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Energy from nuclear fission(*)

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Summary. — The main features of nuclear fission as physical phenomenon will be revisited, emphasizing its peculiarities with respect to other nuclear reactions. Some basic concepts underlying the operation of nuclear reactors and the main types of reactors will be illustrated, including fast reactors, showing the most important differences among them. The nuclear cycle and radioactive-nuclear-waste production will be also discussed, along with the perspectives offered by next generation nuclear assemblies being proposed. The current situation of nuclear power in the world, its role in reducing carbon emission and the available resources will be briefly illustrated.

1. – Physics of fission

What is fission and why can it produce energy? The basic reason is the behaviour of the nuclear mass and nuclear binding energy for different nuclear species. As is known from relativity, the mass of a bound system is given by the sum of the masses of its components, plus the binding energy of the system. The binding energy is by definition a negative quantity, *i.e.* the mass of the composite system is always less than the sum of the masses of its components. When applied to nuclear masses, this concept can be

(*) Most of the material of sects. **1**, **2** and **3** is based on [1] and [2].

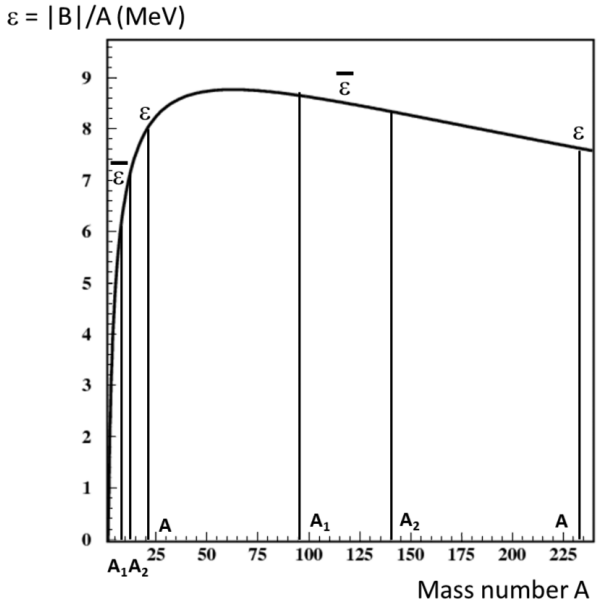

Fig. 1. – Absolute value of binding energy per nucleon, as a function of the mass number A.

expressed through the equation

$$M(Z,A) = ZM_p + (A-Z)M_n + B(Z,A), \tag{1}$$

where $M(Z,A)$ is the mass of a nucleus with Z protons and A protons and neutrons (*i.e.* $A-Z$ neutrons), M_p is the proton mass, M_n is the neutron mass and $B(Z,A)$ is the binding energy of the system, with $B(Z,A) < 0$, *i.e.* $M(Z,A) < ZM_p + (A-Z)M_n$. Z is called the atomic number, while A is called the mass number. The absolute value of the binding energy is the energy that must be supplied from outside to overcome the binding nuclear force and set all nucleons free. We can also define the binding energy per nucleon, *i.e.* the average quantity of energy that must be supplied per each single nucleon when separating all nucleons from one another. As one can see from fig. 1, where the absolute value of the binding energy per nucleon is reported, $\epsilon = \frac{|B|}{A}$, such value increases from the lightest nucleus, the deuteron, to the region of iron, then it decreases towards the heavier nuclei. In other words, iron is more bound than the light nuclei, while *e.g.* uranium is less bound than iron.

1˙1. *Energy balance*. – For a nuclear reaction to give up more energy than it absorbs, it has to move in the direction of stronger binding. Indeed, if a system goes from smaller binding energy to higher binding energy, the total mass will decrease in going from the initial to the final state, so that the difference in mass will correspond to energy being released by the system. Again looking at fig. 1, it is clear that two light nuclei undergoing

nuclear fusion, *i.e.* a nuclear system moving from the left-hand side of the graph in fig. 1 to the right, will move towards higher binding, thereby releasing nuclear energy, *i.e.* giving rise to an *exoenergetic* nuclear reaction, namely a reaction which yields a net amount of energy. This is the mechanism at the basis of nuclear fusion [3], *e.g.* in the reaction $^2\mathrm{H} + {}^3\mathrm{H} \rightarrow {}^4\mathrm{He} + \mathrm{n}$, where about 17.5 MeV are released in the process. On the other hand, heavy nuclei above iron can undergo fission, where the nuclear system splits into two nuclei, moving from the right-hand side of the graph in fig. 1 to the left, again moving towards higher binding. This is called the process of nuclear fission and can also be exoenergetic. Let us consider in detail the energy balance of a fission process where a nucleus with Z protons and $(A - Z)$ neutrons splits into two nuclei

$$(Z, A) \rightarrow (Z_1, A_1) + (Z_2, A_2), \tag{2}$$

where obviously $Z = Z_1 + Z_2$ and $A = A_1 + A_2$ and the masses of the nuclei are $M(Z, A)$, $M(Z_1, A_1)$ and $M(Z_2, A_2)$, respectively. The energy release in the reaction is called the Q-value and it is given by definition by

$$Q = M(Z, A) - M(Z_1, A_1) - M(Z_2, A_2), \tag{3}$$

where, by using eq. (1) we obtain

$$\begin{aligned} Q &= B(Z, A) - B(Z_1, A_1) - B(Z_2, A_2) \\ &= -\epsilon A + \epsilon_1 A_1 + \epsilon_2 A_2 = -\epsilon A + \bar{\epsilon} A = (\bar{\epsilon} - \epsilon) A, \end{aligned} \tag{4}$$

where

$$\bar{\epsilon} = \frac{\epsilon_1 A_1 + \epsilon_2 A_2}{A_1 + A_2}\,. \tag{5}$$

Again looking at fig. 1, it is easy to see that if the initial nucleus is a heavy one (as indicated at the right of the picture), its fission into two medium-mass nuclei is such that $Q > 0$. By repeating a similar calculation for two light nuclei undergoing fusion, it is easy to see that also in that case one can get $Q > 0$.

The fact that fission of heavy nuclei can be energetically favored does not mean that fission will happen easily in a spontaneous manner. Indeed, by looking at fig. 2, imagine to bring the final two nuclei (called *fission fragments*) from far apart to being close to each other: they will experience Coulomb repulsion in form of a Coulomb barrier and their fusion into the original nucleus will happen only via quantum tunneling through the barrier. Viceversa, imagine the two nuclei being bound together in a potential well: to be separated from each other, again they will have to pass through the Coulomb barrier. This is happening in the so-called *spontaneous fission* and the lifetime for the nucleus to decay via spontaneous fission will vary depending on the probability for the two nuclei to pass through the barrier via quantum tunneling. Clearly, if energy is supplied to the initial heavy nucleus from the outside, the system will get to a quantum state closer to

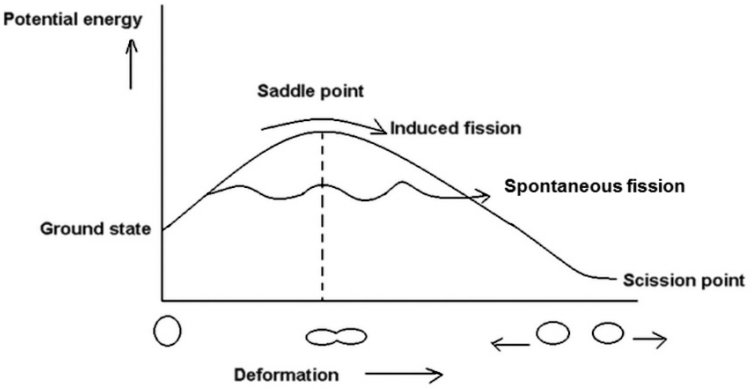

Fig. 2. – Nuclear potential energy as a function of the distance between the two fragments.

the maximum and therefore the probability for the two fragments to penetrate the barrier will increase: this is what we call *induced fission*. Now, suppose that one more neutron is added to a heavy nucleus (Z, A): in general, it will become bound by a significant amount of energy (several MeV) in the newly formed $(Z, A + 1)$ nucleus. This means that when a neutron is captured by a heavy nucleus, almost 8 MeV of energy will be released and made available for the two fragments inside the nucleus to approach the top of the barrier, thereby increasing the probability of the $(Z, A + 1)$ nucleus to undergo fission by orders of magnitude and making fission become a process of relatively high importance.

Fission is not the only exoenergetic nuclear reaction induced by neutrons. For instance, the reaction $\mathrm{n} + {}^3\mathrm{He} \rightarrow {}^3\mathrm{H} + \mathrm{p}$, when initiated by a very slow neutron (whose kinetic energy we can neglect), can release about 0.76 MeV (by the way this is the reaction used in ^{3}He slow neutron detectors) in form of the kinetic energy of the two final nuclei. The reaction $\mathrm{n} + {}^6\mathrm{Li} \rightarrow {}^3\mathrm{H} + {}^4\mathrm{He}$ can release about 4.78 MeV, again taken by the motion of the final nuclei. Other reactions of this kind can occur. So then what is so special about fission of heavy nuclei? It happens that for some selected heavy nuclei, e.g. the specific isotope of uranium ${}^{235}_{92}\mathrm{U}$ (where 92 is the atomic number Z and 235 is the mass number A), fission occurs with a very high probability after capture of a neutron. At the same time, an important feature of the fission process is that it is accompanied by the emission of a few *prompt neutrons*, *i.e.* in addition to the splitting of the initial nucleus into two fragments, a few neutrons are emitted by the fragments in a very short time after the splitting. Such neutrons are fast, *i.e.* they have high speed and high kinetic energy (about 2 MeV on average) and, as we will see shortly, if properly slowed down, can be very effective in inducing another fission, and so on and so forth. Therefore fission, under certain conditions, can give rise to a *chain reaction*, where the neutrons emitted by one fission can produce additional fissions, with a global very significant release of energy.

1˙2. *Energy released and reaction products.* – As one could in principle derive from eq. (4), when a uranium nucleus fissions into two nuclear fragments, about 0.1% of uranium mass appears as fission energy of $\sim 200\,\mathrm{MeV}$, which is much bigger than any other exoenergetic nuclear reaction (just for comparison, in the chemical reaction where a methane molecule is burnt, $CH_4 + 2O_2 \rightarrow CO_2 + 2H_2O$, about 9.2 eV of energy are released, more than 7 orders of magnitude less). Together with the fission fragments, an average of 2.5 prompt neutrons are emitted, with a mean kinetic energy per neutron of about 2 MeV (the total mean kinetic energy of all prompt neutrons being about 4.8 MeV). The average number of neutrons emitted is one of the many important parameters characterizing fission and it is called ν, whose value in general is of the order of 2–3 for various nuclei and incident neutron energies. Out of the 200 MeV released per fission, about 180 MeV appear at the time of fission as kinetic energy of the emerging two nuclei, plus kinetic energy of the prompt neutrons, plus energy in form of prompt gamma rays (photons) emitted when nuclear fragments are produced in a nuclear excited state that decays electromagnetically in a time so short that is considered negligible (say, typically less than a femtosecond, to give an idea). Additional energy is released later because most nuclear fragments undergo β decay (the process caused by the weak interaction where Z of the nucleus increases by one unity, while A stays the same, and concurrently an electron and an antineutrino are emitted by the nucleus; β decay is a very common process for the decay of unstable light and medium mass nuclei; you can see it basically as one neutron inside the nucleus transforming into a proton by emitting an electron and an antineutrino). Most of this energy is carried by the electron, the associated antineutrino and by the delayed gammas emitted when the beta decay transition leads to an excited state of the final nucleus. However, since the antineutrinos will escape from the reactor with negligible interaction with the materials crossed, the amount of energy from β decay actually contributing to the thermal budget of the reactor is about 13 MeV. When radioactive fragment build-up reaches a steady state, these 13 MeV (6.5% of fission energy) actually become a prompt contribution to the reactor thermal power and appear as a residual heat at the moment of reactor shutdown (see later).

1˙3. *Cross sections and flux.* – The cross section is a physical quantity and an observable that characterizes a nuclear reaction (elastic, inelastic scattering, etc.). The cross section is connected to the range of the involved forces and represents the effective area of a nuclear target in a certain type of reaction or collision. Here we will only consider the so-called total cross section, defined as follows. Given a flux of particles incident on a single nucleus (target) per unit area $\mathrm{d}S$ per unit time,

$$\frac{\mathrm{d}N_{in}}{\mathrm{d}S\mathrm{d}t}\,, \tag{6}$$

and given an interaction (or collision) rate, which represents the number of interacting particles (scattered or absorbed projectiles) per unit time,

$$\frac{\mathrm{d}N_{\mathrm{reac}}}{\mathrm{d}t}\,, \tag{7}$$

then the total reaction cross section σ is

$$\sigma = \frac{\frac{\mathrm{d}N_{\mathrm{reac}}}{\mathrm{d}t}}{\frac{\mathrm{d}N_{in}}{\mathrm{d}S\mathrm{d}t}} . \tag{8}$$

As is evident from the above equation, σ has the physical dimensions of a surface, which is logical given its interpretation as an effective target area.

It is possible to show that, given a macroscopic target comprising several nuclei with density ρ (for instance, expressed in gr/cm^3) and thickness x, struck by a particle beam of intensity I (particles/sec), the reaction rate is given by

$$R = \frac{\mathrm{d}N_{\mathrm{reac}}}{\mathrm{d}t} = I\frac{\rho x}{A}N_A\sigma, \tag{9}$$

where A is the target atomic weight (for instance, expressed in grams) and N_A is the Avogadro number. Note that $\frac{\rho x}{A}$ is the number density of nuclei in the target (*i.e.* number of nuclei per unit volume). This is all valid for a small thickness x. For a target of arbitrary thickness, one has first to divide it in thin slices of thickness $\mathrm{d}x$, to obtain that the intensity of the incident particle beam is decreasing with the distance inside the target as

$$I(x) = I(0)\exp\left\{-\frac{\rho}{A}N_A\sigma x\right\}. \tag{10}$$

In nuclear physics, the cross section unit is the barn, corresponding to $10^{-24}\,\mathrm{cm}^2$.

The quantity

$$\Sigma = \frac{\rho}{A}N_A\sigma \tag{11}$$

is called the *macroscopic cross section* and represents the probability of interaction per unit length of material crossed (and therefore has dimensions of the inverse of a length). It is easy to show that $1/\Sigma$ is the mean free path of an incident particle in the material and Σv is the frequency at which reactions occur while crossing the material, if v is the projectile speed.

1˙4. *Types of nuclear reactions*. – The scattering process between two nuclei A and B can be *elastic*, when the reaction is $A+B \to A+B$ (just a collision without change in the form of the two colliding systems), *inelastic* in the case of reactions like $A+B \to A+B*$ (where $B*$ means an excited state of nucleus B), $A+B \to A+C+D$ (where the nucleus B splits into C and D), $A+B \to C+D$ (where A and B, *e.g.*, exchange some protons or neutrons and therefore become different nuclei), etc.

The simplest type of nuclear reaction occurring in a nuclear reactor is the so-called *potential scattering*, where neutrons scatter elastically off the nuclear potential without ever penetrating the nucleus itself (similar to billiard ball collisions). By quantum mechanical arguments, it is possible to show that at low energies the cross section for such

a reaction is essentially just the geometrical cross section of the nucleus. Such type of collision has a rather flat energy dependence from about 1 eV up to the MeV range.

Another very relevant reaction mechanism is *neutron capture*. From our subsection above about nuclear binding energies, one can recall that for heavy nuclei, the addition of one more neutron can provide several MeV from binding energy. This energy can make the nucleus jump to an excited state, from which it will typically either decay by gamma emission or by fission. The first process, followed by gamma emission, is called *radiative capture* and is clearly in competition with fission. By quantum mechanical arguments, it is possible to show that at low energies the neutron capture cross section will follow a $1/v$ law, v being the speed of the neutron.

In the capture process, a neutron is first absorbed by nucleus $X(Z, A)$, forming the compound nucleus $Y(Z, A + 1)$. The formation of a compound nucleus can proceed through a so-called resonance reaction, which occurs when the CM energy of the neutron+nucleus system + the binding energy of the captured neutron match one of the energy levels in the compound nucleus. This phenomenon is indicated by sharp peaks in the capture cross section (see fig. 3). Neutrons which go around in a reactor core with an initial kinetic energy above the resonance region, can then "cross" a resonance when they scatter around and lose energy and can therefore be more strongly absorbed also by elements other than fuel.

Since the nuclear radius is roughly 10^{-12} cm, the geometrical cross sectional area of the nucleus is roughly 10^{-24} cm $= 1$ barn. Hence we might expect that nuclear cross sections are of the order of 1 barn. However, quantum mechanical effects can make nuclear cross sections a lot bigger, while specific dynamical effects from the nuclear forces can make them much smaller. As an example, in fig. 3 are reported the cross sections for the processes of fission (red curves) and radiative capture (green curves) for the two uranium isotopes found in nature and used in nuclear reactors, ^{238}U and ^{235}U, as a function of the neutron energy [4]. It is clearly seen that, for fast neutrons with kinetic energy around 1 MeV, both fission and radiative capture show a cross section in the order of 1 barn, but for lower energies the radiative capture for both nuclei and the fission for ^{235}U grow towards much higher values, following the above-mentioned $1/v$ law.

Fission itself can produce fission fragments with very strong radiative capture cross sections: such specific fission products are called *neutron poisons*, *e.g.* ^{135}Xe ($\sigma \simeq 2 \times 10^6$ barns, which is a *very* large cross section in nuclear physics), as they remove neutrons from the reactor core.

1˙5. *Neutron density and flux*. – We define as neutron density the expected number of neutrons with energy between E and $E + \mathrm{d}E$, in the volume $d^3\vec{r}$ about $\vec{r}$, at a time t at a certain position and at a certain time in a nuclear assembly. This quantity is indicated by $n(\vec{r}, E, t)$ (where $\vec{r}$ is a vector indicating a precise position in space). The neutron density can be measured for instance in neutrons per cm^3 (or in cm^{-3}). The reaction density $R(\vec{r}, E, t)$ is defined as the number of reactions in the volume $\mathrm{d}^3\vec{r}$ about $\vec{r}$, at a time t, initiated by neutrons with energy between E and $E + \mathrm{d}E$ and is given by the

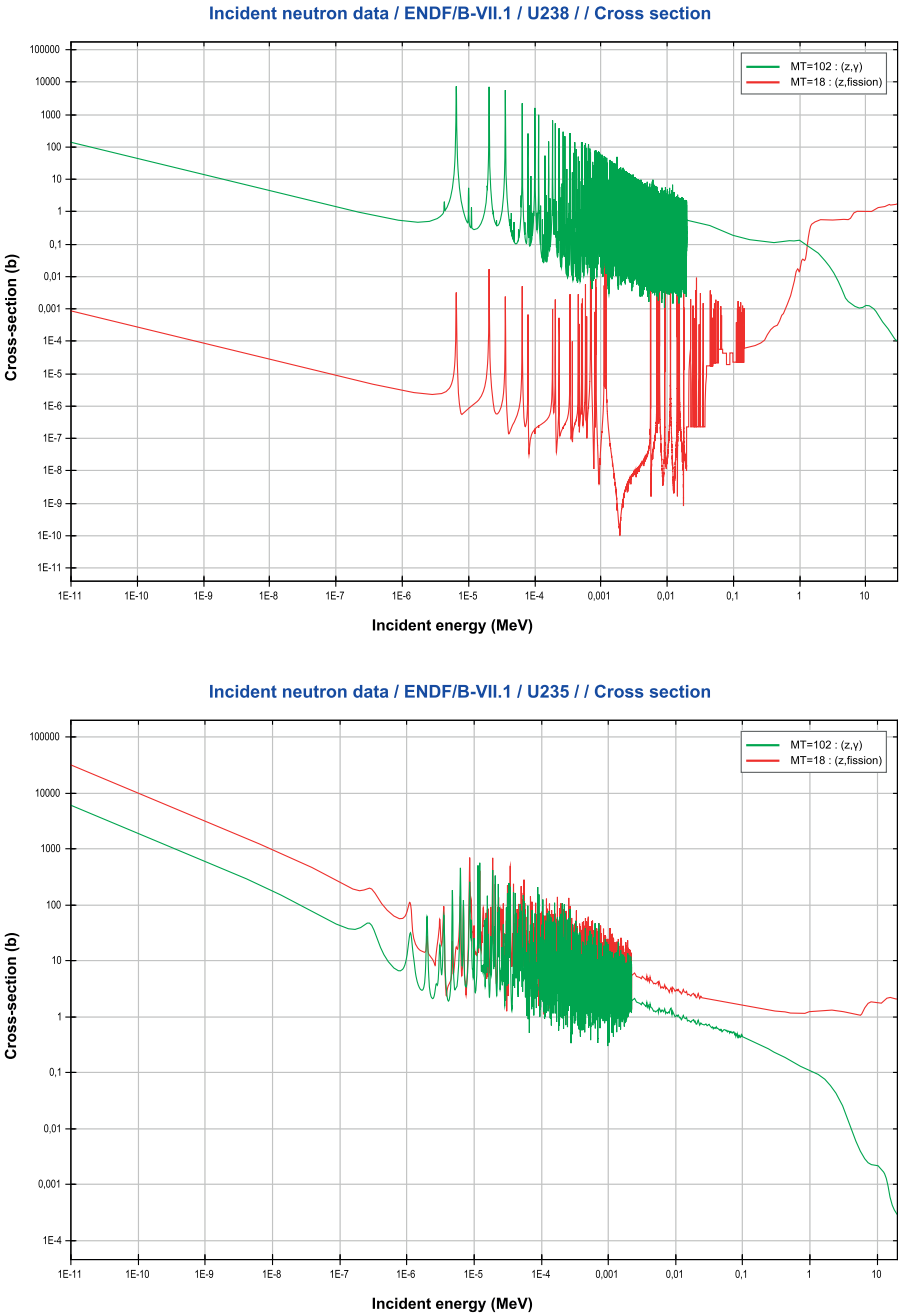

Fig. 3. – Fission (red) and radiative capture (green) cross sections for the two isotopes of uranium.

equation

$$R(\vec{r}, E, t) = n(\vec{r}, E, t)\, \Sigma\, v, \tag{12}$$

where again v is the neutron speed. The reaction density is measured in reactions per cm^3 per second. We give a special name to the quantity $n(\vec{r}, E, t)\ v$: it is called the neutron "flux" $\phi(\vec{r}, E, t)$ and is measured in neutrons per cm^2 per second (this differs from other definitions of flux in physics but we will not discuss these differences here). Therefore, given a neutron flux ϕ and a macroscopic cross section Σ, the reaction density, number of reactions per unit volume, R is simply given by $\Sigma\ \phi$. As an immediate example, consider a thermal reactor (see later) with 3 GW thermal power $= 3 \cdot 10^9$ Joule/s (it is indicated by the symbol 3 GWth and due to the efficiency of the conversion to electric power, this corresponds to 1 GW electric power, indicated by 1 GWe). Assume each fission releases order of 180 MeV energy $= 3 \cdot 10^{-11}$ Joule at the moment of fission. Then it follows immediately that in the whole reactor the fission rate is about 10^{20} fissions/s, which corresponds to 2–$3 \cdot 10^{20}$ neutrons/s emitted and about $6 \cdot 10^{20}$ neutrinos/s from fragments' β decay. By using the known value of the fission cross section for thermal (very slow) neutrons on ^{235}U, one can see that the flux inside the reactor must be in the order of $\phi \simeq 10^{13}$–10^{14} neutrons $\mathrm{cm}^{-2}\,\mathrm{s}^{-1}$.

1˙6. *Fuel*. – Heavy nuclei with a high fission cross section at low (thermal) neutron energies are called *fissile* (*e.g.* ^{233}U, ^{235}U, ^{239}Pu). Those with a sizeable fission cross section only at higher neutron energies (typically above 1 MeV) are called *fissionable* (*e.g.* ^{238}U, see fig. 3). Those that can produce a fissile isotope via neutron radiative capture and subsequent β decay are called *fertile*, meaning that they can be used to produce fuel (essentially ^{238}U, ^{232}Th). As an example, ^{238}U can produce a fissile nucleus via the chain of reactions

$$\mathrm{n} + {}^{238}\mathrm{U} \to {}^{239}\mathrm{U} + \gamma \to {}^{239}\mathrm{Np} + \mathrm{e}^- + \bar{\nu} \to {}^{239}\mathrm{Pu} + \mathrm{e}^- + \bar{\nu}. \tag{13}$$

Natural uranium is composed by 0.7% ^{235}U and 99.3% ^{238}U. Given the cross sections shown in fig. 3, it is easy to understand that in a reactor where neutrons are "slow", *e.g.* have thermal energies < 1 eV, ^{235}U is the uranium isotope that actually plays the role of fuel by contributing very significantly to the fission rate.

Let us go back to our ideal 1 GWe reactor burning only ^{235}U. We already calculated that about 10^{20} fissions/s occur in the reactor, which means that 10^{20} (^{235}U nuclei)/s disappear (actually a bit more because of radiative capture). Therefore roughly 40 mg/s of ^{235}U are "burnt" in the reactor. For 1 year of operation at 80% load factor (the actual working time), this means a consumption of about 1 ton of ^{235}U: in volume of pure metallic ^{235}U, this would be a cube of about 36 cm side. For comparison, the same amount of electric power is obtained by burning 1.6 Mtoe, million tons of oil equivalent (considering a slightly higher conversion efficiency of oil-fired power plants), or about 2 billion cubic meters of natural gas, or 2.2 million tons of coal. In a real fission reactor

^{235}U consumption is partly compensated by Plutonium (^{239}Pu) breeding according to reaction (13), as ^{239}Pu is also a fissile nucleus. In practice, the initial load of a typical 1 GWe PWR (see later) comprises about 27 tons of enriched uranium, of which about 950 kg are ^{235}U. After 1 year 280 kg of ^{235}U remain: about 560 kg of ^{235}U and 380 kg of other isotopes, mostly ^{239}Pu have been burnt, which is approximately the 1 ton we have been estimating above.

1˙7. *Fast and slow neutrons.* – We have seen that the cross sections for nuclear reactions induced by neutrons depend a lot on the energies involved, therefore it is important to introduce some classifications of the neutrons according to their energies. It is customary to adopt the following classification: *slow* neutrons are those with kinetic energy $T_n < 1\,\text{eV}$, in particular thermal neutrons have T_n around 0.025 eV or 25 meV (the value of kT, where k is the Boltzmann constant and T is the temperature of the system); *epithermal* neutrons are those with $1\,\text{eV} < T_n < 100\,\text{keV}$ (0.1 MeV); finally, fast neutrons have $0.1\,\text{MeV} < T_n < 20\,\text{MeV}$. Obviously neutrons in general can have energies above 20 MeV but this is an extreme limit in reactor physics (*e.g.*, neutrons from fusion of a deuteron and a tritium nucleus have 14 MeV fixed energy).

It is easy to show in non-relativistic kinematics that after a scattering off a nucleus with mass number A, *on average* the kinetic energy of the neutron changes according to the ratio

$$\frac{T'_n}{T_n} \simeq \frac{1+A^2}{(1+A)^2} \tag{14}$$

(where T_n and T'_n are the kinetic energies before and after the scattering, respectively). For a heavy nucleus $A \gg 1$, one has that $T'_n \simeq T_n$ or in other words, the neutron has to undergo many collisions in order to significantly lose energy. Consider instead the case $A = 1$ (this is relevant for any medium containing hydrogen, where protons, hydrogen nuclei, will be among the targets). In this case $T'_n = \frac{T_n}{2}$, *i.e.* on average a neutron will lose half of its energy at each collision and therefore few collisions are sufficient to rapidly decrease its energy. For collisions on a light nucleus like carbon ($A = 12$), the situation will be similar to that of hydrogen, although the average loss of energy per collision will be lower. Light materials containing hydrogen (water, paraffin, plastic materials in general etc.) or containing light nuclei (graphite, etc.) are called *moderators*, for their effectiveness in slowing down neutrons. In a thermal reactor, the fuel is immersed in water, which is used both as coolant and as moderator: the fast neutrons from fission will be slowed down very quickly via collisions with protons and oxygen nuclei in water and therefore they will have a very high fission cross section for ^{235}U (see fig. 3).

2. – Chain reaction and simple reactor kinetics

For the reactor to work safely and effectively based on the chain reaction where neutrons emitted upon a fission can produce further fissions, the chain reaction must not diverge (more and more fissions at each step, or "generation") and must not die away

(less and less fissions at each generation). This means that precisely one neutron from each fission, out of the average 2–3 emitted, has to induce another fission event, not more, not less. The remaining fission neutrons will then either be absorbed by radiative capture or will leak out from the system. The character of the chain reaction depends essentially on the reactor composition and on its geometry and is defined through the quantity called k, also called multiplication coefficient

$$k = \frac{\text{number of neutrons in one generation}}{\text{number of neutrons in the preceding generation}}. \tag{15}$$

The condition $k = 1$ corresponds to a *critical* reactor, *i.e.* a reactor actually working and producing thermal power from fission. For $k > 1$ we have a supercritical reactor (fission reactions tend to diverge) and for $k < 1$ we have a subcritical reactor (fission reactions tend to die away). We can write equations for a "simple minded" reaction *kinetics*, meaning by kinetics the evolution of the reactor parameters like neutron flux, etc. with time. If we indicate by $n(t)$ the neutron population at some specific time t, clearly n will vary according to

$$\frac{\mathrm{d}n(t)}{\mathrm{d}t} = P(t) - L(t), \tag{16}$$

where $P(t)$ is the neutron production at time t (mainly due to neutrons emitted as fission products), $L(t)$ is the neutron loss, due to fission, capture and leakage at time t. It is easy to see that

$$k = \frac{P(t)}{L(t)} \tag{17}$$

and that the quantity

$$\tau = \frac{n(t)}{L(t)} \tag{18}$$

represents the average lifetime of a neutron in the system before it disappears due to fission, capture and leakage. Then

$$\frac{\mathrm{d}n(t)}{\mathrm{d}t} = \frac{k-1}{\tau} n(t) \tag{19}$$

and assuming that k and τ are time-independent (not true in general, but can be a reasonable approximation for short enough time spans), we can solve the above equation and find

$$n(t) = n(0) \exp\left(\frac{k-1}{\tau}\, t\right). \tag{20}$$

Here we clearly see that $k = 1$ corresponds to a steady state, while $k > 1$ implies that the system is diverging and $k < 1$ corresponds to a decreasing population. Based on this simple kinetics, we see that the time constant governing the response of the reactor would be $\frac{\tau}{k-1}$. Since the typical neutron lifetime in a thermal power reactor is of the order of 10^{-4} sec, we see that even for $k = 1.001$, the neutron population and therefore the reactor power would increase by a factor 2.7 in 0.1 s, which would not be a manageable situation from a practical point of view. But there is one missing piece in the picture of fission discussed above. Actually, there is a small fraction (less than a percent) of neutrons originating from the fission process that are not emitted immediately, but some time later. These neutrons are emitted by highly excited fission fragments on a time scale from milliseconds to seconds and for this reason are called *delayed neutrons*. In simple terms, a nuclear reactor is made critical thanks to such small fraction of delayed neutrons and therefore they are the ones that dominate the reactor response time making it much longer, thereby allowing reactor control by neutron-absorbing control rods.

2˙1. *A bit more on neutron multiplication and transport.* – Our simple picture of the chain reaction above did not take into account also many other effects occurring in neutron transport. First of all, thinking about what can happen to the 2–3 neutrons emitted in the next generation after a fission, one can come up with a better definition of the multiplication coefficient k, by writing

$$k = \eta f P_{\mathrm{NL}}, \tag{21}$$

where f is the conditional probability that, if neutron will be absorbed, it will be absorbed in the fuel instead of structural elements; η is the average number of neutrons produced per neutron absorbed in the fuel, in other words the average number of neutrons per fission ν multiplied by the probability that the absorption process leads to fission instead of radiative capture $\eta = \nu \frac{\sigma_f^F}{\sigma_a^F}$, where σ_a^F is the absorption cross section in the fuel (absorption=fission+radiative capture) and σ_f^F is the fission cross section in the fuel; finally, P_{NL} is the probability that the neutron does not escape from the reactor (No-Leakage). For an infinite reactor clearly $P_{\mathrm{NL}} = 1$ and we get $k_\infty = \eta f$. If we consider that fissions can occur both from slow and fast neutrons, there is a correction factor $\epsilon > 1$ which simply tells that fissions can be more than just those produced by slow neutrons (as we saw before, in a thermal reactor the latter will be the vast majority). If we also take into account that neutron moderation to small speeds implies a number of collisions, we see that the neutron may not survive such collisions and be absorbed, so we will add a factor p representing the fraction of fission neutrons that survive moderation without being absorbed. For an infinite reactor where $P_{\mathrm{NL}} = 1$, we arrive at the *4 factor formula*

$$k_\infty = p\epsilon\eta f. \tag{22}$$

If we instead consider a finite reactor and for the sake of precision we consider the probability of no-leakage for thermal (slow) and fast neutrons separately, *i.e.* if we write

$P_{\rm NL} = P_{\rm TNL}P_{\rm FNL}$ ("T" for thermal and "F" for fast), we arrive at a formula for the *effective* multiplication coefficient, the *6 factor formula*:

$$k_{\rm eff} = p\epsilon\eta f P_{\rm TNL}P_{\rm FNL}. \tag{23}$$

The name "effective" means that we are not considering an infinite, homogeneous medium but a more realistic situation where obviously $k_{\rm eff} < k_\infty$, *i.e.* in a real reactor one has to consider that the unavoidable leakage due to neutrons escaping from the reactor will make the multiplication in the chain reaction smaller than in an ideal, infinite medium. In practice, to make $P_{\rm TNL}$ and $P_{\rm FNL}$ as close as possible to one and increase $k_{\rm eff}$, a *reflector* is put around the reactore core containing the fuel, *i.e.* a thick blanket of material (graphite, Cu, Pb depending on the reactor type) with the purpose of scattering back (reflecting) the neutrons into the core.

2˙2. *Subcritical assemblies.* – From eq. (15), we see that the multiplication coefficient k must be equal to 1 to attain criticality, *i.e.* to make the reactor work and produce power by a self-sustained chain reaction. What happens if k is less than unity? Clearly, if you start from criticality, then reduce the multiplication by for instance inserting control rods significantly, the chain reaction will die away exponentially fast, as indicated by eq. (20). However, the chain reaction can still be kept going if an *external* neutron source is active within the system. It is possible to solve the appropriate equations with such an external source and find a stationary state where the neutron flux (and therefore the system's power) are constant. If we indicate the external source as S_0 (neutrons/cm^3/s supplied by the external source to the reactor core), for an infinite, homogeneous reactor the solution is

$$\phi = {\rm const} = \frac{S_0}{k_\infty - 1}\Sigma_a, \tag{24}$$

where Σ_a is the macroscopic absorption cross section. We see that the neutron flux and therefore the power is proportional to the intensity of the source and is obviously increasing when the multiplication in the chain reaction k_∞ approaches unity. An external neutron source can be obtained by accelerating electrons or ions (protons, deuterons) in a suitable accelerator, then smashing the accelerated particles onto an appropriate, totally absorbing target. When using high energy protons with kinetic energies in the order of hundreds of MeV to GeV on a heavy nuclear target based for instance on mercury, tungsten, lead, several neutrons per incoming proton are obtained in the nuclear process called *spallation*. At lower energies, other types of targets can be used but spallation by high-energy ions is the most effective process for producing neutrons. A subcritical reactor core will therefore be comprising a suitable accelerator with a beam penetrating into the reactor core and stopped by a target placed in the middle of the core. This is called an *Accelerator-Driven System*. The interest in this type of reactor will be briefly discussed later.

3. – Reactor types

We saw that neutrons slow down when undergoing collisions with nuclei (in particular with light nuclei), therefore their energies in the core can go from about 10 MeV (usually the maximum energy of fission neutrons), down to as low as 10^{-3} eV. We also saw that neutron cross sections have a strong dependence on neutron energy, in particular absorption cross sections such as capture or fission decrease very strongly with energy for particular species like ^{235}U. When using fuels containing these particular species, it is easiest to maintain a fission chain reaction using slow neutrons. Hence most nuclear reactors until now (the so-called Generation I to III+) use low-mass-number materials such as water or graphite to slow down or moderate the fast fission neutrons, thereby having the neutrons slow down to energies comparable to the thermal energies of the nuclei in the reactor core. Therefore, by definition a *thermal reactor* is one where the average neutron energy is comparable to thermal energies (order of 25 meV). Because of the very high fission cross section at thermal energies, this type of reactor requires the minimum amount of fissile material for fueling. As an example, a Light Water Reactor (LWR) can start with 3% ^{235}U + 97% ^{238}U (a mixture enriched with respect to natural uranium). Burn-up (consumption) of ^{235}U is partly compensated by breeding of ^{239}Pu via neutron radiative capture and subsequent β decays and after 1 year operation, the core may contain something like 1% ^{235}U + 1% ^{239}Pu. However, it turns out that the number of neutrons emitted per neutron absorbed in the fuel is largest for fast neutrons, therefore if one could make a reactor work with fast neutrons, the "extra" neutrons may be used to convert or breed new fuel. On the other hand, σ_f is smaller for fast neutrons, which means that much more fuel is needed to sustain the chain reaction. Based on the kinematic considerations seen above, to keep the neutron energy high, the core should contain mostly high mass-number materials (including the necessary coolant for which a liquid metal should be chosen like sodium, lead or a lead-bismuth mixture, or alternatively the coolant should be a low-density material like Helium gas). In such a *fast reactor* the average neutron energies in the core are typically above 100 keV. Later on we will consider some possible advantages of fast reactors over thermal reactors. In summary, the typical scheme of a thermal and a fast reactor are the ones illustrated in fig. 4.

3˙1. *Decay heat*. – Decay heat is the heat produced in the reactor core as a result of radioactive decay: the energy of the alpha, beta or gamma radiation is converted into atomic motion, but heat production comes mostly from β and γ decay of fission products. Decay of the short-lived radioisotopes created by fission continues for some time after shut down. A practical approximation is given by the formula

$$\frac{P}{P_0} = 6.6 \cdot 10^{-2} \left[\frac{1}{(\tau - \tau_s)^{0.2}} - \frac{1}{\tau^{0.2}} \right], \tag{25}$$

where P is the decay power, P_0 is the reactor power before shutdown, τ is the time since reactor startup and τ_s is the time of reactor shutdown measured from the time of startup

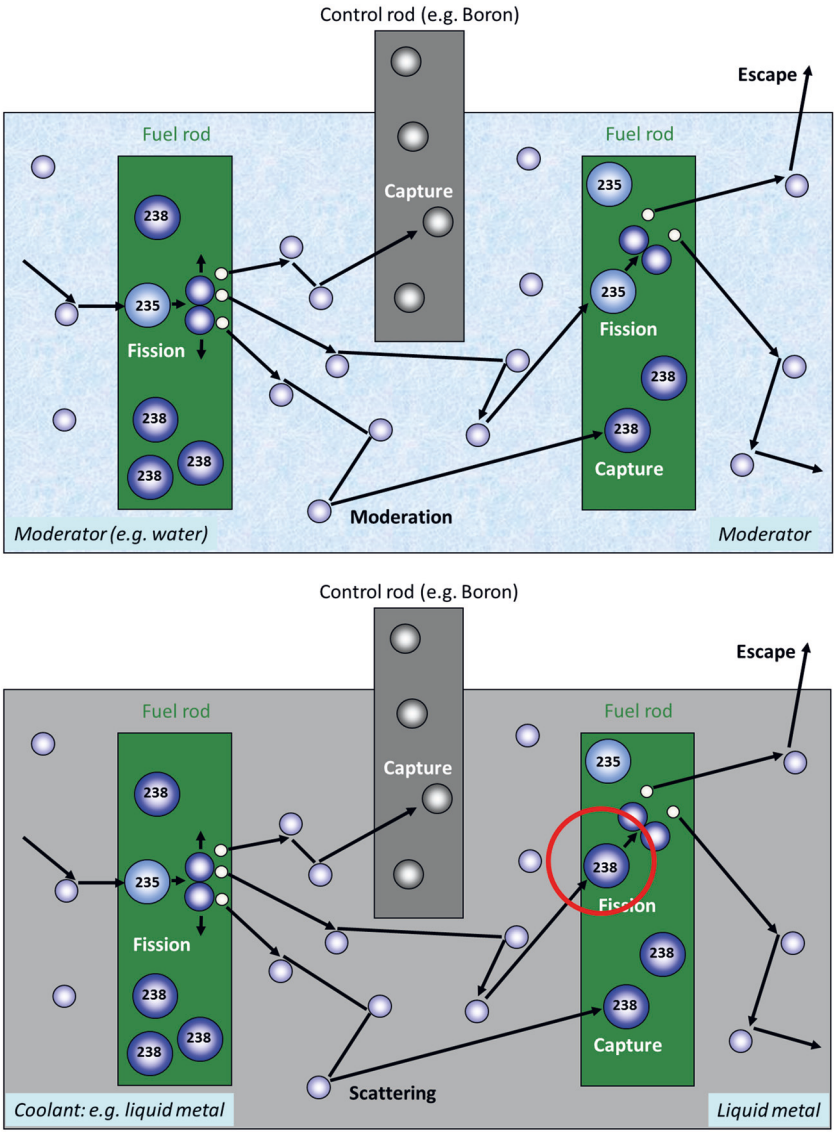

Fig. 4. – Conceptual schemes of thermal (top) and fast reactor (bottom).

(in seconds). At shutdown, the heat power is about 6.5% of the previous core power ($\simeq$ 200 MWth for a 1 GWe reactor), which is sufficient to melt the core. About 1 hour after shutdown, the decay heat will be about 1.5% of the previous core power, after 1 day it will fall to 0.4% and after a week it will be only 0.2%. For this reason, spent fuel rods are kept for long time in a special pool of water, before being further processed. Removal of decay heat is therefore a very important aspect in designing a reactor and implementing safety measures, as the Fukushima accident has dramatically stressed.

3˙2. *Nuclear reactor types*. – Most current reactors are based on fissile fuels, ^{235}U and ^{239}Pu, with fission induced by slow neutrons at thermal energies. Ordinary water serves as both coolant and moderating material in the reactor. There are two major types of Light Water Reactors (LWR): Pressurized Water Reactors (PWR) and Boiling Water Reactors (BWR). In a PWR, the primary coolant is water maintained under very high pressure so that the coolant reaches a high temperature without steam formation within the reactor. Heat transported out of the reactor core by the primary coolant is then transferred to a secondary loop connected to a steam generator. In a BWR, the primary coolant water is maintained at lower pressure, so that boiling and steam formation occur within the reactor core itself, which serves directly as the steam generator, while secondary loop and heat exchanger are not present. In both PWR and BWR, the nuclear reactor itself and the primary coolant are contained in a large steel pressure vessel designed to accomodate the high pressures and temperatures. The heavy water (D_2O) reactor takes advantage of the lower neutron capture capture cross section in deuteron with respect to hydrogen. This allows to have more neutrons around in the core and to use low-enrichment uranium fuels (including natural uranium). This concept has been developed for instance in Canada in the CANDU (CANadian Deuterium uranium) series of power reactors. This type of reactors is indicated by the acronym PHWR (Pressurized Heavy Water Reactor). Yet another design is based on gas as coolant. For instance the early MAGNOX reactors developed in the UK used low-pressure CO_2 as cooling gas. In general this reactor class is indicated by the acronym GCR (Gas Cooled Reactor). A more recent design is the High-Temperature Gas-cooled Reactor (HTGR, USA), that uses high-pressure helium as coolant. Gas-cooled reactors generally use graphite as moderator material to slow down neutrons to thermal energies.

4. – Fission energy in perspective

4˙1. *Worldwide figures*. – As illustrated in fig. 5, from the 1950's, when the first commercial reactors started to appear, until 1985 the number of operating reactors as well as the nuclear net generating capacity (*i.e.* the available power) has been increasing about linearly with time, then the number of reactor has remained approximately stable, while the generating capacity has been still increasing somewhat due to the improved efficiency of existing plant and some replacement of older installations [5]. At the end of 2013, there were 434 reactors in operation in the world, for a total net nuclear generating capacity of about 372 GWe, and 72 under construction, for a total capacity of about 69.4 GWe [6]. In fig. 6, the top and bottom panels show the distribution of reactor types in operation and under construction in the world, respectively. Figure 7 shows the share of nuclear power in electricity production at the end of 2013 [6] country by country, while fig. 8 shows the share of nuclear power in the world electricity generation [7].

4˙2. *Nuclear fuel cycle, natural resources, cost of electricity and carbon emission scenarios*. – How much uranium is consumed by the nuclear reactor fleet worldwide? And how long will uranium resources last? This is clearly an important question for project-

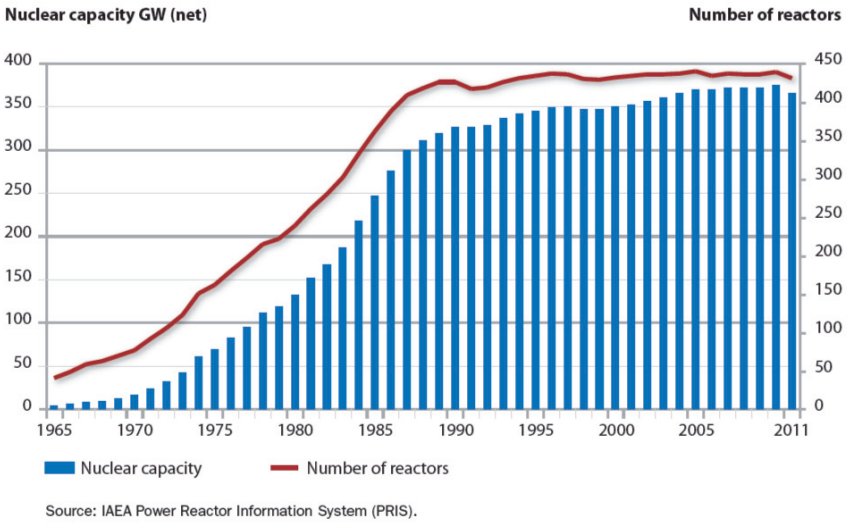

Fig. 5. – Worldwide nuclear generating capacity and number of operating reactors (1965-2011) [5]. © OECD 2012.

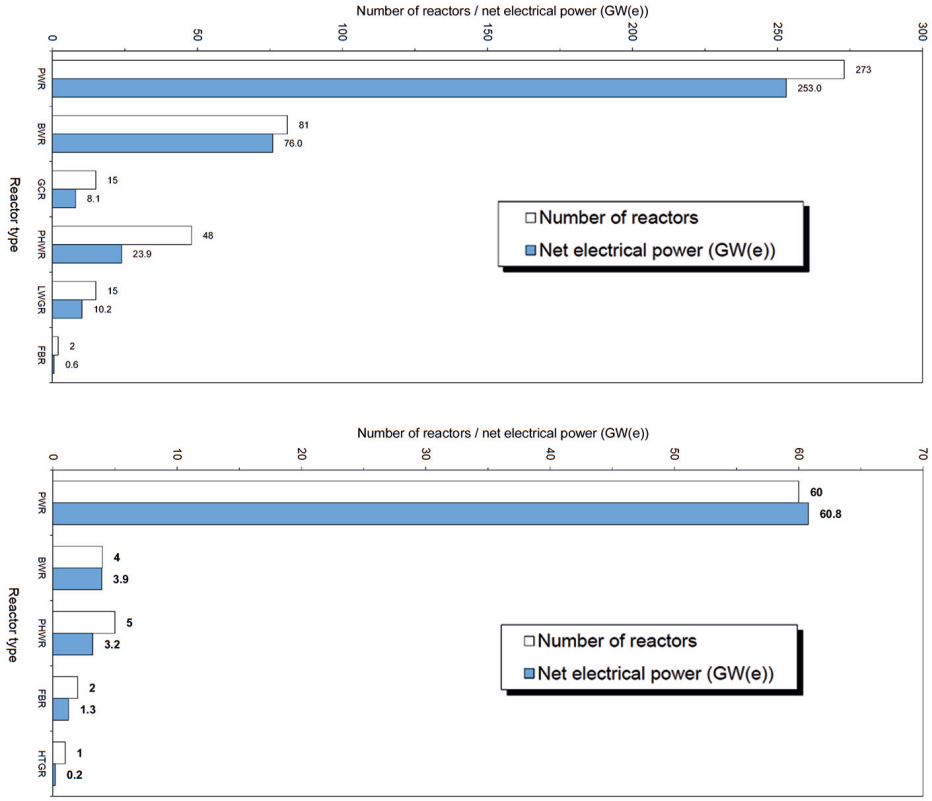

Fig. 6. – Number of operational reactors by type and net electrical power (Top, as of 31 Dec. 2013). Reactors under construction by type and net electrical power (Bottom, as of 31 Dec. 2013) [6]. © IAEA 2014.

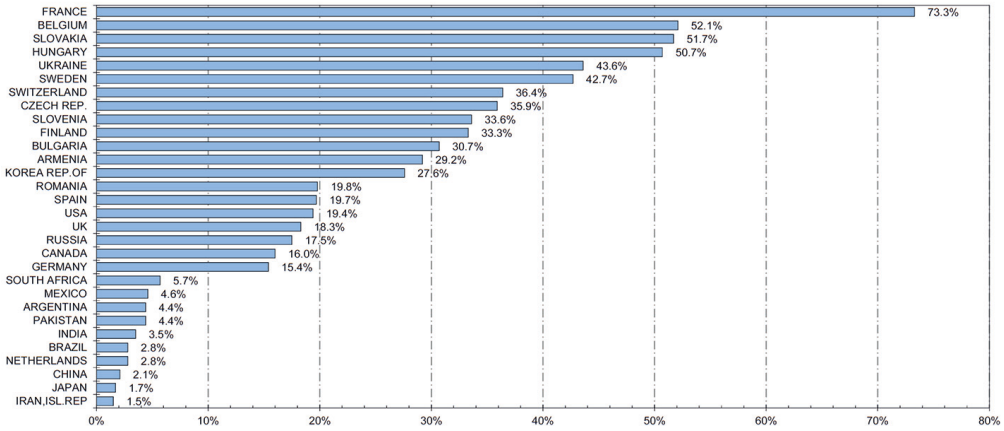

Fig. 7. – Nuclear share of electricity generation in each country (as of 31 Dec. 2013). Note: The nuclear share of electricity supplied in Taiwan, China was 19.1% of the total [6]. © IAEA 2014.

ing the use of nuclear power from fission to the future. The whole series of activities starting from uranium mining down to disposal of radioactive waste is called the *nuclear fuel cycle*. In the case of the so-called *open fuel cycle* the spent fuel is extracted from the reactor, then cooled in specific pools, then transferred to a storage infrastructure. In the case of the so-called *closed fuel cycle*, the spent fuel undergoes a reprocessing procedure, where plutonium is extracted from the fuel rods and used to produce new fuel containing both uranium and plutonium. On 1 January 2013, a total of 437 commercial nuclear

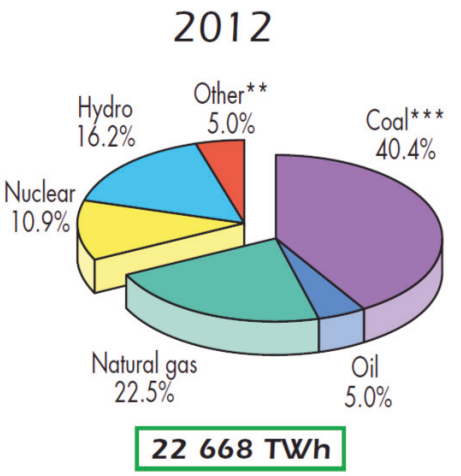

Fig. 8. – World electricity generation (2012) [7]. © OECD 2014.

reactors were connected to the grid in 30 countries. The global reactor fleet generated a total of about 2 323 TWh (Terawatthours) of electricity in 2012. World annual uranium requirements amounted to 61 600 tU (tons uranium) in 2012 [8]. The so-called identified resources appear to be sufficient for over 120 years, based on the 2012 uranium consumption [8]. Consideration of the entire amount of conventional resources would increase the time scale to well over 300 years, but this would require significant efforts in securing all resources for an effective use. Moving to advanced-technology reactors and recycling fuel could increase the long-term availability of nuclear energy from hundreds to thousands of years. In addition, thorium, which is more abundant than uranium in the Earth's crust, is another potential source of alternative fuel [9].

When considering the so-called Levelized Cost Of Electricity, levelized over the estimated plant lifetime, for various world areas, assuming specific discount rates and cost of carbon emissions, especially for the low discount rate scenario (5%) the cost of electricity from nuclear power appears to be still competitive with other sources [5].

As far as carbon and other greenhouse gas emissions are concerned, for nuclear energy from fission an accurate estimate of the CO_2 equivalent emissions should include all activities in the whole plant lifetime, from fuel mining/extraction to the final decommissioning of the plant. Such a detailed study has been performed for instance in [10], showing that corresponding greenhouse gas emissions can be estimated as about 66 gCO_2equiv/kWh, while a 2.5 MW offshore wind farm sets at 9, a 3.1 MW hydroelectric dam scores 10 as does a 1.5 MW onshore wind farm, solar photovoltaic based on polycrystalline silicone goes up to 32, while fossile fuels are all above 400. Considering the capability to deploy a large quantity of electric power per plant (typically a true power of around 1 GWe with high availability) and the relatively low associate emission level per kWh, it is clear why nuclear energy is considered one of the players in meeting greenhouse gas reduction targets. Indeed in the World Energy Outlook 2012 so-called New Policies Scenario [11], nuclear power production is expected to grow from the 2 323 TWh of 2012 to 3 443 TWh in 2020 and 4 366 TWh in 2035. In the alternative and aggressive "450" Scenario (whose name comes from the limit of 450 ppm atmospheric level of CO_2 equivalent), among many strong measures a rapid growth in nuclear power is assumed and the expected growth of nuclear power sees 3 601 TWh in 2020 and 5 968 TWh in 2035.

4˙3. *Radioactive waste.* – Among many important safety aspects of fission reactors, a particularly relevant one is the production of radioactive waste. There are two mechanisms for the production of radioactive species in addition to those already present in the fuel (*e.g.* the two uranium isotopes are radioactive in the first place): the first mechanism is fission itself, which is producing many radioactive fission products (or fragments), *i.e.* unstable nuclei that decay by β decay at a later time, with the β decay being followed very often by emission of gamma rays. Radioactive decay follows the exponential law

$$N(t) = N(0) \exp\left(-\frac{t}{\tau}\right), \tag{26}$$

where $N(0)$ is the initial number of radioactive nuclei of a specific isotopic species, $N(t)$ is the average number of surviving nuclei after a time t and τ is the lifetime of the species. A radioactive isotope *half-life* is the time at which on average 50% of the nuclei have survived. The half-lives of fission products for β decay is enormously variable, ranging from fractions of a second to million years. Fission products are continuously produced in the fuel material inside a reactor and can be found in the spent fuel once it is removed from the reactor at the end of a power production cycle. Fission products with a half-life until several days are responsible for the large amount of decay heat that is produced within the core immediately after reactor shutdown, *i.e.* after stopping the chain reaction. However, their radioactivity dies away in a few days or weeks, which means that in case of accidental release of radioactive products they pose a threat only for such a limited amount of time and that when the spent fuel is stored in the cooling pools they do not produce anymore heat after a short amount of time. Other Short-Lived Fission Products with half-lives until a few years can instead be a safety concern also after an accordingly longer period of time, as are Medium-Lived Fission Products with half-lives until about 100 years (*e.g.* ^{137}Cs and ^{90}Sr, with half-life of about 30 years). Such isotopes contribute to heat production in the spent fuel and can be a safety concern if accidentally released into the environment, for decades. Finally, Long-Lived Fission Products (LLFP) have half-lives over about 200 000 years (it happens that no fission product has half-life between about a 100 years and 200 000 years). There is a second mechanism producing radioactive isotopes. We saw that in the radiative capture process a nucleus can capture a neutron thereby emitting one or more gamma rays. When this process occurs in particular on ^{238}U, it not only leads to the production of fissile plutonium ^{239}Pu, but also to the production of a series of nuclides including other plutonium isotopes (like, *e.g.*, ^{240}Pu), americium, curium, etc., with increasing atomic number $Z > 92$. Capture on ^{235}U leads also to formation of ^{237}Np. Neptunium, americium, curium and other similar isotopes are called *Minor Actinides, MA*. Many of these MA decay predominantly by α decay and have half-lives ranging from a few hundreds of years up to above 2 million years (^{237}Np).

The typical inventory from a 1 GWe PWR fuelled with about 27 tons of U (3.5% ^{235}U) at discharge (when the spent fuel is removed from the core) is approximately 280 kg of ^{235}U, 266 kg of Pu of which 56 kg of fissile ^{239}Pu, 20 kg of MA, 13 kg of ^{90}Sr and 30 kg of ^{137}Cs (both with 30 years half-life), and 63 kg of LLFP. The very long decay time of both LLFP and MA is the reason why disposal of spent fuel and similar waste requires very long-term storage, for which one possible solution envisaged is storage deep underground in the so-called geological repositories. Some countries adopt the so-called reprocessing of the spent fuel, where the plutonium is extracted from the fuel rods and used to produce new fuel containing both uranium and plutonium.

4˙4. *Fast reactors, subcritical reactors and fuel breeding.* – In the effort to improve the safety, security and efficiency of nuclear plants, new concepts of reactors have been developed with goals including the minimization of the production of MAs, a better and more efficient use of the fuel, a better thermodynamic efficiency, the possible production of hydrogen at high temperatures and finally improved safety features to minimize the risk

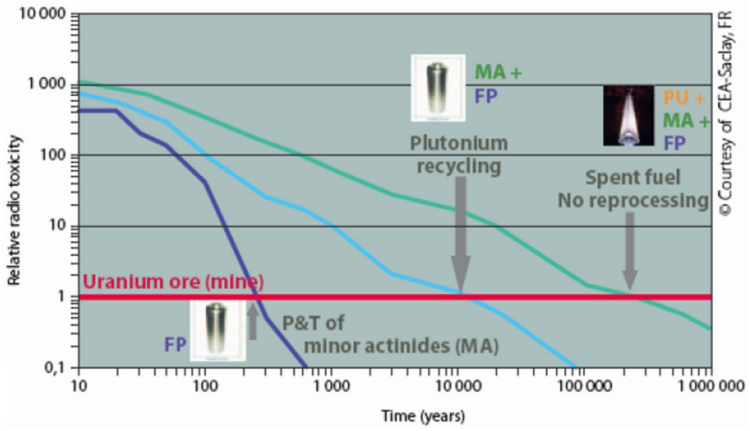

Fig. 9. – Radiotoxicity originating from used fuel in the different fuel cycle scenarios. © Courtesy of CEA-Saclay, France.

of accidents. All these various concepts are considered within the so-called Generation-IV reactors, which are the subject of an international initiative [12]. As an example, we saw that in a fast reactor, by using *e.g.* a liquid metal or gas as coolant, the energy spectrum of the neutrons is harder. In such a reactor core, it becomes possible to burn not only fissile elements like ^{235}U and ^{239}Pu, but also fissionable elements like ^{238}U for which fission occurs significantly only above a certain energy threshold around 0.5 MeV, as can be seen in fig. 3. Although given the much lower fission cross section typically the fuel has to be richer in fissile content, this means that also ^{238}U can be to some extent considered to be a fuel as well, which has obvious implications in terms of how long uranium resources will last. Another important aspect in fast neutron systems is that, for energies, say, above 0.5 MeV, fission becomes dominant over capture, as again seen in fig. 3. This means that the MA production, which occurs namely via capture, is relatively suppressed. Yet another important fact is that among the MA, some are fissionable, *i.e.* they can be burnt by fission for neutron energies above about 0.5 MeV. This means that in a reactor core with a hard neutron energy spectrum, not only less MA are produced but those produced can be partly destroyed in the reactor itself. This is why worldwide, currently especially in Europe and Asia, a lot of design studies are carried on whose goal is to determine realistic fast reactor configurations. An important physics aspect when considering fission of MA is that the amount of delayed neutrons produced upon their fission is significantly less than for ^{235}U and ^{239}Pu. This means that for instance fueling a reactor purely with MA is not possible, as one would lose the crucial safety feature of the reactor response times being dominated by the relatively long delayed neutron emission times. However, for Accelerator Driven Systems based on a subcritical reactor core that requires an external neutron source, the role of delayed neutrons is not so crucial and therefore there is the possibility to mix a larger quantity of MA into the fuel, thereby offering the possibility to use the apparatus to perform

transmutation of the MA into fission products and reduce the radiotoxicity of the final materials in the fuel. Figure 9 shows the radiotoxicity of the material originated from the spent fuel, *i.e.* its potential danger to human health, estimated according to three scenarios of fuel cycle. In the open cycle without reprocessing, the radiotoxicity of the spent fuel reaches the level of natural uranium ore in hundreds of thousands of years. In the closed fuel cycle where reprocessing is used to extract the Pu and form new fuel, that level is reached in about 10 000 years and finally, if the MAs are separated and burnt in appropriate assemblies (P&T, partioning and transmutation), the radiotoxicity could be reduced to the ore level in a few hundred years, a time span that can be conceivably managed by a developed society.

While nuclear fusion is continuining its path towards actual energy production with the ITER and DEMO projects [3], deployment of new-generation fission reactor prototypes is expected between 2020 and 2025, which will represent an important step in the future development of this form of energy.

REFERENCES

[1] Duderstadt J. J. and Hamilton L. J., *Nuclear Reactor Analysis* (John Wiley & Sons, Inc.) 1976.

[2] Nifenecker H., Meplan O. and David S., *Accelerator Driven Subcritical Reactors* (Institute of Physics Publishing, Bristol and Philadelphia) 2003.

[3] Ongena J., contribution to the present volume.

[4] *Java-based Nuclear Data Information System*, http://www.oecd-nea.org/janis/.

[5] *Nuclear Energy Today*, Second Edition, NEA, No. 6885, (OECD) 2012, p. 9, http://www.oecd-nea.org/pub/nuclearenergytoday/6885-nuclear-energy-today.pdf.

[6] *Nuclear Power Reactors in the World*, 2014 Edition, (IAEA), 2014, p. 76, http://www-pub.iaea.org/MTCD/Publications/PDF/rds-2-34_web.pdf.

[7] *OECD/IEA 2014 Key World Energy Statistics*, (OECD/IEA) 2014, p. 24, http://www.iea.org/publications/freepublications/publication/KeyWorld2014.pdf, Terms and Conditions in http://www.iea.org/t&c/termsandconditions/.

[8] *Uranium 2014: Resources, Production and Demand*, NEA, No. 7209, (OECD) 2014, http://www.oecd-nea.org/ndd/pubs/2014/7209-uranium-2014.pdf.

[9] Revol J. P., contribution to the present volume.

[10] Sovacool B. K., *Energy Policy*, **36** (2008) 2940.

[11] *World Energy Outlook 2012*, (OECD/IEA) 2012, http://www.iea.org/publications/freepublications/publication/WEO2012_free.pdf, Terms and Conditions in http://www.iea.org/t&c/termsandconditions/.

[12] *Generation IV International Forum*, https://www.gen-4.org/.