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Rotational character of the ^{12}C spectrum investigated through inelastic cross sections via photon emission

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Abstract. In this work the bremsstrahlung and photon dissociation cross sections for transitions between 0^+, 2^+, and 4^+ states in ^{12}C are computed. The nucleus is described within the three-alpha model, and the wave functions are computed by means of the hyperspherical adiabatic expansion method. The continuum states are discretized by imposing a box boundary condition. The transition strengths are obtained from the cross sections, and compared to schematic rotational model predictions. The computed results strongly suggest that the two lowest bands are made, respectively, by the states with angular momentum and parity $\{0^+_1, 2^+_1, 4^+_2\}$ and $\{0^+_2, 2^+_2, 4^+_1\}$. The transitions between the states in the first band are consistent with the rotational pattern corresponding to three alphas in an equilateral triangular structure. For the second band, the transitions are also consistent with a rotational pattern, but with the three alphas in an aligned distribution.

1 Introduction

The properties of the spectrum of ^{12}C have attracted a lot of attention along the years. In particular, the presence in the energy spectrum of at least two sequences of states with angular momentum and parity $\{0^+, 2^+, 4^+\}$ immediately suggests that these states could correspond to states in a rotational band. At least two rotational bands could exist in ^{12}C, one of them sitting on the ground state, and another one sitting on the Hoyle state.

The same rotational sequence has also been observed in ^{8}Be, where the ground state and the two first excited states follow as well the angular momentum and parity sequence 0^+, 2^+, and 4^+ (together with very broad 6^+ and 8^+ states, which are found numerically as poles of the $\mathcal{S}$-matrix). All the states in the ^{8}Be spectrum are unbound (resonances), with the ground state only about 0.1 MeV above the threshold for emission of two alpha particles. This spectrum was recently investigated in [1–3] by computation of the electric quadrupole cross sections for bremsstrahlung emission after transitions between the ^{8}Be-states, which were described as two-body systems made of two alpha particles. These calculations required a careful treatment of the continuum wave functions and clarification of the definition of the cross section for transitions between states with a non well-defined energy.

In this work we extend the investigations to the spectrum of ^{12}C, which is described as a three-alpha system. In particular, we shall investigate whether the behavior of the $\{0^+_1, 2^+_1, 4^+_1\}$ and $\{0^+_2, 2^+_2, 4^+_2\}$ sequences of states agree with the predictions of the rotational model. This will be done

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by means of the the electric quadrupole γ-emission bremsstrahlung cross sections for the different transitions between the states. These cross sections contain the information about the corresponding transition strengths, which in the case of transitions between states within a rotational band must be given by very simple expressions. In particular, these expressions contain the intrinsic quadrupole moment, which should be the same for all the states in the band. The consistency of the intrinsic quadrupole moment with specific (linear or triangular) structures of the three-alpha system are also investigated.

2 Method

Let us consider the three-body system made of particles a, b, and c, and the continuum to continuum γ-emission process $A^* \rightarrow A + \gamma$, where A^* and A represent three-body continuum states with energies E and E' ($E > E'$), respectively. The cross section for this reaction is given by

$$\frac{d\sigma^{(\lambda)}}{dE'}(E) = \nu! \frac{2(2J'+1)}{(2J_a+1)(2J_b+1)(2J_c+1)} \frac{32\pi}{\kappa^5} \left(\frac{E_\gamma}{\hbar c}\right)^2 \frac{d\sigma_\gamma^{(\lambda)}}{dE'}(E_\gamma), \tag{1}$$

where $E_\gamma = E - E'$ is the photon energy, J and J' are the initial and final angular momenta, ν is the number of identical particles in the three-body system, J_a, J_b, and J_c are the total angular momenta of particles a, b, and c, respectively, and κ is the three-body momentum, which is defined as $\kappa = \sqrt{2mE/\hbar^2}$. The mass m is the normalization mass used to define the Jacobi coordinates. Finally, the differential photo-absorption cross section is given by:

$$\frac{d\sigma_\gamma^{(\lambda)}}{dE'}(E) = \frac{(2\pi)^3(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda-1} \frac{d\mathcal{B}^{(\lambda)}}{dEdE'}(J' \rightarrow J). \tag{2}$$

In this work the continuum spectrum is discretized by imposing a box boundary condition. When done, the differential transition strength contained in the equation above can be written as:

$$\frac{d\mathcal{B}^{(\lambda)}}{dEdE'}(J' \rightarrow J) = \frac{1}{2J'+1} \sum_{i,j} \left|\langle \Phi_J^{(i)} \| \hat{O}_\lambda \| \Phi_{J'}^{(j)} \rangle\right|^2 \delta(E - E_i)\delta(E' - E'_j), \tag{3}$$

where the indeces i and j run over the initial and final discrete continuum states with energies E_i and E'_j, respectively. The corresponding continuum wave functions are denoted by $\Phi_J^{(i)}$ and $\Phi_{J'}^{(j)}$. The electromagnetic operator with multipolarity λ is denoted by $\hat{O}_\lambda$. It is important to note that the summation over i and j in the Eq.(3) is not unrestricted, but limited to the initial and final energy ranges of experimental interest [4].

2.1 Transition strength

As shown in Ref.[3], in the case of continuum to continuum transitions, a direct calculation of the transition strength from Eq.(3) is not possible. It is in fact necessary to extract it directly from the cross section, which, as evident from Eq.(2), contains all the information about the transition strength. In particular, in this work two different methods will be used.

In the first method the strength is obtained from the total (integrated) cross section. More precisely, for a transition between some initial and final energy ranges, typically around resonances in the initial and final states, integration of Eq.(1) over E and E' within those two windows will provide the total cross section for the transition. If the photon energy, E_γ, were constant, one could immediately get the

Table 1. Experimental and computed energies E_R (in MeV) and widths Γ_R (in keV) of the ^{12}C resonances for different J^π and the Buck α-α potential. For each of the calculations, the strength S (in MeV) used in the gaussian effective three-body potential is also given (the range is taken to be $b = 6.0$ fm). The energies are referred to the three-alpha threshold.

J^π	$E_{R,\mathrm{exp}}$ (MeV)	$\Gamma_{R,\mathrm{exp}}$ (keV)	$E_{R,\mathrm{Buck}}$ (MeV)	$\Gamma_{R,\mathrm{Buck}}$ (keV)	S
0^+_1	$-7.275^{(a)}$	–	-7.27	–	-22.0
0^+_2	$0.380^{(a)}$	$0.009 \pm 0.001^{(a)}$	0.38	$\lesssim 0.05$	-18.0
0^+_3	$4.20 \pm 0.14^{(b)}$	$3440 \pm 220^{(b)}$	4.12	700	-18.0
2^+_1	$-2.8356 \pm 0.0003^{(a)}$	0.0	-2.85	–	-12.1
2^+_2	$2.76 \pm 0.11^{(c)}$	$800 \pm 130^{(c)}$	1.72/2.35	161/920	$-12.1/-3.0$
	$2.86 \pm 0.05^{(d)}$	$2080 \pm 300^{(d)}$			
2^+_3	$3.88 \pm 0.05^{(a)}$	$430 \pm 80^{(a)}$	5.04/5.48	570/3600	$-12.1/-3.0$
4^+_1	$6.0 \pm 0.2^{(e)}$	$1700 \pm 200^{(e)}$	5.48	3800	13.4
4^+_2	$6.81 \pm 0.02^{(a)}$	$258 \pm 15^{(a)}$	6.81	800	13.4
4^+_3			13.1	2060	13.4

(*a*) Ref.[5]; (*b*) Ref.[6]; (*c*) Ref.[7]; (*d*) Ref.[8], (*e*) Ref.[9]

total transition strength simply after division of the total cross section by the constants that multiply the transition strength. However, since this is not correct, we must use an average value for $E_\gamma^{2\lambda+1}$. In particular, the photon energy will be taken as $E_\gamma = E_c - E'_c$ where E_c is the energy of the cross section peak corresponding to the resonance in the initial state and E'_c is the center of the final energy window (usually the resonance energy in the final state). Of course, for this procedure to be applied, some initial information about the resonance positions is needed. Furthermore, this procedure can be sensitive to rather small variations around a chosen E_γ due to the $(2\lambda + 1)$ exponent for the $\mathcal{B}(E\lambda)$ transitions, see Ref.[1] for details.

The second method exploits the fact that in the vicinity of a resonance the cross section of the photo-dissociation reaction $A + \gamma \to a + b + c$ takes the form

$$\sigma_\gamma(E_\gamma) = \frac{2J+1}{2(2J_A+1)} \frac{\pi\hbar^2c^2}{E_\gamma^2} \frac{\Gamma_R\Gamma_\gamma}{(E-E_R)^2 + \Gamma^2/4}, \tag{4}$$

where after the collision the particles a, b, and c are assumed to populate a resonance with angular momentum J, energy E_R, and width for decay into three particles Γ_R. J_A is the angular momentum of A and $\Gamma = \Gamma_R + \Gamma_\gamma$, where Γ_γ is the γ-decay width of the three-body resonance. In this method the value of Γ_γ in Eq.(4) is used as a parameter in order to fit the peak of the computed cross section corresponding to the three-body resonance at $E = E_R$. The Γ_γ value obtained in this way is then used to extract the transition strength by means of the well-known expression:

$$\Gamma_\gamma = \frac{8\pi(\lambda+1)}{\lambda[(2\lambda+1)!!]^2} \left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda+1} \mathcal{B}^{(E\lambda)}(J \to J_A). \tag{5}$$

In the following, the transition strengths computing using these two methods will be denoted by $\mathcal{B}_\sigma$ and $\mathcal{B}_\gamma$, respectively.

3 The three-alpha system

The resonances of the three-alpha system are obtained by use of the hyperspherical adiabatic expansion combined with the the complex scaling method. The three-body resonances appear then formally

Table 2. $\mathcal{B}(E2)$ transition strengths in $e^2\text{fm}^4$ for different transitions between ^{12}C states. For transitions from continuum states, $\mathcal{B}_\gamma$ and $\mathcal{B}_\sigma$ denote the two different methods used (see text). The second, third, and fourth columns give the known experimental values, and the results obtained with the microscopic cluster model calculation described in Ref.[10] and with the Antisymmetrized Molecular Dynamics (AMD) method.

Transition	Exp.	α-cluster	AMD	$\mathcal{B}(E2)$ ($e^2\text{fm}^4$)	
$2_1^+ \to 0_1^+$	$7.6 \pm 0.4^{(a)}$	$9.16^{(a)}$	$8.4^{(d)}$	9.9	
				$\mathcal{B}_\gamma$	$\mathcal{B}_\sigma$
$4_2^+ \to 2_1^+$			$15.8^{(d)}$	12.5	9.9
				$\mathcal{B}_\gamma$	$\mathcal{B}_\sigma$
$2_2^+ \to 0_2^+$			$102^{(d)}$	158	136
$4_1^+ \to 2_2^+$			$595^{(d)}$	220	145
$2_1^+ \to 0_2^+$	$2.6 \pm 0.4^{(a)}$	$0.84^{(a)}$	$5.1^{(d)}$	0.96	
				$\mathcal{B}_\gamma$	$\mathcal{B}_\sigma$
$4_1^+ \to 2_1^+$				1.2	1.0
$2_2^+ \to 0_1^+$	$0.73 \pm 0.13^{(b)}$	$1.99^{(a)}$	$0.4^{(d)}$	2.3	2.2
	$1.57 \pm 0.13^{(c)}$				
$2_2^+ \to 2_1^+$				5.5	5.4
$4_2^+ \to 2_2^+$			$7.5^{(d)}$	7.5	4.7

(*a*) Ref.[10]; (*b*) Ref.[7]; (*c*) Ref.[8]; (*d*) Ref.[11]

as ordinary bound states with complex energy, whose real and imaginary parts give the resonance energy and half the width of the resonance, respectively. In order to reproduce the known experimental energies in the ^{12}C-spectrum a fine tuning with a short-range three-body force is required. This is done by an effective three-body potential. The results obtained with the α-α Buck potential are shown in Table 1.

The use of the complex-rotated three-body wave functions permits to compute very easily the complex expectation value $\rho_{rms} = \langle\rho^2\rangle^{1/2}$, whose real part gives a feeling of the spatial distribution of the system (the imaginary part is related to the uncertainty). States arising as the result of a rotation of a reasonable rigid structure should show similar values of ρ_{rms}. It is remarkable that, on the one side, the 0_1^+ and 2_1^+ states have similar sizes, both of them in the vicinity of $\rho_{rms} \sim 6.8$ fm and $r_{rms} \sim 2.5$ fm, and, on the other side, the same happens with the 0_2^+ and 2_2^+ resonances, with a value of $\rho_{rms} \sim 10$ fm and $r_{rms} \sim 3.4$ fm. This may suggest that these two sets of 0^+ and 2^+ states could correspond to two different rotational bands, each of them with a reasonably well-"frozen" structure. It is interesting to note that for the 4_1^+ and 4_2^+ states the computed values of ρ_{rms} are precisely about 10 fm and 6 fm, respectively, which might indicate a crossing of the first and second 4^+ states, in such a way that the 4_1^+ state should belong to the second band and the 4_2^+ state to the first one. This is consistent with the fact that the 4_2^+ state becomes actually the first 4^+ and the 4_1^+ state becomes the second 4^+ when the repulsion in the three-body force is diminished [4].

In Table 2 we give the computed electric quadrupole transition strengths for the transitions between the 0^+, 2^+, and 4^+ states shown in Table 1. In those cases where the initial state corresponds to a continuum state we give in the table the two values, $\mathcal{B}_\sigma$ and $\mathcal{B}_\gamma$, corresponding to the two different computation procedures described in the previous section.

For transitions between states within a schematic rotational band, and assuming axial symmetry for the system, the quadrupole transition strength is given by

$$\mathcal{B}^{(E2)}(J \to J') = \frac{5}{16\pi} Q_0^2 \langle J0; 20|J'0\rangle^2, \tag{6}$$

Table 3. Absolute value of the intrinsic transition quadrupole moments $|Q_0|$ (in $e$fm^2) obtained from the $\mathcal{B}(E2)$ transition strengths given in Table 2 and Eq.(6). $|Q_0|_\sigma$ and $|Q_0|_\gamma$ denote the intrinsic quadrupole moments obtained from the transition strengths $\mathcal{B}_\sigma$ and $\mathcal{B}_\gamma$, respectively.

Transition	Exp.	α-cluster	AMD	$\|Q_0\|$ ($e$fm^2)	
$2_1^+ \to 0_1^+$	19.5 ± 0.5	21.46	20.5	22.3	
				$\|Q_0\|_\gamma$	$\|Q_0\|_\sigma$
$4_2^+ \to 2_1^+$			23.6	21.0	18.7
				$\|Q_0\|_\gamma$	$\|Q_0\|_\sigma$
$2_2^+ \to 0_2^+$			72	89	83
$4_1^+ \to 2_2^+$			145	88	71
$2_1^+ \to 0_2^+$	11.4 ± 0.9	6.50	16.0	6.9	
				$\|Q_0\|_\gamma$	$\|Q_0\|_\sigma$
$4_1^+ \to 2_1^+$				6.5	5.9
$2_2^+ \to 0_1^+$	6.03 ± 0.54 8.88 ± 0.37	10.0	4.5	10.8	10.5
$2_2^+ \to 2_1^+$				13.9	13.8
$4_2^+ \to 2_2^+$			16.2	16.2	12.9

where the projection, K, of the angular momentum on the intrinsic symmetry axis has been assumed to be zero. The intrinsic quadrupole moment Q_0 is given by:

$$Q_0 = \langle \sum_i q_i (2z_i^2 - x_i^2 - y_i^2) \rangle \ , \tag{7}$$

where i runs over all the charged particles with charge q_i and whose center of mass coordinates are denoted by (x_i, y_i, z_i), where the z-axis is chosen along the intrinsic symmetry axis. The expectation value is taken in the intrinsic body-fixed coordinate system. The quadrupole moment is a measure of the deformation and it has ideally one characteristic value for a sequence of states belonging to a given rotational band. From each of the calculated transition strengths given in Table 2, it is then easy, to obtain from Eq.(6) the absolute value of the intrinsic quadrupole moments. The $|Q_0|$-values obtained in this way are given in Table 3.

4 Discussion

The transition strengths obtained from the cross sections are in general consistent with each other. The results are similar no matter what method is used to extract the transition strengths ($\mathcal{B}_\sigma$ or $\mathcal{B}_\gamma$), especially when taking into account the uncertainties inherent to each of the methods. Although not shown, it has also been observed that the results are, to a large extent, independent of the potential used [4].

An important result is that the 4_1^+ and 4_2^+ states have actually crossed, in such a way that the 4_1^+ state belongs to the second band and the 4_2^+ to the first one. This is first suggested by the fact that the computed values of ρ_{rms}, which should be similar for systems having the same 'frozen' spatial structure, provides values for the 4_1^+ state similar to the ones of the 0_2^+ and 2_2^+ states, and values for the 4_2^+ state similar to the ones of the 0_1^+ and 2_1^+ states. Furthermore, investigating how the 4_1^+ and 4_2^+ resonances move in the complex energy plane when making the effective three-body force more and more attractive, we have seen that the first 4^+ state becoming bound would actually be the 4_2^+ state.

The assignment of the 4_1^+ state to the second band and the 4_2^+ to the first is also confirmed by the analysis of the transition strengths and the intrinsic quadrupole moments. The values of Q_0 are

obtained assuming a rotational character for the two bands in ^{12}C under investigation, in such a way that the transition strength is basically the square of the intrinsic quadrupole moment multiplied by some geometrical factor depending on the initial and final angular momenta of the transition. Doing like this we have seen that for the $4^+_2 \rightarrow 2^+_1$ reaction the computed transition strength is clearly more consistent with the previous AMD calculation than the one corresponding to the $4^+_1 \rightarrow 2^+_1$. The large discrepancy still remaining between our result and the AMD calculation can be understood from the fact that, as shown in Ref.[4], the interference between the 4^+_1 and 4^+_2 resonances and the removal of the so-called soft-photon contribution introduce in the calculation a series of uncertainties that lead us to consider our result in Table 3 as a lower limit for the transition strength (and the intrinsic quadrupole moment). Furthermore, the intrinsic quadrupole moments derived from the transition strengths for the $2^+_1 \rightarrow 0^+_1$ and $4^+_2 \rightarrow 2^+_1$ reactions are rather similar and consistent with previous calculations and the experimental value. This consistency would disappear if the reaction $4^+_1 \rightarrow 2^+_1$ were the one taken into account. The same happens for the reactions in the second band. Although the computed transition strength for the $4^+_1 \rightarrow 2^+_2$ is even a factor of 3 smaller than the previous AMD calculation, this discrepancy is certainly less important than when considering the $4^+_2 \rightarrow 2^+_2$ reaction, and it can actually be understood from the uncertainties associated to the method used to extract the strength. Also, the intrinsic quadrupole moments for the $2^+_2 \rightarrow 0^+_2$ and $4^+_1 \rightarrow 2^+_2$ are reasonably stable, stability that would be clearly broken if assuming the $4^+_2 \rightarrow 2^+_2$ transition as belonging to the second band. Therefore, from the stability of the intrinsic quadrupole moments for each of the $\{0^+_1, 2^+_1, 4^+_2\}$ and $\{0^+_2, 2^+_2, 4^+_1\}$ bands we can conclude that each of the bands correspond to states having a rather well preserved rigid structure.

The rotational character of two bands is confirmed when comparing with the prediction obtained from an axially symmetric rotating structure made by three point-like alpha particles. Assuming values for ρ_{3b} similar to the ones computed numerically for the states in the first and second bands ($\rho_{rms} \sim 6.7$ fm and ~ 10 fm, respectively) we have seen that the Q_0 values previously computed for the states in the first band are consistent with a equilateral triangular structure rotating around an axis perpendicular to the plane holding the three particles. For the states in the second band, the computed Q_0 values are mostly consistent with a linear distribution of the particles. The same results are obtained from the analyses of the moments of inertia.

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