

Probing the BFKL dynamics in inclusive three jet production at the LHC

F. Caporale^{1,a}, F. G. Celiberto^{1,2,b}, G. Chachamis^{1,c}, D. Gordo Gómez^{1,d}, and A. Sabio Vera^{1,e}

¹*Instituto de Física Teórica UAM/CSIC, Nicolás Cabrera 15 & Universidad Autónoma de Madrid, E-28049 Madrid, Spain.*

²*Dipartimento di Fisica, Università della Calabria & Istituto Nazionale di Fisica Nucleare, Gruppo Collegato di Cosenza, I-87036 Arcavacata di Rende, Cosenza, Italy.*

Abstract. We propose the study of new observables in LHC inclusive events with three tagged jets, one in the forward direction, one in the backward direction and both well-separated in rapidity from the each other (Mueller-Navelet jets), together with a third jet tagged in central regions of rapidity. Since non-tagged associated mini-jet multiplicity is allowed, we argue that projecting the cross sections on azimuthal-angle components can provide several distinct tests of the BFKL dynamics. Realistic LHC kinematical cuts are introduced.

1 Introduction

The Balitsky-Fadin-Kuraev-Lipatov (BFKL) resummation program in the leading logarithmic (LL) [1–6] and next-to-leading logarithmic (NLL) approximation [7, 8] may be applied for phenomenological studies at hadronic colliders when the final-state is characterised by jets that are produced at large relative rapidities. Mueller-Navelet jets [9] is an key example, specifically, for observables that are based on the azimuthal angle formed by the two outermost in rapidity tagged jets, ϕ . The precise form of the observables is built by considering ratios of projections on the azimuthal angle $\mathcal{R}_n^m = \langle \cos(m\phi) \rangle / \langle \cos(n\phi) \rangle$. Comparison of different NLL predictions against LHC experimental data for these observables has been quite successful [10–30].

New LHC observables, that may be seen as a generalisation of the Mueller-Navelet jets, were proposed recently for inclusive three-jet [31, 32] and four-jet production [33, 34]. In this work we discuss only the observables for inclusive three-jet production. These are defined by the generalised ratios [31]

$$\mathcal{R}_{PQ}^{MN} = \frac{\langle \cos(M\phi_1) \cos(N\phi_2) \rangle}{\langle \cos(P\phi_1) \cos(Q\phi_2) \rangle}, \quad (1)$$

^ae-mail: francesco.caporale@uam.es

^be-mail: francescogiovanni.celiberto@fis.unical.it

^ce-mail: chachamis@gmail.com

^de-mail: david.gordo@csic.es

^ee-mail: a.sabio.vera@gmail.com

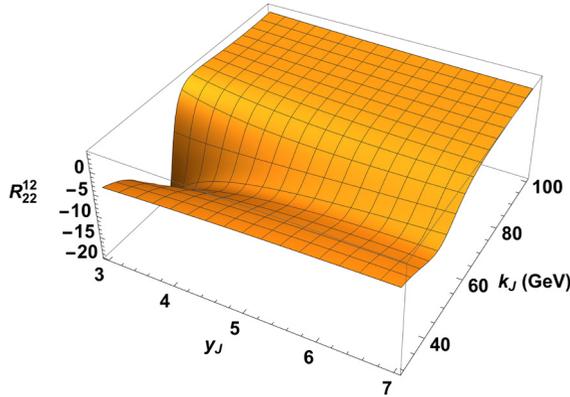


Figure 1. 3D plot of the partonic \mathcal{R}_{22}^{12} as a function of the rapidity y_J and the momentum k_J of the central jet for $k_A = 40$ GeV, $k_B = 50$ GeV and $\Delta Y_{A,B} = 10$.

where ϕ_1 is the azimuthal angle difference between the forward and the central jet and ϕ_2 the azimuthal angle difference between the central jet and the backward in rapidity jet. The ratios \mathcal{R}_{PQ}^{MN} in Eq. (1) are actually partonic level quantities and therefore, cannot be readily compared to experimental data. Therefore, we define the hadronic level observables R_{PQ}^{MN} [32] and study their stability once we introduce corrections beyond the LL accuracy. For that, we produce the two outermost in rapidity jets within the collinear factorization scheme, each of them associated with a forward “jet vertex” [35]. Then we link these vertices and the central jet using two BFKL gluon Green’s functions. At the end, the partonic differential cross-section is convoluted with collinear parton distribution functions and is integrated over the momenta of all produced jets in order to calculate the ratios R_{PQ}^{MN} . For the integration over the momenta of the jets we use standard LHC experimental cuts. The rapidity of the central jet takes values close to the middle of the rapidity distance between the two outermost tagged jets.

2 Hadronic inclusive three-jet production in multi-Regge kinematics

Let us first remember some of the notation defined in [31, 32]. Assuming that the transverse momenta of the outermost jets are $\vec{k}_{A,B}$, their rapidity difference, Y , is large and the central jet has transverse momentum \vec{k}_J . We allow for mini-jet activity between the three tagged jets so that the process¹ we need to study is

$$\text{proton}(p_1) + \text{proton}(p_2) \rightarrow \text{jet}(k_A) + \text{jet}(k_J) + \text{jet}(k_B) + \text{minijets} . \quad (2)$$

Firstly, we define the two relative azimuthal angles between the outermost jets and the central jet as $\Delta\theta_{AJ} = \theta_A - \theta_J - \pi$ and $\Delta\theta_{JB} = \theta_J - \theta_B - \pi$ respectively. Then the projection on azimuthal-angle components gives the mean value

$$C_{MN} = \langle \cos(M(\theta_A - \theta_J - \pi)) \cos(N(\theta_J - \theta_B - \pi)) \rangle \quad (3)$$

$$= \frac{\int_0^{2\pi} d\theta_A d\theta_B d\theta_J \cos(M(\theta_A - \theta_J - \pi)) \cos(N(\theta_J - \theta_B - \pi)) d\sigma^{3\text{-jet}}}{\int_0^{2\pi} d\theta_A d\theta_B d\theta_J d\sigma^{3\text{-jet}}}$$

¹Another interesting idea, suggested in [36] and investigated in [37, 38], is the study of the production of two charged light hadrons, π^\pm , K^\pm , p , \bar{p} , with large transverse momenta and well separated in rapidity.

where M, N are positive integers) and $d\sigma^{3\text{-jet}}$ the partonic differential cross-section for three-jet production defined in [31].

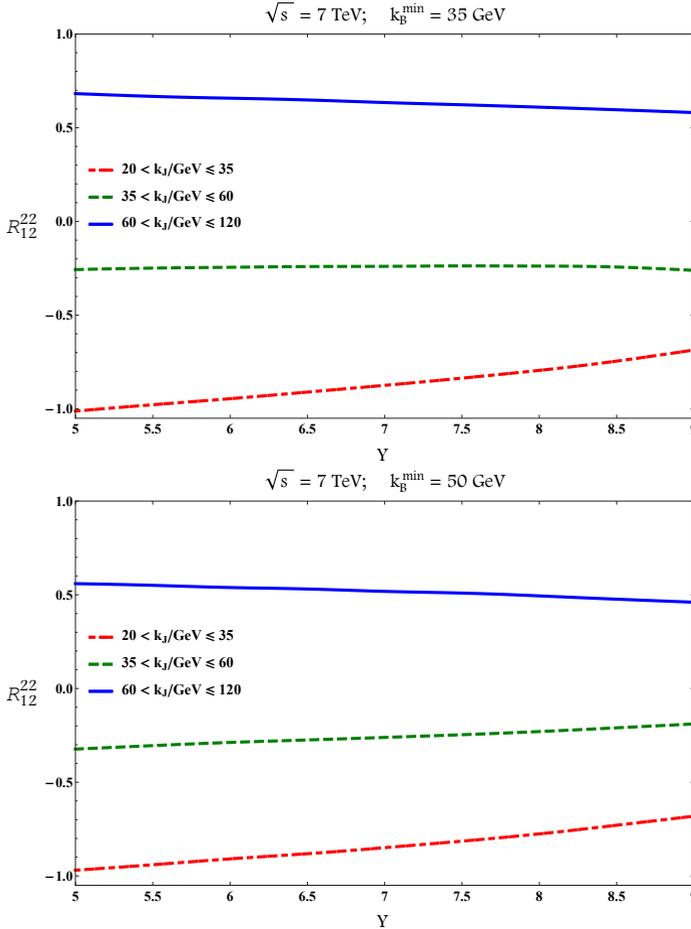


Figure 2. Y -dependence of the LL R_{12}^{22} for $\sqrt{s} = 7$ TeV. Symmetric cut $k_B^{\min} = 35$ GeV (top) and asymmetric cut $k_B^{\min} = 50$ GeV (bottom).

In order to compute theoretical estimates that may be compared against current and future experimental data, we integrate $C_{M,N}$ over the momenta of the tagged jets in the form

$$C_{MN} = \int_{Y_A^{\min}}^{Y_A^{\max}} dY_A \int_{Y_B^{\min}}^{Y_B^{\max}} dY_B \int_{k_A^{\min}}^{k_A^{\max}} dk_A \int_{k_B^{\min}}^{k_B^{\max}} dk_B \int_{k_J^{\min}}^{k_J^{\max}} dk_J \delta(Y_A - Y_B - Y) C_{MN}, \quad (4)$$

where the rapidity of the forward jet takes values in the range $Y_A^{\min} = 0$ and $Y_A^{\max} = 4.7$ and that of the backward jet in the range $Y_B^{\min} = -4.7$ and $Y_B^{\max} = 0$ while their difference $Y \equiv Y_A - Y_B$ is kept fixed at definite values in the range $5 < Y < 9$. We calculate C_{MN} for two different center-of-mass energies, $\sqrt{s} = 7$ and $\sqrt{s} = 13$ TeV and we introduce two typical kinematical cuts previously used in the study of Mueller-Navelet jets at the LHC. Specifically, we use both a symmetric and an asymmetric cut [19, 29]:

1. $k_A^{\min} = 35 \text{ GeV}$, $k_B^{\min} = 35 \text{ GeV}$, $k_A^{\max} = k_B^{\max} = 60 \text{ GeV}$ (symmetric);
2. $k_A^{\min} = 35 \text{ GeV}$, $k_B^{\min} = 50 \text{ GeV}$, $k_A^{\max} = k_B^{\max} = 60 \text{ GeV}$ (asymmetric).

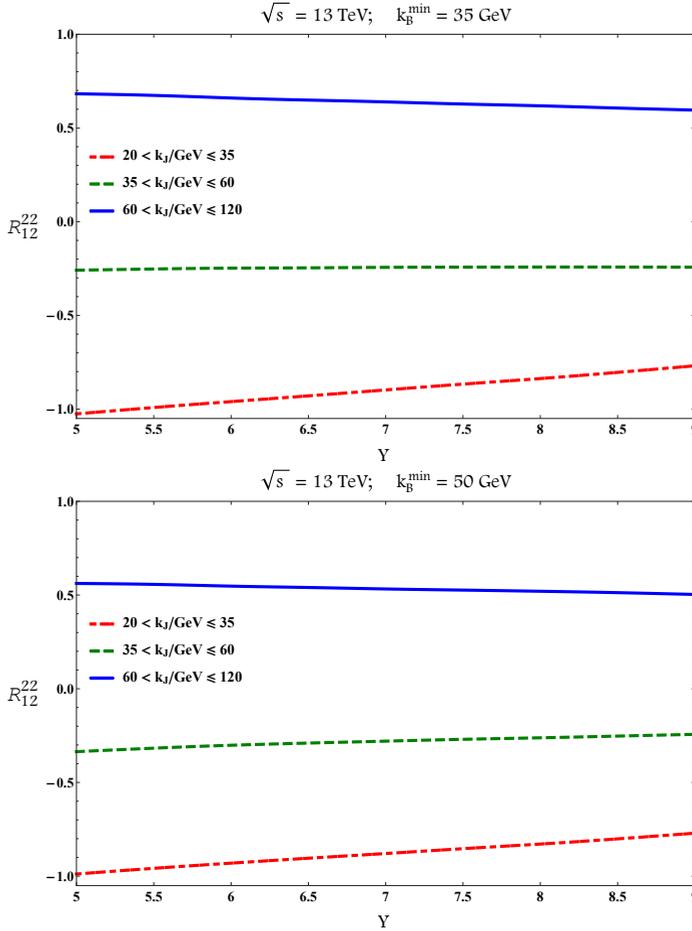


Figure 3. Y -dependence of the LL R_{12}^{22} for $\sqrt{s} = 13 \text{ TeV}$. Symmetric cut $k_B^{\min} = 35 \text{ GeV}$ (top) and asymmetric cut $k_B^{\min} = 50 \text{ GeV}$ (bottom).

We are interested in maximising the stability with respect to higher order effects (beyond LL) in our results (see [15]), therefore, we remove the zeroth conformal spin contribution of the BFKL kernel by considering the ratios

$$R_{PQ}^{MN} = \frac{C_{MN}}{C_{PQ}}, \quad M, N, P, Q > 0, \quad (5)$$

which have no $n = 0$ dependence. Thus, we can study the ratios $R_{PQ}^{MN}(Y)$ in Eq. (5) as functions of the rapidity difference Y between the outermost jets for some typical values of M, N, P, Q . We define three different p_T ranges (bins) for the allowed momentum of the central jet: bin-1 = $[20 \text{ GeV} < k_J < 35 \text{ GeV}]$ (k_J ‘smaller’ than k_A, k_B),

bin-2 = [35 GeV < k_J < 60 GeV] (k_J ‘similar’ to k_A, k_B),
 bin-3 = [60 GeV < k_J < 120 GeV] (k_J ‘larger’ than k_A, k_B).

This permits the discrimination of different behaviours of the $R_{PQ}^{MN}(Y)$ by using as a criterion the relative size of the central jet. In Fig. 1 we show the behaviour of \mathcal{R}_{22}^{12} as we change the size of the central jet and its position in rapidity. We notice that while a small variation in y_J around the central rapidity value $\Delta Y_{A,B}/2 = 5$ does not result in significant changes for a fixed k_J , a change in the value of k_J may have a big impact for a fixed y_J . A number of different ratios was presented in [32], here

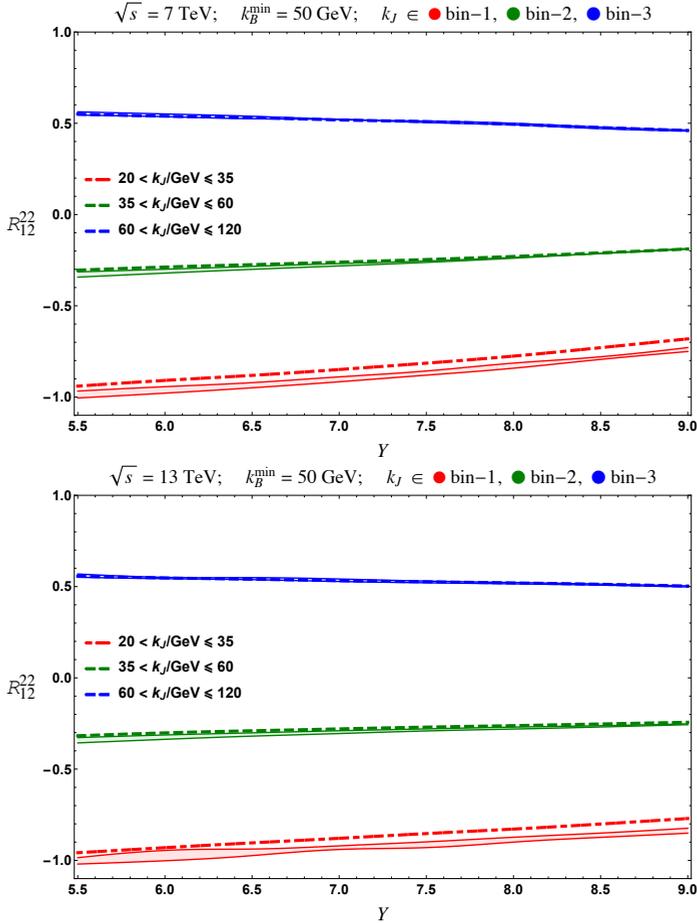


Figure 4. Y -dependence of the LL (dashed lines) and NLL (BLM) predictions (continuous bands) for R_{12}^{22} in the asymmetric cut at $\sqrt{s} = 7$ TeV (top) and $\sqrt{s} = 13$ TeV (bottom). The blue NLL band is very narrow and lies on top of the LL line.

we are focussing on ratios that involve the coefficients C_{12} and C_{22} . In Figs. 2 and 3 we see the LL accuracy results for R_{12}^{22} . Generally, the dependence of the different observables on the rapidity difference between k_A and k_B is rather smooth whereas the slope of the three curves depends on the particular observable. For R_{12}^{22} we see that shifting from a symmetric to an asymmetric cut makes no noticeable difference. Moreover, there are no important changes when we change the colliding energy

from $\sqrt{s} = 7$ TeV to $\sqrt{s} = 13$ TeV. The latter is crucial as it suggests that R_{12}^{22} is already within some sort of asymptotic regime for the specific kinematical configurations.

Apart from the stability of the observable with regard to an increase of the colliding energy, another important question is the stability with respect to effects that go beyond the LL approximation [39]. A first important step towards a full NLL computation is to take into account the NLL contributions to the two gluon Green's functions that connect the three jets. In Fig. 4 we present exactly these corrections obtained by using the Brodsky-Lepage-Mackenzie (BLM) prescription [40] for the R_{12}^{22} coefficient in the asymmetric cut. In particular, we have used the MOM scheme and chosen the renormalisation scale such that the β_0 -dependence of the given observable vanishes, following the BLM prescription. The dashed lines represent the LL predictions and the coloured bands represent the NLL BLM predictions. It is impressive that the NLL values are almost on top of the LL ones which gives us great confidence that the observables R_{PQ}^{MN} are indeed excellent BFKL probes at the LHC.

3 Summary & Outlook

We have presented a first BFKL driven hadronic-level phenomenological work on the recently proposed observables R_{PQ}^{MN} that depend on the azimuthal-angles of the jets in inclusive three-jet production at the LHC. In particular, we concentrated on the ratio R_{12}^{22} . Our major task was to study the variation of R_{12}^{22} when we set the rapidity difference Y between the outermost jets to different fixed values in the range $5 < Y < 9$. Generally, we notice a smooth functional dependence of the ratio R_{12}^{22} on Y . A key observation is that R_{12}^{22} and other similar ratio observables do not exhibit a significantly different behaviour when one changes the energy configuration from 7 to 13 TeV. This gives us confidence that these ratios pinpoint the most important characteristics of the azimuthal behaviour of the tagged jets within the BFKL framework. Moreover, one of the two big parts of radiative corrections beyond the LL approximation, namely, the NLL contributions to the two gluon Green's functions that link the three tagged jets, do not significantly change the functional behaviour of R_{12}^{22} on Y . The other big part of the beyond the LL corrections would be the NLO corrections to the jet vertices and these need also to be taken into account. In addition, it is crucial to investigate whether fixed order calculations and studies with the BFKL inspired Monte Carlo **BFKLex** [41–48] give similar results. Predictions from the usual all-purpose collinear Monte Carlo codes are also needed to complete the picture from the theoretical side. To conclude with, a dedicated experimental analysis on the proposed ratio observables R_{PQ}^{MN} based on existing (and future) LHC data will answer the question of whether these observables qualify as new probes for the BFKL dynamics.

Acknowledgements

GC acknowledges support from the MICINN, Spain, under contract FPA2013-44773-P. ASV acknowledges support from Spanish Government (MICINN (FPA2010-17747,FPA2012-32828)) and, together with FC and FGC, to the Spanish MINECO Centro de Excelencia Severo Ochoa Programme (SEV-2012-0249). FGC thanks the Instituto de Física Teórica (IFT UAM-CSIC) in Madrid for warm hospitality.

References

- [1] L. N. Lipatov, Sov. Phys. JETP **63** (1986) 904 [Zh. Eksp. Teor. Fiz. **90** (1986) 1536].
- [2] I. I. Balitsky and L. N. Lipatov, Sov. J. Nucl. Phys. **28** (1978) 822 [Yad. Fiz. **28** (1978) 1597].

- [3] E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP **45** (1977) 199 [Zh. Eksp. Teor. Fiz. **72** (1977) 377].
- [4] E. A. Kuraev, L. N. Lipatov and V. S. Fadin, Sov. Phys. JETP **44** (1976) 443 [Zh. Eksp. Teor. Fiz. **71** (1976) 840] [Erratum-ibid. **45** (1977) 199].
- [5] L. N. Lipatov, Sov. J. Nucl. Phys. **23** (1976) 338 [Yad. Fiz. **23** (1976) 642].
- [6] V. S. Fadin, E. A. Kuraev and L. N. Lipatov, Phys. Lett. B **60** (1975) 50.
- [7] V. S. Fadin and L. N. Lipatov, Phys. Lett. B **429** (1998) 127 [hep-ph/9802290].
- [8] M. Ciafaloni and G. Camici, Phys. Lett. B **430** (1998) 349 [hep-ph/9803389].
- [9] A. H. Mueller and H. Navelet, Nucl. Phys. B **282** (1987) 727.
- [10] V. Del Duca and C. R. Schmidt, Phys. Rev. D **49** (1994) 4510 [hep-ph/9311290].
- [11] W. J. Stirling, Nucl. Phys. B **423** (1994) 56 [hep-ph/9401266].
- [12] L. H. Orr and W. J. Stirling, Phys. Rev. D **56** (1997) 5875 [hep-ph/9706529].
- [13] J. Kwiecinski, A. D. Martin, L. Motyka and J. Outhwaite, Phys. Lett. B **514** (2001) 355 [hep-ph/0105039].
- [14] M. Angioni, G. Chachamis, J. D. Madrigal and A. Sabio Vera, Phys. Rev. Lett. **107**, 191601 (2011) [arXiv:1106.6172 [hep-th]].
- [15] F. Caporale, B. Murdaca, A. Sabio Vera and C. Salas, Nucl. Phys. B **875** (2013) 134 [arXiv:1305.4620 [hep-ph]].
- [16] F. Caporale, D. Y. Ivanov, B. Murdaca and A. Papa, Nucl. Phys. B **877** (2013) 73 [arXiv:1211.7225 [hep-ph]].
- [17] C. Marquet and C. Royon, Phys. Rev. D **79**, 034028 (2009) [arXiv:0704.3409 [hep-ph]].
- [18] D. Colferai, F. Schwennsen, L. Szymanowski and S. Wallon, JHEP **1012**, 026 (2010) [arXiv:1002.1365 [hep-ph]].
- [19] B. Ducloue, L. Szymanowski and S. Wallon, JHEP **1305**, 096 (2013) [arXiv:1302.7012 [hep-ph]].
- [20] B. Ducloue, L. Szymanowski and S. Wallon, Phys. Lett. B **738**, 311 (2014) [arXiv:1407.6593 [hep-ph]].
- [21] A. H. Mueller, L. Szymanowski, S. Wallon, B. W. Xiao and F. Yuan, JHEP **1603**, 096 (2016) [arXiv:1512.07127 [hep-ph]].
- [22] N. Cartiglia *et al.* [LHC Forward Physics Working Group Collaboration], CERN-PH-LPCC-2015-001, SLAC-PUB-16364, DESY-15-167.
- [23] G. Chachamis, arXiv:1512.04430 [hep-ph].
- [24] A. Sabio Vera, Nucl. Phys. B **746** (2006) 1 [hep-ph/0602250].
- [25] A. Sabio Vera and F. Schwennsen, Nucl. Phys. B **776** (2007) 170 [hep-ph/0702158 [HEP-PH]].
- [26] B. Ducloue, L. Szymanowski and S. Wallon, Phys. Rev. Lett. **112** (2014) 082003 [arXiv:1309.3229 [hep-ph]].
- [27] F. Caporale, D. Y. Ivanov, B. Murdaca and A. Papa, Eur. Phys. J. C **74** (2014) 3084 [arXiv:1407.8431 [hep-ph]].
- [28] F. Caporale, D. Y. Ivanov, B. Murdaca and A. Papa, Phys. Rev. D **91** (2015) 11, 114009 [arXiv:1504.06471 [hep-ph]].
- [29] F. G. Celiberto, D. Yu. Ivanov, B. Murdaca and A. Papa, Eur. Phys. J. C **75** (2015) 292 [arXiv:1504.08233 [hep-ph]].
- [30] F. G. Celiberto, D. Y. Ivanov, B. Murdaca and A. Papa, Eur. Phys. J. C **76**, no. 4, 224 (2016) [arXiv:1601.07847 [hep-ph]].

- [31] F. Caporale, G. Chachamis, B. Murdaca and A. Sabio Vera, *Phys. Rev. Lett.* **116**, no. 1, 012001 (2016) [arXiv:1508.07711 [hep-ph]].
- [32] F. Caporale, F. G. Celiberto, G. Chachamis, D. G. Gomez and A. Sabio Vera, arXiv:1603.07785 [hep-ph].
- [33] F. Caporale, F. G. Celiberto, G. Chachamis and A. Sabio Vera, *Eur. Phys. J. C* **76**, no. 3, 165 (2016) [arXiv:1512.03364 [hep-ph]].
- [34] F. Caporale, F. G. Celiberto, G. Chachamis, D. G. Gomez and A. Sabio Vera, arXiv:1606.00574 [hep-ph].
- [35] F. Caporale, D. Yu. Ivanov, B. Murdaca, A. Papa, A. Perri, *JHEP* **1202** (2012) 101; [arXiv:1212.0487 [hep-ph]].
- [36] D. Y. Ivanov and A. Papa, *JHEP* **1207**, 045 (2012) [arXiv:1205.6068 [hep-ph]].
- [37] F. G. Celiberto, D. Y. Ivanov, B. Murdaca and A. Papa, arXiv:1604.08013 [hep-ph].
- [38] F. G. Celiberto, D. Y. Ivanov, B. Murdaca and A. Papa, work to be released soon.
- [39] F. Caporale, F. G. Celiberto, G. Chachamis, D. G. Gomez and A. Sabio Vera, work to be released soon.
- [40] S. J. Brodsky, G. P. Lepage and P. B. Mackenzie, *Phys. Rev. D* **28**, 228 (1983). doi:10.1103/PhysRevD.28.228
- [41] G. Chachamis, M. Deak, A. Sabio Vera and P. Stephens, *Nucl. Phys. B* **849** (2011) 28 [arXiv:1102.1890 [hep-ph]].
- [42] G. Chachamis and A. Sabio Vera, *Phys. Lett. B* **709** (2012) 301 [arXiv:1112.4162 [hep-th]].
- [43] G. Chachamis and A. Sabio Vera, *Phys. Lett. B* **717** (2012) 458 [arXiv:1206.3140 [hep-th]].
- [44] G. Chachamis, A. Sabio Vera and C. Salas, *Phys. Rev. D* **87** (2013) 1, 016007 [arXiv:1211.6332 [hep-ph]].
- [45] F. Caporale, G. Chachamis, J. D. Madrigal, B. Murdaca and A. Sabio Vera, *Phys. Lett. B* **724** (2013) 127 [arXiv:1305.1474 [hep-th]].
- [46] G. Chachamis and A. Sabio Vera, arXiv:1511.03548 [hep-ph].
- [47] G. Chachamis and A. Sabio Vera, *JHEP* **1602** (2016) 064 [arXiv:1512.03603 [hep-ph]].
- [48] G. Chachamis and A. Sabio Vera, *Phys. Rev. D* **94**, no. 3, 034019 (2016) [arXiv:1606.07349 [hep-ph]].