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Systematic study of shell-model effective interaction in sd shell

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Abstract. The spin-tensor decomposition method has been used to analyze the shell model effective interactions in *sd* shell systematically. Almost all the interactions have been studied, including the microscopic interactions and phenomenological ones. It can be noticed that the discrepancies between the central force of microscopic interactions with the ones of empirical interactions are remarkable.

1 Introduction

As is well known, effective interaction, valence space and algorithm are three pillars in shell model [1]. The effective interaction, as the starting point, can be derived by a microscopic theory from the free nucleon-nucleon (NN) interaction or given by a phenomenologically determined interaction [1–3]. The famous Kuo-Brown interaction is built from a realistic NN potential by means of many-body perturbation techniques [4]. The phenomenological ones, also called empirical interactions, can be obtained by least-square fitting experimental binding energies and spectroscopies [2]. The most successful empirical interactions might be USDA and USDB interactions [5].

Because the universal interaction is still lacking in shell model, the predictive power is restricted in small regions [1–3, 6]. It has been demonstrated in Refs. [1, 7] that one may obtain a reliable multipole Hamiltonian from G-matrix, but the monopole Hamiltonian is often incorrect. However, the monopole Hamiltonian has large influence on the large-scale shell model calculations because of the significant role at shell structure evolution [1]. Otsuka *et al.* suggested that the spin-isospin $(\sigma \cdot \sigma)(\tau \cdot \tau)$ component of nucleon-nucleon interaction between spin-orbit partners, $j = l \pm \frac{1}{2}$, is important for the shell evolution [8]. In a later publication [9], they pointed out that the tensor term of NN interaction interacting between spin-flip proton-neutron partners can have a large influence on shell structures. They also proved that bare tensor force has persistency against renormalization and is well established in microscopic interactions [10]. The effect of tensor interaction on shell structure evolution is stressed also in Refs. [11, 12]. The same results can also be obtained by mean-field model calculations [13]. Umeya and Muto showed, by employing the spin-tensor-decomposition method, that central-triplet-even components dominate at the monopole parts of shell-model effective interactions [14, 15]. The universality of central force has been demonstrated by Ref. [10].

2 Spin-tensor decomposition of the effective interaction

One of the key methods to analyze effective interaction is spin-tensor-decomposition method [11, 14, 16–18]. To do this, we first transform the jj-coupled two-body-matrix-elements (TBMEs) into LS-coupled representations, in which various components of the interaction (central, tensor, spin-orbit, and antisymmetric spin-orbit) can be explicitly separated [19]. It is a unique transformation and the only assumption is that the interaction is based on a two-body operator [19]. Following this transformation we can do a spin-tensor decomposition of these elements. The most general two-body interaction $V(1,2)$ can be decomposed as [19, 20]

$$V(1,2) = \sum_{k=0,1,2} (S^k \cdot Q^{(k)}) = \sum_{k=0,1,2} V^{(k)}, \tag{1}$$

where S and Q are irreducible tensors of rank k in spin and space coordinates, respectively. The ranks 0, 1, and 2 correspond to central, spin-orbit, and tensor forces, respectively.

Two-body interaction of central, spin-orbit, and tensor forces can be described in details by the singlet-triplet even-odd representation [14, 15, 19]. When antisymmetrizing two-nucleon states, the condition $L + S + T =$ odd has to be fulfilled, which restricts the channels to be singlet-odd (SO), triplet-even (TE), singlet-even(SE), and triplet-odd (TO) [14, 15, 19]. The central force has all these four channels [14, 15, 19]. The spin-orbit force $\hat{V}_{\mathrm{LS}}$ and the tensor force $\hat{V}_{\mathrm{T}}$ only have spin triplet states [14, 15, 19]. NN interaction in the vacuum conserves relative parity. The effective interaction for the shell model, which is defined in a restricted configuration space, does not necessarily conserve it, because the translational invariance is violated. This component is called anti-symmetric spin-orbit force (ALS). The possible origins of ALS may be core-polarization, higher order terms of the perturbation or three body interaction [19, 21].

3 Shell Model Hamiltonian in sd shell

Effective interactions in a specific model space need to include all possible combinations of orbitals and have dependence on spin and isospin. In the sd shell, involving the $1s1/2$, $1d3/2$, and $1d5/2$ levels, the single-particle-energy (SPE) of 3 levels and 63 TBMEs have to be known [5]. There are many sets of sd shell Hamiltonians. Through the spin-tensor-decomposition method, we can analyze them in a systematic way. As shown in Fig. 1, Bonn-A [3], Kuo-Brown [4] interactions are microscopic ones from G matrix. Bare Kuo-Brown [4] and modified Kuo-Brown (Kuosdh) [22] interactions are also given. M3Y [23], HKT [24], Schiffer-True [6] interactions are based on phenomenological potentials, the purpose of which is searching for a universal interaction. All of them are compared to USDA and USDB interactions [5]. Original USD interaction [25] is also included.

In the singlet-triplet even-odd representation, the monopoles of these interactions are given in Fig. 1. It can be seen that central-triplet-even parts are the strongest among all these monopoles, same as the discoveries in Refs. [14, 15]. As can be expected, USDA and USDB interactions are quite alike, because the only difference between them is the number of linear combinations when doing the fitting [5]. The discrepancies of original USD from

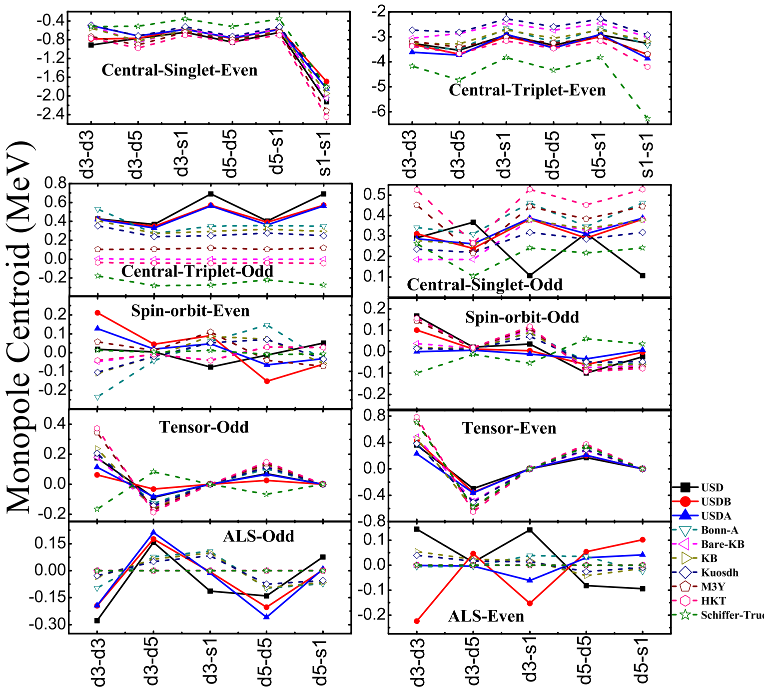

Figure 1. Comparisons of monopole-centroid components among USD [25], USDA [5], USDB [5], Bonn-A [3], bare-Kuo-Brown [4], Kuo-Brown [4], Kuosdh [22], M3Y [23], HKT [24], and Schiffer-True [6] interactions decomposed by spin-tensor-decomposition method. The triplet-singlet even-odd representation is used to distinguish different channels of the interactions. The left and right panels are T=1 and T=0 interactions, respectively.

USDA and USDB are remarkable in the odd channels of central force. The interactions from G matrix, Bonn-A and Kuo-Brown interactions, are less attractive and less repulsive than USDA and USDB in T=0 and T=1 central force respectively, which is in accordance with the trend for the fp-shell interactions [26]. The discrepancies in non-central channels can also be noticed. However, the magnitudes of differences are smaller than those in central force. For the phenomenological ones, Schiffer-True interaction has large discrepancies with USDA and USDB interactions in all the channels. The practical application of this interaction can encounter problems. M3Y interaction is closer to empirical and G matrix interactions than HKT interaction. It might be a helpful tool when reliable Hamiltonians are lacking. Interestingly, all these interactions have similar tensor-even parts. The empirical interactions have larger ALS channels than interactions obtained by G matrix. It may explain the need of three-body force which is one important source of ALS interactions [19, 21].

4 Summary

In summary, by spin-tensor-decomposition technic, we analyze shell model Hamiltonian in sd shell systematically. The central-triplet-even channel dominates the monopoles. The discrepancies between microscopic interactions and empirical interactions are remarkable in central forces. The differences in non-central force can be noticed but has smaller magnitudes than the central force.

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