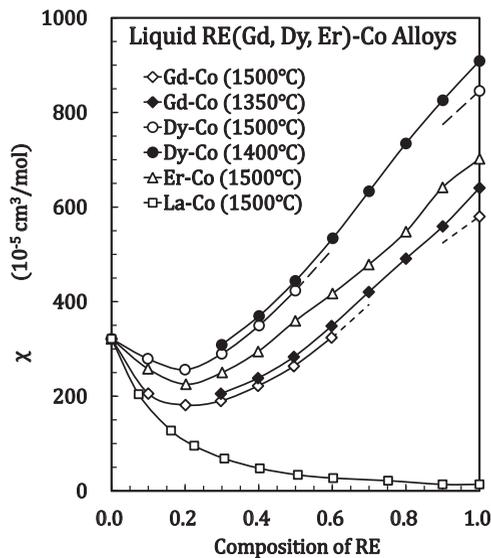
As shown in Fig. 1(b), the magnetic susceptibilities  $\chi$  of liquid Er-Co alloys also decrease with increasing Er content up to 20 at% Er and these temperature coefficients of  $\chi$  become weak up to 30 at% Er. With further increasing Er content, similarly, the magnetic susceptibilities of liquid Er-Co alloys increase with a large and negative temperature coefficient of  $\chi$ . This suggests that the Co and Er ions in the liquid Er-Co alloys are in magnetic states. The limit of utility of MgO crucible may be up to the temperature of 1570 °C for liquid Er.

Figure 2 shows the compositional dependence of  $\chi$  for liquid Dy-Co and Er-Co alloys at 1400 and 1500 °C. We added the compositional dependence of  $\chi$  of liquid Gd-Co and La-Co alloys at 1350 and 1500 °C to do some more investigating. The compositional

dependences of  $\chi$  for liquid Dy-Co and Er-Co alloys decrease with decreasing Co content and exhibit a minimum at the compositions of 20 at% Dy and Er, respectively. The magnetic susceptibilities of liquid Dy, Er, and Gd are approximately  $901$ ,  $693$ , and  $663 \times 10^{-5} \text{ cm}^3/\text{mol}$  at the melting point, respectively.



**Fig. 1.** Magnetic susceptibility of liquid  $\text{RE}_c\text{Co}_{1-c}$  alloys as a function of temperature. (a) liquid  $\text{Dy}_c\text{Co}_{1-c}$  alloys; (b) liquid  $\text{Er}_c\text{Co}_{1-c}$  alloys. The arrows indicate the melting point of the alloys. Experimental error is about 2.5%.



**Fig. 2.** Dependence of  $\chi$  on composition for  $\text{RE}_c\text{Co}_{1-c}$  (ER = Dy, Er, Gd) alloys. The compositional dependence of  $\chi$  for liquid  $\text{La}_c\text{Co}_{1-c}$  alloys was reported by Singer et al. [3]. The solid lines and dashed lines are drawn as guides for the eye.

## 4 Discussion

The magnetic susceptibility of a liquid RE-Co alloy is given by [5, 7]

$$\chi_1 = [(1-c)\chi_{\text{dia}}(\text{Co}^+) + c\chi_{\text{dia}}(\text{RE}^{3+})] + \chi_{\text{el}} + \chi_{3d} + \chi_{4f} + \chi_{5d} \quad (1)$$

where  $\chi_{\text{dia}}(\text{Co}^+)$  and  $\chi_{\text{dia}}(\text{RE}^{3+})$  are the diamagnetic susceptibilities due to the  $\text{Co}^+$  and  $\text{RE}^{3+}$  (RE = Dy, Er, Gd) ions, respectively. The diamagnetic susceptibilities of  $\text{Co}^+$ ,  $\text{Dy}^{3+}$ ,  $\text{Er}^{3+}$ , and  $\text{Gd}^{3+}$  ions have been estimated by Bain and Berry [8], and Mendelsohn et al. [9], respectively. The diamagnetic susceptibility of Co ions in metals has been obtained by Banhart et al. [10]. The values of  $\chi_{\text{dia}}$  for  $\text{Co}^+$ ,  $\text{Dy}^{3+}$ ,  $\text{Er}^{3+}$ , and  $\text{Gd}^{3+}$  ions employed in this work are -1.82, -1.90, -1.80, and  $-1.80 \times 10^{-5} \text{ cm}^3/\text{mol}$ , respectively. We assume that the total  $\chi_{\text{dia}}$  for liquid Dy-Co, Er-Co, and Gd-Co alloys varies linearly with composition.

$\chi_{\text{el}}$  is the paramagnetic susceptibility due to conduction electrons, expressed as [7, 11]

$$\chi_{\text{el}} = \alpha(m^*, r_s) \mu_B^2 N(E_F), \quad (2)$$

where  $N(E_F)$  is the density of states at  $E_F$ , which can be estimated from the nearly free electron model as follows: [7, 11]

$$N(E_F) = (V_a/2\pi^2)(2m/\hbar^2)^{3/2} E_F^{1/2}. \quad (3)$$

Here,  $V_a$  is the atomic volume and  $E_F$  is given by [7, 11]

$$E_F = (\hbar^2/2m)(3\pi^2 N_c/V_a)^{2/3}, \quad (4)$$

where  $N_c$  is the number of conduction electrons.

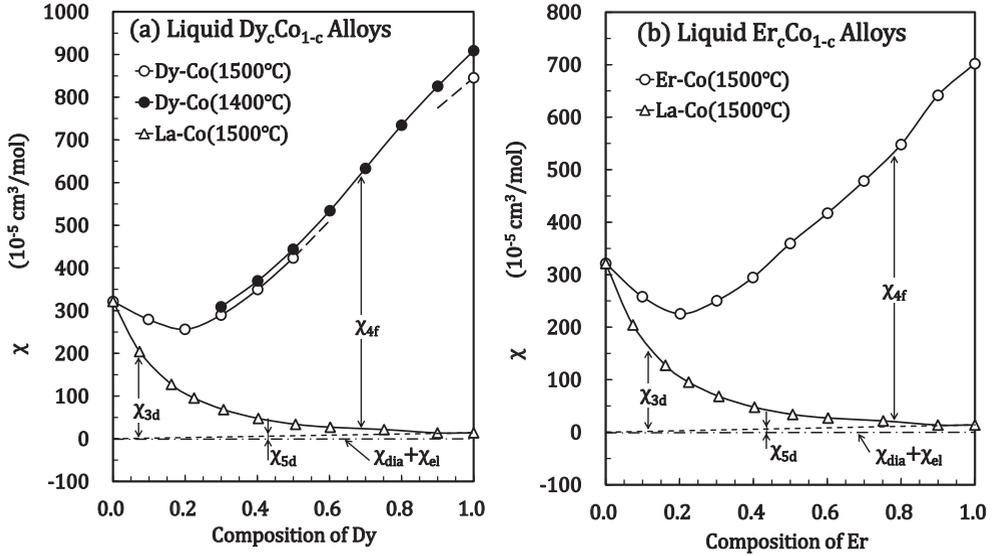
We assume that the  $E_F$  and  $N(E_F)$  values are estimated by the linear interpolation of  $N_c$  and  $V_a$  from the RE side to the Co side. The 4s electrons of Co ions and the 6s<sup>2</sup> electrons of RE ions are simply treated as conduction electrons. It is assumed that the 5d electrons of RE ions form independently the density of 5d-electron states.

We assume that the  $m^*/m$  ratio is 1.0 in liquid Co, RE, and their alloys. In this assumption, the enhancement factor for nearly free electrons is given by [12, 13]

$$\alpha(m, r_s) = \alpha(r_s) - 1/3, \quad (5)$$

where  $\alpha(r_s)$  is the enhancement factor of free-electron theory. The dimensionless quantity  $r_s$  is defined as  $(3.64 \times 10^8)/k_F$ , where  $k_F$  is the Fermi wave number. The  $\alpha(r_s)$  estimated by the random phase approximation increases slightly with increase  $r_s$ , and were determined to be 1.28, 1.28, 1.28, and 1.27 for liquid Dy, Er, Gd, and Co, respectively.

The compositional dependences of  $\chi_{\text{el}}$  estimated using this approach deviate above slightly from the straight line connected on the Co side and RE (RE = Dy, Er, Gd) side, respectively. Assuming that the value of  $\chi_{\text{dia}}$  for liquid RE-Co varies linearly with composition, we estimated the compositional dependence of  $(\chi_{\text{dia}} + \chi_{\text{el}})$ , as shown in Fig. 3. As a result, the compositional dependence of  $(\chi_{\text{dia}} + \chi_{\text{el}})$  obtained in this procedure approximates a straight line on the scale shown in Fig. 3. As a result, the compositional dependence of  $(\chi_{\text{dia}} + \chi_{\text{el}})$  obtained in this procedure approximates a straight line on the scale shown in Fig. 3. The values of  $V_a$ ,  $\chi_{\text{dia}}$ , and  $\chi_{\text{el}}$  are shown in Table 1.



**Fig. 3.** Dependence of  $\chi$  on composition for liquid  $RE_cCo_{1-c}$  alloys. (a) liquid  $Dy_cCo_{1-c}$  alloys; (b) liquid  $Er_cCo_{1-c}$  alloys. The compositional dependence of  $\chi$  for liquid  $La_cCo_{1-c}$  alloys was reported by Singer et al. [3]. The base line is the compositional dependence of  $(\chi_{dia} + \chi_{el})$ . The solid lines and dashed lines are drawn as guides for eye.

**Table 1.**  $V_a$ ,  $\chi_{dia}$ , and  $\chi_{el}$  for  $Co^+$ ,  $Dy^{3+}$ ,  $Er^{3+}$ ,  $La^{3+}$ ,  $Gd^{3+}$ ,  $Pr^{3+}$ , and  $Ce^{3+}$  ions.

	$Co^+$	$Dy^{3+}$	$Er^{3+}$	$La^{3+}$	$Gd^{3+}$	$Pr^{3+}$	$Ce^{3+}$
$V_a$ ( $10^{-23}$ $cm^3/mol$ )	1.26	3.32	3.32	3.94	3.80	3.63	3.56
$\chi_{EXP}$ ( $10^{-5}$ $cm^3/mol$ )	321	846	702	14	579	147	85
$\chi_{dia}$ ( $10^{-5}$ $cm^3/mol$ )	-1.82	-1.90	-1.80	-2.00	-2.00	-2.00	-2.00
$\chi_{el}$ ( $10^{-5}$ $cm^3/mol$ )	0.68	1.65	1.65	1.86	1.81	1.75	1.87
$\chi_{dia} + \chi_{el}$ ( $10^{-5}$ $cm^3/mol$ )	-1.14	-0.25	-0.15	-0.14	-0.19	-0.25	-0.13

As mentioned above, the 5d-electrons of RE and La ions seems to form independently the density of 5d-electron states. Therefore, the magnetic susceptibility of  $\chi_{5d}$  due to 5d-electron is given by [14]

$$\chi_{5d} = N_A \mu_B^2 \rho_{5d}(E_F) / [1 - U \rho_{5d}(E_F) / 2], \quad (6)$$

where  $U$  is the intra-atomic Coulomb interaction. The experimental  $\chi$  value of liquid La corresponds to  $(\chi_{dia} + \chi_{el} + \chi_{5d})$  value. We can obtain the  $\rho_{5d}(E_F)$  value of liquid La under the assumption that  $U = 2.0$  eV. The  $\rho_{5d}(E_F)$  obtained is  $0.81$   $eV^{-1} \cdot atom^{-1}$ , which is larger than the  $\rho_{6s}(E_F)$  value of  $0.60$   $eV^{-1} \cdot atom^{-1}$ . The  $\rho_{5d}(E_F)$  value may be really admixed by  $\rho_{6s}(E_F)$ , which forms the width of 5d-state. The magnetic susceptibilities of liquid TM-La alloys consists of  $(\chi_{dia} + \chi_{el})$  value and  $\chi_{5d}$  of eq. (7). The dependence of  $(\chi_{dia} + \chi_{el} + \chi_{5d})$  on composition obtained in this procedure approximates a straight line on the scale shown in Fig. 3.

According to their suggestion of March and Sayers [15], the magnetic susceptibility of liquid Dy-Co, Er-Co, and Gd-Co alloys consists of the two large terms due to the 3d-electron and 4f-electron states on the TM-rich side and RE-rich side. We compare the magnetic susceptibility of liquid RE-Co and La-Co alloys. The magnetic susceptibility of La-Co alloy is given by [5, 7]

$$\chi_2 = [(1 - c)\chi_{\text{dia}}(\text{Co}^+) + c\chi_{\text{dia}}(\text{La}^{3+})] + \chi_{\text{el}} + \chi_{3\text{d}} + \chi_{5\text{d}}, \quad (7)$$

where  $\chi_{\text{dia}}(\text{La}^{3+})$  are the diamagnetic susceptibilities due to  $\text{La}^{3+}$  ions. Roughly speaking, we can obtain the  $\chi_{4\text{f}}$  value by subtracting eq. (7) from eq. (1) as follows;

$$\chi_{4\text{f}} \approx \chi_1 - \chi_2. \quad (8)$$

Assuming that  $(\chi_{\text{dia}} + \chi_{\text{el}})$  is considerably smaller than  $\chi_{3\text{d}}$  and  $\chi_{4\text{f}}$ , we can obtain the  $\chi_{4\text{f}}$  values of eq. (8) with a high degree of accuracy.

Figure 3(a) shows the dependence of  $\chi$  on composition for liquid Dy-Co alloys. Assuming that the  $\chi_{3\text{d}}$  of Co ions in liquid Dy-Co alloys may be little different from that in liquid La-Co alloys, we also plotted the dependence of  $\chi$  on composition for liquid La-Co reported by Singer et al. [3]. This difference between the lines of liquid Dy-Co and La-Co alloys corresponds to the dependence of  $\chi_{4\text{f}}$  on composition due to 4f-electrons. The base line exhibits the dependence of  $(\chi_{\text{dia}} + \chi_{\text{el}})$  on composition due to the Dy and Co ions, and conduction electrons, which is a very small value compared with  $\chi_{4\text{f}}$  and  $\chi_{3\text{d}}$ . The dependence of  $\chi_{3\text{d}}$  on composition decreases rapidly with decreasing Co content. The variation of  $\chi_{4\text{f}}$ ,  $\chi_{5\text{d}}$ , and  $\chi_{3\text{d}}$  are reasonable for the composition of Dy and Co.

Figure 3(b) shows the dependence of  $\chi$  on composition for liquid Er-Co alloys. The dependence of  $\chi$  on composition for liquid La-Co alloys exhibits the data reported by Singer et al. [3]. As shown in Fig. 3(b), the significant point of comparison is that the difference between the lines for liquid Er-Co and La-Co increases with increasing Er (or La) content. This difference corresponds to the dependence of  $\chi_{4\text{f}}$  on composition due to 4f-electrons. The base line exhibits the dependence of  $(\chi_{\text{dia}} + \chi_{\text{el}})$  on composition due to the Er (or La) and Co ions, and conduction electrons, which is a very small value compared with  $\chi_{4\text{f}}$  and  $\chi_{3\text{d}}$ . The dependence of  $\chi_{3\text{d}}$  on composition decreases rapidly with decreasing Co content. The variations of  $\chi_{4\text{f}}$  and  $\chi_{3\text{d}}$  obtained in Fig. 3(b) are also reasonable for the composition of Er and Co.

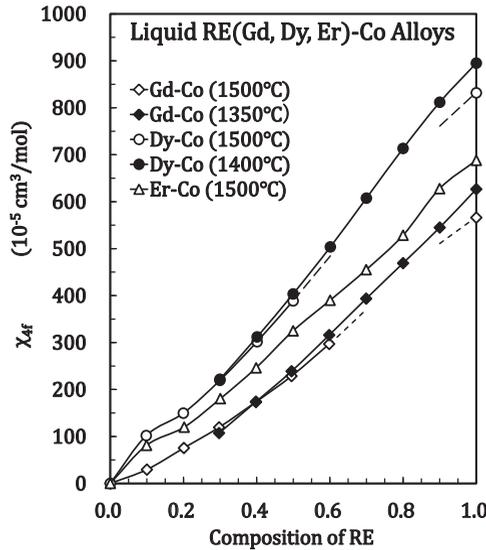
Figure 4 shows the dependence of  $\chi_{4\text{f}}$  on composition for liquid RE-Co (RE = Dy, Er, Gd) alloys. For liquid Gd-Co alloys, the dependence of  $\chi_{4\text{f}}$  on composition was obtained by similar manner. Liquid RE-Co alloys have similar dependence of  $\chi_{4\text{f}}$  on composition, which deviates below from straight line. The  $\chi_{4\text{f}}$  values of liquid RE-Co alloys decrease with increasing temperature, which reflects the temperature dependence of the experimental  $\chi$  value. It suggests that the RE ions in liquid RE-Co alloys are in the magnetic state over the wide composition range. It is known that the Curie-Weiss law describes the magnetic susceptibility in the magnetic state fairly well. For liquid RE-Co in the magnetic state, the third and fourth terms on the right-hand side of eq. (1) is given by [5, 7]

$$\chi_{4\text{f, or } 3\text{d}} = [N_{\text{A}}P_{\text{eff}}^2\mu_{\text{B}}^2/3k_{\text{B}}(T - \theta)] + \alpha, \quad (9)$$

where  $N_{\text{A}}$  is the number of RE or Co atoms per mole.  $P_{\text{eff}}$  is the effective number of Bohr magnetons. The symbols  $\theta$  and  $\alpha$  are the paramagnetic Curie temperature and a residual susceptibility at infinite temperature, respectively [9].

In this work, the  $P_{\text{eff}}$  values of RE ions in liquid RE-Co alloys were obtained from the equation of  $P_{\text{eff}}(\text{RE}^{3+}) = (3k_{\text{B}}T\chi_{4\text{f}}/N_{\text{A}})^{1/2}\mu_{\text{B}}$  under the assumptions that  $\theta = 0$  and  $\alpha = 0$ . On

the Co side, the  $P_{\text{eff}}$  and  $\theta$  for the Co ions in liquid RE-Co (RE = Dy, Er, Gd) alloys were obtained from the plots of  $(1/\chi_{3d})$  vs  $T$ . The  $\theta$  values obtained from this approach are 1450, 1340, and 920 K for liquid Co, Dy<sub>0.1</sub>Co<sub>0.9</sub>, and Dy<sub>0.2</sub>Co<sub>0.8</sub> alloys, respectively. The  $\theta$  values obtained for liquid Er<sub>0.1</sub>Co<sub>0.9</sub>, Er<sub>0.2</sub>Co<sub>0.8</sub>, Gd<sub>0.1</sub>Co<sub>0.9</sub>, and Gd<sub>0.2</sub>Co<sub>0.8</sub> alloys are 1340, 790, 1140, and 730 K, respectively. The rapid decrease in  $\chi$  of liquid RE-Co (RE = Dy, Er, Gd) alloys on the Co-rich side corresponds to the rapid decrease in  $\theta$  which results from the reduction of exchange d-d interaction between Co ions.

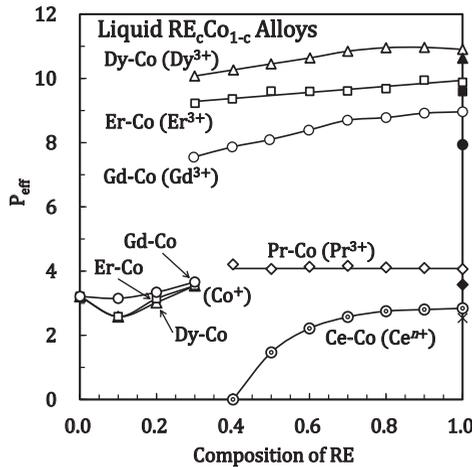


**Fig. 4.** Dependence of  $\chi_{4f}$  on composition for liquid RE<sub>c</sub>Co<sub>1-c</sub> (RE = Dy, Er, Gd) alloys. The solid lines and dashed lines are drawn as guides for eye.

As shown in Fig. 5, we integrate the trends of  $P_{\text{eff}}$  (RE ions) in liquid RE-Co (RE = Dy, Er, Gd, Pr, Ce) alloys. The  $P_{\text{eff}}$  values of Ce and Pr ions were estimated from their data [2, 3] by the same manner. The  $P_{\text{eff}}$  values of Dy and Er ions are approximately 10.9 and 9.9, which are somewhat larger than the theoretical prediction of 10.6 and 9.6 estimated for Dy<sup>3+</sup> and Er<sup>3+</sup> ions [16], respectively. The  $P_{\text{eff}}$  value of Gd<sup>3+</sup> ions is approximately 8.9, which is larger than the theoretical prediction of 7.9 estimated for Gd<sup>3+</sup> ions [16]. A swell in  $P_{\text{eff}}$  for Gd<sup>3+</sup> ions on the Gd-rich side may be attributable to the assumption of  $\theta = 0$ . If we can obtain an accurate  $\theta$  value, we apply the general equation of  $P_{\text{eff}} = [3k_B(T - \theta)\chi_{4f}/N_A]^{1/2}$  for the analysis of  $P_{\text{eff}}$ .

It is supposed that metallic RE alloys can be represented by an assembly of RE<sup>3+</sup> ions embedded in a sea of conduction electrons [17]. In the case of heavy RE-TM alloys [17], the spin of TM ion lying near Gd<sup>3+</sup> ion take the orientation contrary to the total spin of Gd<sup>3+</sup> ion. Therefore, a gradual decrease in  $P_{\text{eff}}$  for liquid RE-Co (RE = Dy, Er, Gd) alloys may be attributed to the alignment of antiparallel moment due to the indirect d-f interaction by way of the conduction electrons [17]. It is well known that liquid Ce-Co alloys give rise to the continuous transition of Ce<sup>n+</sup> ions with increasing Co content [2, 3]. The trend of  $P_{\text{eff}}$  for Ce<sup>n+</sup> ions in liquid Ce-Co alloys is closely related to the continuous transition of Ce ions into the nonmagnetic state. We confirmed the transition of Ce<sup>3+</sup> to Ce<sup>4+</sup> ions by this treatment. On the other hand, the Pr ions are stable in liquid Pr-Co alloys. On the Co-rich side, the  $P_{\text{eff}}$  values correspond to the intermediate state between Co<sup>+</sup> and Co<sup>2+</sup> ions with the

total spin  $S = 1$  and  $3/2$ , respectively. This indicates that the s-d interaction dominates the magnetic susceptibilities of liquid RE-Co alloys on the Co-rich side.



**Fig. 5.** Dependence of  $P_{\text{eff}}$  on composition for RE (RE = Dy, Er, Gd, Pr, Ce) and Co ions obtained in liquid RE-Co alloys. The closed triangle, closed square and so on plotted on the RE side are the calculated values due to Van Vleck [16]. The solid lines are drawn as guides for the eye.

## 5 Conclusions

Based on the compositional dependence of  $\chi$  for liquid La-Co alloys, those for liquid Dy-Co and Er-Co alloys can be divided into the two main terms due to the 4f- and 3d-electrons. Using the Curie and the Curie-Weiss law,  $P_{\text{eff}}$  for Dy, Er, and Co ions can be estimated from the temperature dependence of  $\chi$  divided partially. By the addition of Co, the gradual decrease in  $P_{\text{eff}}$  of Dy and Er ions in these liquid alloys may be attributed to the indirect d-f interaction by way of the conduction electrons. In the liquid RE-Co systems, the 4f-electron states involved with the s-f interaction predominates steadily over the wide composition range.

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