

Recent progress in the theoretical modelling of Cepheids and RR Lyrae stars

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Abstract. Cepheids and RR Lyrae are among the most important primary distance indicators to calibrate the extragalactic distance ladder and excellent stellar population tracers, for Population I and Population II, respectively. In this paper I first mention some recent theoretical studies of Cepheids and RR Lyrae obtained with different theoretical tools. Then I focus the attention on new results based on nonlinear convective pulsation models in the context of some international projects, including VMC@VISTA and the Gaia collaboration. The open problems for both Cepheids and RR Lyrae are briefly discussed together with some challenging future application.

1 Introduction

An important challenge for next decade astrophysics is the determination of stellar distances with a precision better than 5% from the Local Group to about 200 Mpc as the crucial ingredient not only to constrain the Universe geometry but also to characterize several physical properties of stars and galaxies (including dynamical time scales, masses, ages, linear sizes, etc.). In the Milky Way and in photometrically resolved galaxies, the study of the different types of hosted pulsating stars (in different evolutionary phases) allows us to trace the intrinsic properties (age and chemical composition) and, through individual distance estimates, the spatial distribution of the associated stellar populations. This approach also permits to point out the possible presence of radial trends, haloes and/or streams, and in turn to reconstruct the star formation history of the investigated galaxy and to obtain information on galactic formation and evolution mechanisms. Pulsating stars such as Classical Cepheids and RR Lyrae are excellent primary distance indicators and stellar population tracers ([5, 10, 21, 22, 26–28]). This twofold role is particularly challenging in the era of present and future space astrometric missions. Through a multi-epoch monitoring of the full sky, Gaia ([11, 12]) will get the position, the parallax, the proper motion and the time series photometry of thousands of pulsating stars in the Milky Way and its surroundings, down to a faint magnitude limit of $G \sim 20.7$ mag. In particular, Gaia's complete census of the Galactic Cepheids and RR Lyrae will allow a breakthrough in our understanding of the Galactic structure by tracing variable stars of various ages in the Galactic bulge, disc, halo, likely revealing new streams and faint satellites as the signatures of the Milky Way's hierarchical build-up. Moreover Gaia parallaxes will provide an unprecedented absolute calibration of the extragalactic distance scale ([11, 12]). Similarly, in the near future, missions such as LSST will allow

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us to extend Gaia capabilities to Local Group and external galaxies. Finally, using JWST and subsequently MICADO@E-ELT (with spatial resolution similar to the one of the *Hubble Space Telescope*) it will be possible to extend Classical Cepheid observations up to the crowded fields of galaxy clusters well beyond Virgo and RR Lyrae observations in elliptical and giant spiral galaxies up to 6 Mpc. In order to properly exploit the results of these crucial missions, accurate hydrodynamical models are needed in order to constrain the relation between the observed pulsation properties and the intrinsic stellar parameters, that is the physical basis of the cosmic distance scale calibration through pulsating stars ([2]). On the other hand, the comparison between theory and observation is crucial to constrain the physical and numerical assumptions adopted in both evolutionary and pulsational models and, in turn, to improve our knowledge of stellar physics ([14, 19, 20, 22]).

2 Some recent theoretical studies of Cepheids and RR Lyrae

To properly exploit the importance of pulsating stars as property tracers and standard candles of different stellar populations, accurate hydrodynamical models are needed. During the last few decades there has been an increasing interest in predicting the observed properties of pulsating stars by adopting different theoretical approaches: linear non adiabatic hydrodynamical models, non linear convective hydrodynamical 1D models, multidimensional hydrodynamical simulations. Among the most recent results: [1] adopted linear non adiabatic hydrodynamical models to investigate the effect of rotation on Cepheid observables as a function of metallicity (see also Anderson et al., this volume); [29] used non linear convective hydrodynamical 1D models to investigate Type II Cepheid dynamical instability and period doubling as a function of stellar parameters (see also Smolec et al., this volume); assuming a similar theoretical scenario our team developed powerful theoretical tools to use RR Lyrae and Classical Cepheids as distance indicators and stellar population tracers (see, e.g., [5, 10, 20, 21], and references therein); finally, with pioneering multidimensional hydrodynamical simulations, [23, 24] produced realistic simulations of the convection-pulsation interaction in Cepheids, as powerful guidelines for improving the description of convection in 1D modelling. In the following we will focus on some results of recent nonlinear convective 1D pulsation models.

3 The model fitting of light and radial velocity curves

One of the advantages of nonlinear convective models is the possibility to directly compare observed and predicted light, radial velocity, radius variations. The first application of such a comparison was performed by [30] who compared the predicted and the observed light curve of a Magellanic Cepheid. The method was then extended to a Galactic field RR Lyrae by [3]. Since then, many applications to Cepheids and RR Lyrae followed in the literature (see, e.g., [16, 18–20, 22, 25], and references therein). In particular in [22] we performed the model fitting of light and radial velocity curves for a sample of 12 Small Magellanic Cloud (SMC) Cepheids with optical data from the OGLE collaboration and near-infrared data from the ESO Public Survey VMC@VISTA (see Cioni's paper in this volume and [22] for details). Some interesting implications of this analysis were: i) the deviation from a canonical mass-luminosity relation for SMC Cepheids possibly due to a combination of mass-loss, overshooting, rotation; ii) a period-radius relation in agreement with previous radius measurements in the SMC; iii) a mean value of the SMC distance modulus in agreement with the literature; iv) period-luminosity and period-Wesenheit relations in agreement with empirical OGLE and VMC relations. A similar analysis but for Large Magellanic Cloud (LMC) Cepheids is in progress (Ragosta et al., in preparation). Preliminary results are shown in Figure 1 for the inferred mass-luminosity relation. We note that, in agreement with previous results in the Milky Way ([8]) and the SMC ([22]), the predicted

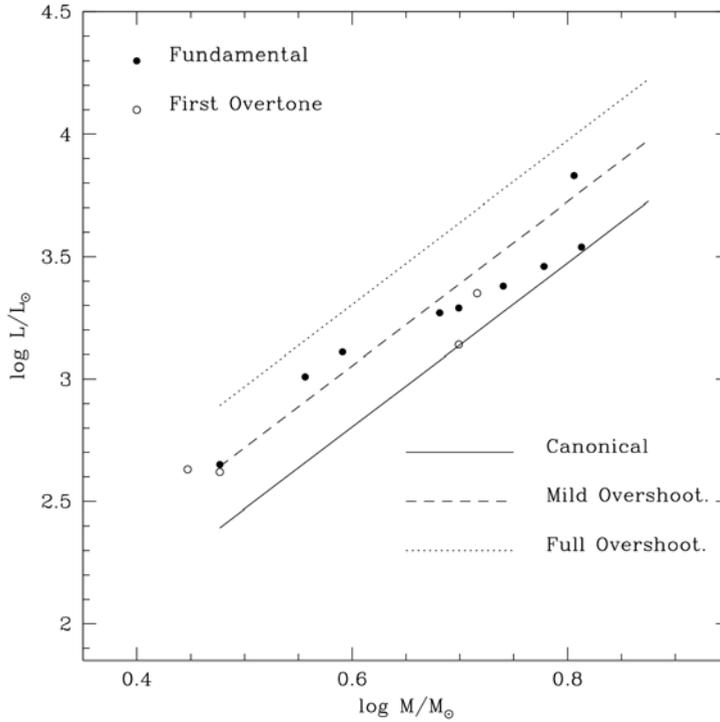


Figure 1. The inferred mass-luminosity relation from the model fitting of a sample of LMC Cepheids with OGLE optical and VMC@VISTA near-infrared photometry (Ragosta et al., in prep).

mass-luminosity relation is somewhat intermediate between the canonical and the full overshooting evolutionary relations. We also took into account a number of Galactic Cepheids for which the Gaia first data release ([11, 12]) provided new trigonometric parallaxes and optical and near-infrared data were available from the literature. We applied the multi-filter light curve model fitting to them and compared the inferred model parallaxes with Gaia values, finding a satisfactory agreement. An example is shown in Figure 2 for the Galactic Cepheid RS Pup. For this star, characterized by a pulsation period of 41.5 d, an independent geometric parallax has been provided on the basis of the circumstellar nebula reflecting the light from the central star ([15]). This value corresponds to $\Pi_{K08} = 0.502 \pm 0.007$ mas and is consistent with the Gaia first data release value $\Pi_{TGAS} = 0.63 \pm 0.26$ mas. On the other hand, the model fitting technique illustrated in Figure 2 provides an intrinsic distance modulus of 11.1 ± 0.1 mag, that implies $\Pi_{FIT} = 0.58 \pm 0.03$ mas, in excellent agreement with both the previous estimates.

4 The role of the helium content in Cepheid models

Cepheid pulsation properties are affected not only by metallicity but also by the helium content ([7, 17]). This implies that once the metallicity of a Cepheid sample is known, e.g. from spectroscopic measurements, the investigation of Cepheid pulsation properties can be used to constrain the helium content and in turn the helium to metal enrichment ratio. An example could be the well known

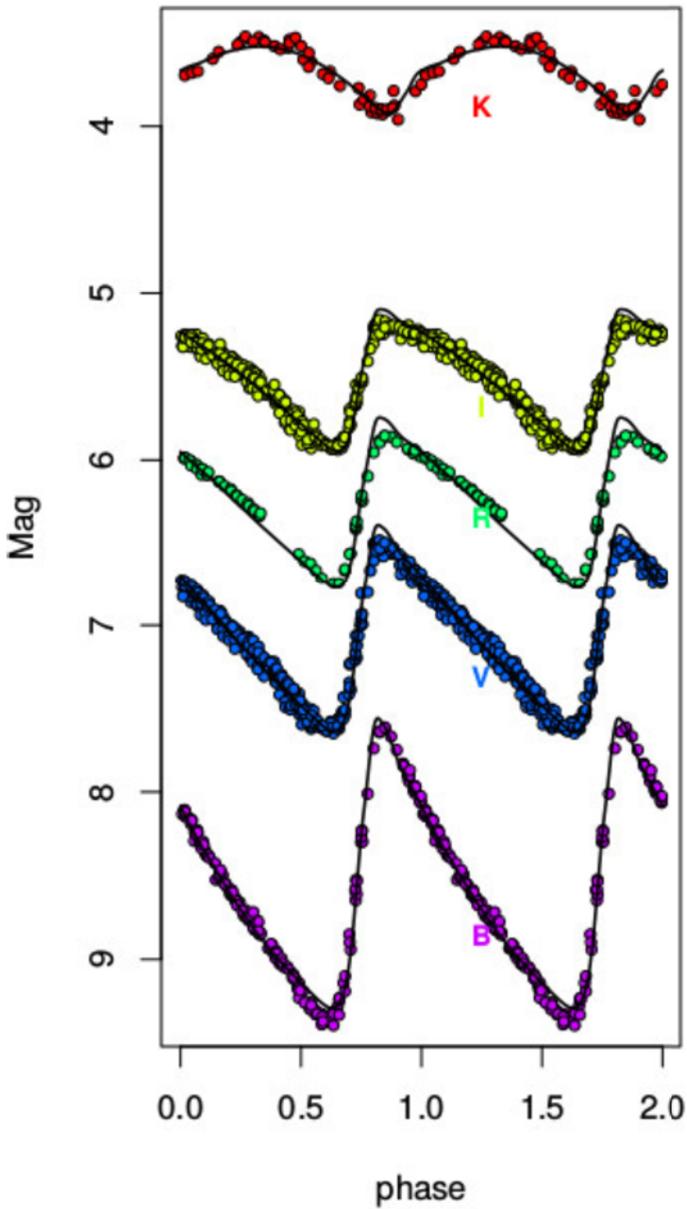


Figure 2. Model fitting of the multi-filter light curve of Gaia Galactic Cepheid RS Pup.

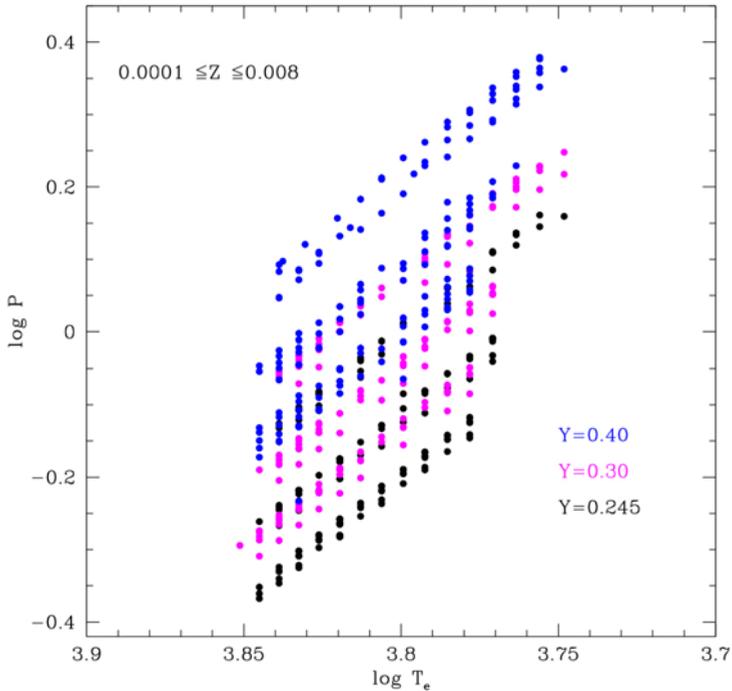


Figure 3. Predicted period distribution of fundamental RR Lyrae as a function of the assumed helium content.

Hertzsprung progression ([13]) of Cepheid light curves with periods roughly between 6 and 16 d. The pulsators in this period range show a bump along both the light and the velocity curves. The phase of this secondary feature varies with the period: it is on the curve descending branch for periods up to about 9 days, then it appears close to maximum light up to about 12 d, to move to the rising branch for longer periods (see, e.g., [4], and references therein). Current non linear convective models support the observational evidence that the Hertzsprung progression central period (P_{HP}) moves to longer values as the metallicity decreases. On the other hand, fixing the metallicity and increasing the H₂ content moves P_{HP} to longer values. For example assuming the LMC metallicity $Z=0.008$ and increasing the He content from the standard $Y = 0.25$ value to $Y = 0.35$, one again obtains $P_{HP} \approx 13$ d (see [7]). This implies that once known the metallicity, the comparison between observed and predicted light curves, for bump Cepheids, might help constrain the helium content and in turn the helium to metal enrichment ratio of the associated stellar population.

5 A new theoretical scenario for RR Lyrae stars: The dependence on chemical composition

During the last few years we have updated the theoretical scenario for RR Lyrae stars as based on a new set of metal-dependent non linear convective pulsation models (see [21] for details). One of the most important results of this investigation was the predicted universality of the period-Wesenheit

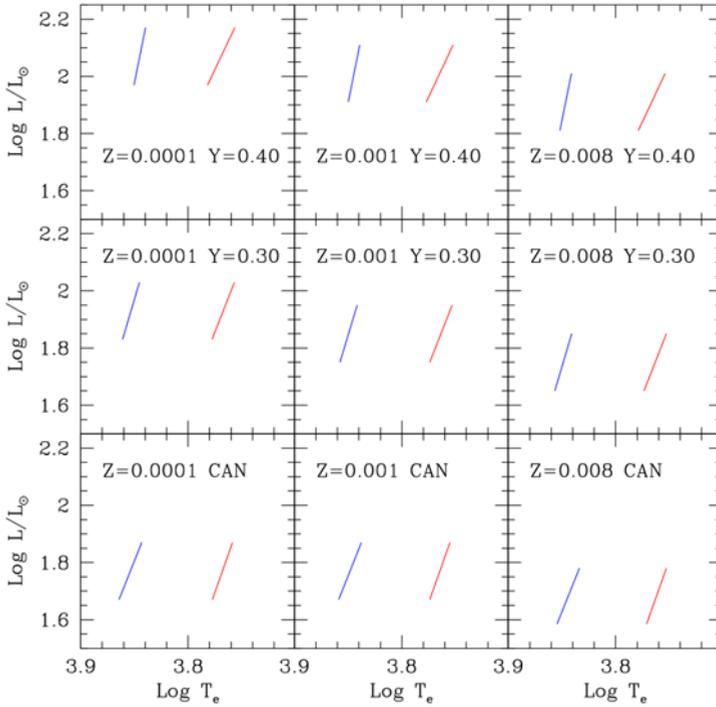


Figure 4. Predicted instability strip extreme boundaries (first overtone blue edge and fundamental red edge) as a function of both metallicity and helium content (see labels).

relation in V , $(B - V)$ for RR Lyrae stars. Indeed this relation was found to show a negligible dependence on the assumed model metal content, whereas for other band combinations a metallicity effect is expected. This theoretical framework was applied with success to observational samples, e.g. in the Galactic globular cluster M4 (see [6]) and in the dwarf spheroidal galaxy Carina (see [9]). An important new prediction of these models was an analytical relation to infer double mode RR Lyrae masses as a function of the period ratio and the metallicity. This relation is

$$\log(M/M_{\odot}) = -0.85(\pm 0.05) - 2.8(\pm 0.3) \log(P_{FO}/P_F) - 0.097(\pm 0.003) \log Z,$$

where P_{FO} and P_F are the first overtone and fundamental pulsation period, respectively (see [21] for details).

There are also some new results on the dependence of RR Lyrae evolutionary and pulsation properties on the He abundance (Marconi et al., in preparation). The main effect of changing the helium content from the canonical value, for a given metal abundance, to values as high as $Y = 0.30$ or $Y = 0.40$, is that the evolutionary zero age horizontal branch luminosity significantly increases, with the consequent shift of predicted pulsation periods towards longer values. In Figure 3 we show the dependence of the predicted period distribution of fundamental RR Lyrae on the assumed helium content. The period distribution is shifted to longer values as Y increases. An interesting implication of the behaviour shown in Figure 3 is that also the minimum RR_{ab} period is predicted to increase

with the helium abundance, thus representing a powerful observable to constrain the helium to metal enrichment ratio of a stellar population.

On the other hand, we expect a quite small effect on the predicted instability strip. As shown in Figure 4, the location of the extreme boundaries of the RR Lyrae instability strip, namely the first overtone blue edge and the fundamental red edge, as a function of the chemical composition, shows that the width of the instability strip is almost unaffected by abundance variations, with the main effect related to the increase of the corresponding luminosities.

6 Some (not all) open problems and possible future developments

Even if the presented theoretical scenario is able to predict all the relevant pulsation observables and has been successfully applied to both Galactic and extragalactic samples, there are a number of open problems that deserve further theoretical investigation. Just to quote a few of them:

- Current non linear convective 1D models are not completely able to reproduce the light curve morphology of the reddest Cepheids and RR Lyrae due to extreme sensitivity to details of the convective turbulent model;
- It is very difficult to disentangle mass loss and core overshooting (and rotation) effects on the mass-luminosity relation of Classical Cepheids;
- Metallicity and helium variations simultaneously affect Cepheid and RR Lyrae light curves, amplitudes, instability strips, period-luminosity and period-luminosity-colour relations and it is hard to disentangle the two contributions.

In order to clarify these important issues we need to improve the treatment of turbulent convection in pulsation models and to provide accurate independent constraints on physical and numerical assumptions. In this context we expect that Gaia and future LSST measurements will have a crucial impact. For example if accurate distances are available from these missions and metallicities are known from complementary spectroscopic surveys, we will be able to constrain the helium content and the stellar mass.

7 Conclusions

Classical Cepheids and RR Lyrae continue to be of great interest for theoreticians. The model fitting of light and radial velocity curves of Magellanic Classical Cepheids with OGLE optical and VMC@VISTA NIR data provides interesting results for the mass-luminosity relation, the period-radius, the period-luminosity and the Wesenheit relations. The application to Galactic Cepheids with parallaxes obtained from the Gaia first data release also gives promising results. The investigation of the dependence of RR Lyrae nonlinear convective models on metallicity has predicted the universality of the period-Wesenheit relation, a 3D version of the Petersen diagram and distances in agreement with the literature. The He content also affects the pulsation properties of both Cepheids and RR Lyrae. In spite of the power of the current theoretical scenario in predicting all the relevant pulsation observables, there are a number of open problems that require further theoretical work. On the other hand, we expect that both Gaia and LSST results will represent a benchmark for pulsation theories and a challenging opportunity to constrain numerical and physical assumptions in both evolutionary and pulsation models.

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