

About fractal models of clouds

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Abstract. In order to deal concretely with the full diversity and complexity of clouds, we propose here a model of clouds issued from fractals and able to deal with both gravity and wind anisotropy. The goal of this approach is to fit with the observations of infrared resonances and diffraction on clouds. These simple models derive from the well-known model of Menger sponges and takes into account both gravity and wind effect. The analysis of the connectivity matrix of this fractal model for different vertical behaviors enables us to derive a classification of these fractal clouds.

1 Introduction

Clouds are fascinating everyday complex large objects. The cloud diversity as well as their permanently mobile presence exhibits a deep complexity. At microscopic level as well as at macroscopic level everybody sees both permanence and change of clouds.

The interest in clouds recently increased with the observation of the new climatic change in which they take an effective part. The recent observation of extra-terrestrial clouds deepens the interest in clouds and extends the knowledge on their chemistry to other materials than water [1, 2]. On earth, the recent observations of atmospheric rivers evidence the presence in some clouds of water as a vapor and not only in liquid or solid phases [3].

Clouds escape to a simple analysis because of their large size, of their permanent evolution under powerful constraints, such as gravity, or temperature and pressure gradients, and of the difficulties of an in situ analysis of their complex topology. This complex status is beneficial for dreams and mystery but does not favour an effective scientific approach.

Numerous old classifications of clouds exist according to their visual shapes and to their expansion in altitude [4, 5]. The large number of different classes and different classifications of clouds is another proof of their complexity.

At a global level, thermodynamics and hydrodynamics act on the formation and evolution of clouds. Moreover, the careful study of the detailed and complex perimeter of clouds and rainfalls as a function of frame size gives evidence to a fractal Hausdorff dimension of clouds [1, 2, 6]. This is a partial proof of the global internal self-similarity of clouds.

At a local level, aggregation-segregation occurs in the formation of water molecule clusters, often localized

around some very small parts of different nature. A catalytic effect of dusts drives the formation of water clusters. According to the local temperature and pressure, water clusters can take the shape of molecular clusters, or of liquid droplets or of solid matter such as ice snowflakes. As shown in many papers [7, 8], such an aggregation-segregation process leads often to self-similar structures which are now known as fractals [9]. The classical example of percolation clusters [10] is stimulating for mind as an example.

These local and global observations suggest a strong link between fractals and the complex structure of clouds, in the lack of detailed observation at the atomic level. The idea suggested by the analysis of the aggregation-segregation process under potential interaction [11] is the emergence of a continuous set of water clusters of different sizes. Of course, if numerous enough hydrometeors with size larger than 0.2 mm occur there will be rain. This is a phase transition. Another suggestion from the results of [11] is the occurrence of very small hydrometeors with rough surfaces. This means that rainbow can appear only if there are enough large hydrometeors as spherical droplets.

Quite obviously, clouds contains many different sizes and shapes of droplets as well as voids. Consequently, the comparison of clouds with fractals sounds clear, even if clouds are not simple fractal objects. Thus, the present consideration of basic fractal models can help the scientific community in order to look at global and local cloud properties in a first step.

The vertical extension of clouds is often very large, up to several kilometers, so gravity effects deeply occur. Quite similarly, temperature and pressure strongly vary with altitude and give rise to vertical motions of the so numerous clusters. Thus, cloud modelling must account for vertical anisotropy as already suggested by Mandelbrot and coworkers [12, 13]. There is another cause of anisotropy of clouds. In atmosphere, many winds occur. Earth rotation and friction induces some basic extended

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winds as well as the full hydrodynamic complexity. Pressure or thermal gradients also induce more local winds. With these numerous winds, horizontal anisotropy happens. Thus, there is a full 3D anisotropy.

This hydrodynamic contribution of winds is comparable to the situation of sand in deserts where dune ripples occur orthogonally to a unique wind [14]. When several winds compete, the resulting dune structure is more complex [15]. The similarity between dunes and clouds both formed of nearly independent microscopic grains suggests some comparison. This is especially interesting since the observation of such cloud ripples in the sky is frequent.

Another main feature of clouds is the part of randomness. Dusts are responsible of water clustering around them in a catalytic process. These clustering seeds occur randomly, and move freely at a local scale. Mandelbrot and co-workers [12] already noticed this random contribution.

From these remarks, a simple model of clouds must involve random fractals in an anisotropic 3D space. This is the goal of the present work in a simple framework. Neglecting the part of wind, the basic problem reduces to a 2D anisotropic one. This study is the main goal of this paper in order to give a tool for in situ diffraction observations. We consider here two basic kinds of vertical anisotropy, a linear one and an exponential one. In these cases, we are considering Menger sponges which are natural 3D extensions of Sierpiński carpets [16, 17], here with different partitions. A simple partition induces some order while a large partition emphasizes the part of randomness. This is the interesting effect of random level. We also study here this random level and look carefully at the connectivity point when comparing structures. The basic tool of analysis of connectivity considered here is the analysis of the connectivity matrix. The connectivity matrix evidences a large number of eigenvalues and eigenstates.

This reveals an underlying structure of this model to compare with experiment. A first experimental comparison between models and reality comes from cloud diffraction patterns. These patterns are sensitive to the long ranged effects of fractals as shown in [18, 19]. Another experimental comparison comes from infrared resonances as already observed for water clusters in clouds [20] where oxygen resonances are sensitive to the oxygen environment and thus to the details of the water clusters. Another observation concerns the vibrational spectra of these clusters. These spectra are quite sensitive to the self similarity [21].

2 Fractal models of clouds

2.1 The Menger sponges

The Menger sponges are three dimensional fractal structures built in a similar way as the planar Sierpiński carpets. A cube is divided into n^3 subcubes (also called sites) among which p selected ones are removed either in a deterministic or a random way. The process is iterated k times (k is called the segmentation step), the fractal structure being the limit of the process when $k \rightarrow \infty$.

The local structural properties may be characterized by a connectivity matrix M which links the numbers (or mean numbers for random structures) of sites with each possible environment between two successive segmentation steps, then which links the steps 1 to $k+1$ after k iterations of the matrix: $M \rightarrow M^k$. Subdimensions may be defined from eigenvalues λ_i of the connectivity matrix with similar formulas as the Hausdorff dimensions $\delta_i = \ln \lambda_i / \ln n$ [22], with the Hausdorff dimension d_f as the largest subdimension [17]. Eigenvalues and subdimensions are meaningful characteristic of these cloud models, as they are for other random fractal models [19].

Complete calculations have already been made for planar random Sierpiński carpets [17] and extended to planar as well as higher dimensional random carpets and sponges at various kind of vicinity (nearest neighbors, next nearest neighbors...) [17]. We presently investigate the three dimensional case with a vertical anisotropic random probability distributions at the nearest neighbors, with a special emphasis to distributions with an anisotropy along the vertical axis (simulating gravity effects) and to the extrapolation to the continuous limit $n \rightarrow \infty$.

2.2 Connectivity matrix of anisotropic Menger sponges

2.2.1 Eigenvalues and fractal subdimensions

In dimension $d = 3$, the cube has six faces, twelve edges and eight vertices, the dimension of the connectivity matrix is then $2^6 = 64$. It has 1 (cube) + 6 (faces) + 12 (edges) + 8 (vertices) = 27 non zero eigenvalues. The multiplicity of the eigenvalue 0 is then $64 - 27 = 37$, leading to 37 ghost states (in the meaning of [17, 19]). The non zero eigenvalues are calculated from the site occupancy probabilities.

For a vertical anisotropy, the distribution of the number of sites is varying along a single axis (say Oz) and are uniform in each plane orthogonal to Oz . Then the four lateral faces are equivalent involving a 4 times multiplicity for the corresponding eigenvalue; the four bottom edges are equivalent (multiplicity 4); the four lateral edges are equivalent (multiplicity 4); the four upper edges are equivalent (multiplicity 4); the four bottom vertices are equivalent (multiplicity 4); the four upper vertices are equivalent (multiplicity 4). At most nine remaining non zero eigenvalues are distinct.

Let m_j the number of occupied sites on the layer (or plane) j (layers are numbered from bottom to top) for a single step of segmentation of a single site and $c_j = m_j/n^2$ the probability for a site of the layer j to be occupied. c_j is also the planar concentration of the layer j (after a single step of segmentation of a single site). And let $c = p/n^3$ the volumic concentration (after a single segmentation step of a single site). Then

$$\sum_{j=1}^n m_j = p \quad \text{and} \quad \sum_{j=1}^n c_j = \frac{p}{n^2} = nc \quad (1)$$

and the fractal dimension is $d_f = \ln p / \ln n$.

Table 1. The eigenvalues and subdimensions of the connectivity matrix of Menger sponges with vertical anisotropy.

Simplex	Dimension ∂	Position i	Eigenvalues $\lambda_{\partial,i}$	$F(n, \partial, i)$	Subdimensions $\delta_{\partial,i}$	Multiplicity
Site	3	0	$n^3 c = p$	$\frac{c}{c_1}$	d_f	1
Bottom face	2	1	$n^2 c_1 c_n = \frac{m_1 m_n}{n^2}$	$f(n)$	$d_1 + d_n - 2$	1
Lateral face	2	4	$n \sum_{j=1}^n c_j^2 = \frac{m_1^2}{n^2} F(n, 2, 4)$	$\frac{1}{n} \sum_{j=1}^n f(j)^2$	$2d_1 - 2 + \frac{\ln(F(n, 2, 4))}{\ln n}$	4
Top face	2	1	$n^2 c_1 c_n = \frac{m_1 m_n}{n^2}$	$f(n)$	$d_1 + d_n - 2$	1
Bottom edges	1	1	$n c_1^2 c_n = \frac{m_1^2 m_n}{n^5}$	$f(n)$	$2d_1 + d_n - 5$	4
Lateral edges	1	4	$\sum_{j=1}^n c_j^3 = \frac{m_1^3}{n^5} F(n, 1, 4)$	$\frac{1}{n} \sum_{j=1}^n f(j)^3$	$3d_1 - 5 + \frac{\ln(F(n, 1, 4))}{\ln n}$	4
Top edges	1	2	$n c_1 c_n^2 = \frac{m_1 m_n^2}{n^5}$	$f(n)^2$	$d_1 + 2d_n - 5$	4
Bottom vertices	0	1	$c_1^3 c_n = \frac{m_1^3 m_n}{n^8}$	$f(n)$	$3d_1 + d_n - 8$	4
Top vertices	0	3	$c_1 c_n^3 = \frac{m_1 m_n^3}{n^8}$	$f(n)^3$	$d_1 + 3d_n - 8$	4

We will note $d_j = \ln m_j / \ln n$ the effective fractal dimensions of each layer, calculated on the segmentation of a single site. Only d_1 is the true fractal dimension of the first layer since only m_1 is integer. The other m_j are mean values of sites numbers on the layer j and then are generally not integers. This is the reason of the word “effective dimension” used above.

The eigenvalues of the connectivity matrix are the number of simplices (cubes=sites, faces, edges, vertices) generated through one segmentation step by a simplex of the same kind [17, 19]. They are calculated in table 1. The fourth column of table 1 contains the expressions of the non zero eigenvalues of the connectivity matrix versus the concentrations c_j . The fifth column gives their expression renormalised by the maximal numbers of sites corresponding to the dimension of the corresponding simplex: n^3 , n^2 , n and 1 for, respectively the sites, faces, edges and vertices of the segmented cube, to switch from mean numbers to proportions.

Eigenvalues will be noted $\lambda_{\partial,i}$ where ∂ is the dimension of the corresponding simplex (first column of table 1) and i is an index which indicates its position with the following convention, $i = 0$ for the site, $i = 1$ for the bottom position, $i = 3 - \partial$ for top position and $i = 4$ for lateral positions (second column of table 1). With this convention, eigenvalues of bottom and top simplices ($i = 1$ and $i = 3 - \partial$) writes

$$\lambda_{\partial,i} = n^\partial c_1^{4-\partial-i} c_n^i \quad (1 \leq i \leq 3) \quad (2)$$

Subdimensions will be noted $\delta_{\partial,i}$: $\delta_{\partial,i} = \ln \lambda_{\partial,i} / \ln n$, and we have always $\delta_{3,0} = \ln p / \ln n = d_f$.

Only lateral positions ($i = 4$) involve all layers in the calculation of eigenvalues, the others eigenvalues involve only the bottom and top layers. And clearly an additional degeneracy occurs for $\partial = 2$ since in this case $3 - \partial = 1$: the bottom and the top faces are equivalent, leading to eight distinct non zero eigenvalues instead of nine.

2.2.2 The vertical distribution of sites

We choose for the vertical distribution of sites the following expression: $m_j = m_1 f(j)$ where f is a decreasing function of the altitude j . Then $f(1) = 1$ and for $j > 1$, $0 < f(j) < 1$. The concentrations of each layer evidently follows the same expression $c_j = c_1 f(j)$. The decay of f implies $n m_1 > p$ then $c_1 > c$. From equation (1), the total number of sites is

$$\sum_{j=1}^n m_j = m_1 \sum_{j=1}^n f(j) = p \rightarrow \sum_{j=1}^n f(j) = \frac{p}{m_1} = \frac{nc}{c_1} \quad (3)$$

From equation (2) top and bottom eigenvalues may be written

$$\lambda_{\partial,i} = n^\partial c_1^{4-\partial-i} c_1^i f(n)^i = n^\partial c_1^{(4-\partial)} f(n)^i. \quad (4)$$

The eigenvalues corresponding to lateral simplices ($i = 4$) write

$$\lambda_{\partial,4} = n^{(\partial-1)} \sum_{j=1}^n c_k^{(4-\partial)} = n^\partial c_1^{(4-\partial)} \left(\frac{1}{n} \sum_{j=1}^n f(j)^{(4-\partial)} \right) \quad (5)$$

And finally, from equations (4) and (5) all the eigenvalues may be written in a unified expression

$$\lambda_{\partial,i} = n^\partial c_1^{4-\partial} F(n, \partial, i) \quad (6)$$

where

$$F(n, \partial, i) = \begin{cases} c/c_1 & \text{if } i = 0 \quad (\partial = 3) \\ f(n)^i & \text{if } 1 \leq i \leq 3 \\ \frac{1}{n} \sum_{j=1}^n f(j)^{(4-\partial)} & \text{if } i = 4 \end{cases} \quad (7)$$

and then subdimensions are

$$\begin{aligned} \delta_{\partial,i} &= \frac{\ln(\lambda_{\partial,i})}{\ln n} = \frac{\ln \left(n^\partial \frac{m_1^{(4-\partial)}}{n^{2(4-\partial)}} F(n, \partial, i) \right)}{\ln n} \\ &= \frac{\ln \left(m_1^{(4-\partial)} n^{(3\partial-8)} F(n, \partial, i) \right)}{\ln n} \\ &= (4-\partial)d_1 + 3\partial - 8 + \frac{\ln(F(n, \partial, i))}{\ln n} \end{aligned} \quad (8)$$

To calculate in real terms $F(n, \partial, i)$, the eigenvalues of the connectivity matrix and the fractal subdimensions, we need a tangible expression of the sites altitudinal distribution f . This will be achieved in sections 2.3 and 2.4 for respectively decreasing linear and exponential distributions.

2.3 Linear vertical anisotropy

The vertical distribution of sites writes $f(j) = 1 - a(j-1)$ with $a > 0$. From equation (3)

$$\sum_{j=1}^n (1 - a(j-1)) = n - a \sum_{j=1}^{n-1} j = n - a \frac{n(n-1)}{2} = \frac{nc}{c_1}$$

then
$$a = \frac{2(c_1 - c)}{c_1(n-1)}$$

The expression of f becomes

$$f(j) = 1 - 2 \left(1 - \frac{c}{c_1} \right) \frac{j-1}{n-1}$$

and then
$$c_n = c_1 f(n) = 2c - c_1 \quad (9)$$

As $c_n > 0$, we have $2c > c_1$ which leads, considering also $c_1 > c$, to $2c > c_1 > c$.

The two sums involved in eigenvalues are

$$\begin{aligned} n F(n, 2, 4) &= \sum_{j=1}^n f(j)^2 = \sum_{j=0}^{n-1} (1 - a j)^2 \\ &= \sum_{j=0}^{n-1} (1 - 2a j + a^2 j^2) \\ &= n - a n(n-1) + a^2 \frac{n(n-1)(2n-1)}{6} \\ &= n - 2n \left(\frac{1-c}{c_1} \right) + \frac{2n(2n-1)}{3(n-1)} \left(\frac{1-c}{c_1} \right)^2 \end{aligned}$$

and

$$\begin{aligned} n F(n, 2, 4) &= \sum_{j=1}^n f(j)^3 = \sum_{j=0}^{n-1} (1 - a j)^3 \\ &= \sum_{j=0}^{n-1} (1 - 3a j + 3a^2 j^2 - a^3 j^3) \\ &= n - 3a \frac{n(n-1)}{2} + 3a^2 \frac{n(n-1)(2n-1)}{6} - a^3 \frac{n^2(n-1)^2}{4} \\ &= n \left(\frac{3c}{c_1} - 2 \right) + 2n \left(1 - \frac{c}{c_1} \right)^2 \frac{(2n-1)}{(n-1)} - 2 \left(\frac{1-c}{c_1} \right)^3 \frac{n^2}{(n-1)} \end{aligned}$$

The expressions of the eigenvalues are, from equations (6) and (7),

$$\begin{aligned} &n^3 c, \quad n^2 c_1 (2c - c_1), \quad n^2 [c_1 (2c - c_1) + \frac{2(2n-1)}{3(n-1)} (c_1 - c)^2], \quad n^2 c_1 (2c - c_1), \dots \\ &\dots n c_1^2 (2c - c_1), \quad n [c_1^2 (3c - 2c_1) + 2c_1 (c - c_1)^2 \frac{(2n-1)}{(n-1)} - (c_1 - c)^3 \frac{2n}{(n-1)}], \dots \\ &\dots n c_1 (2c - c_1)^2, \quad c_1^3 (2c - c_1), \quad c_1 (2c - c_1)^3 \end{aligned}$$

and of subdimensions according to (8).

2.4 Exponential vertical anisotropy

For an exponential vertical distribution of sites $f(j) = b^{(j-1)}$, with $0 < b < 1$, the normalisation condition (3) becomes:

$$\sum_{j=1}^n b^{(j-1)} = \sum_{j=0}^{n-1} b^j = \frac{1 - b^n}{1 - b} = \frac{nc}{c_1}$$

which gives

$$b^n - \frac{nc}{c_1} b + \frac{nc}{c_1} - 1 = 0 \quad (10)$$

This equation should be solved in b which could be possible for $n \leq 4$ and also for large values of n , when the term in b^n becomes negligible, the expression of b becomes

$$b \sim 1 - \frac{c_1}{nc} = 1 - \frac{m_1}{p} = 1 - m_1 n^{-d_f} \quad (11)$$

For n large enough, the expression of f is

$$f(j) \sim \left(1 - \frac{c_1}{nc} \right)^{(j-1)} = \left(1 - m_1 n^{-d_f} \right)^{(j-1)}$$

The two sums involved in eigenvalues are

$$n F(n, 2, 4) = \sum_{j=1}^n f(j)^2 = \sum_{j=0}^{n-1} b^{2j} = \frac{1 - b^{2n}}{1 - b^2}$$

and

$$n F(n, 1, 4) = \sum_{j=1}^n f(j)^3 = \sum_{j=0}^{n-1} b^{3j} = \frac{1 - b^{3n}}{1 - b^3}$$

leading to the following eigenvalues

$$\begin{aligned} &n^3 c, \quad n^2 c_1^2 b^{(n-1)}, \quad n c_1^2 \frac{1 - b^{2n}}{1 - b^2}, \quad n^2 c_1^2 b^{(n-1)}, \quad n c_1^3 b^{(n-1)}, \dots \\ &\dots c_1^3 \frac{1 - b^{3n}}{1 - b^3}, \quad n c_1^3 b^{2(n-1)}, \quad c_1^4 b^{(n-1)}, \quad c_1^4 b^{3(n-1)} \end{aligned}$$

and of subdimensions according to (8).

Table 2. The subdimensions of fractal structures illustrated in figures 2 to 5.

Structure	Decay	δ_1	δ_2	δ_3	δ_4	δ_5	δ_6	δ_7	δ_8
$n = 6, m_1 = 3$	linear	-1.39	-1.16	-1.39	-3.77	-3.35	-4.39	-6.16	-7.39
$n = 10, m_1 = 4$	linear	-1.38	-1.14	-1.38	-3.80	-3.56	-4.40	-6.19	-7.40
$n = 32, m_1 = 7$	linear	-1.44	-1.15	-1.44	-3.88	-3.41	-4.44	-6.32	-7.44
$n = 55, m_1 = 9$	linear	-1.45	-1.15	-1.45	-3.90	-3.44	-4.45	-6.36	-7.45
$n = 7, m_1 = 4$	exponentiel	-1.29	-1.08	-1.29	-3.58	-3.51	-4.29	-5.86	-7.29
$n = 10, m_1 = 5$	exponentiel	-1.30	-1.09	-1.30	-3.60	-3.52	-4.30	-5.90	-7.30
$n = 22, m_1 = 8$	exponentiel	-1.33	-1.10	-1.33	-3.65	-3.54	-4.33	-5.98	-7.33
$n = 49, m_1 = 13$	exponentiel	-1.34	-1.10	-1.34	-3.68	-3.53	-4.34	-6.02	-7.34

3 Discussion

The values of the fractal dimension of clouds previously obtained by area/perimeter measurements [12] or numerical simulations [6] are respectively $d_f = 1.35$ and $d_f = 1.40$. We will illustrate anisotropic Sierpiński structures with the fractal dimension of 1.4. Many sets of parameters n, p, m_1 correspond to a single fractal dimension. For the purpose of illustrations, it may be suitable to fixe the number of occupied sites on the highest layer m_n instead of the total number of occupied sites p . This allows to avoid the upper layer (layer n) to be empty otherwise most of the eigenvalues vanishes ($c_n = 0$), and subdimensions become infinite.

For a linear vertical anisotropy and versus n, m_1, m_n , equation (9) gives

$$m_n = \frac{2p}{n} - m_1 = 2n^{(d_f-1)} - m_1 \quad (12)$$

but also $m_n = m_1 (1 - a(n-1))$

then $a = \left(1 - \frac{m_n}{m_1}\right) \left(\frac{1}{n-1}\right)$

For an exponentiel vertical anisotropy and versus n, m_1, m_n

$$m_n = m_1 b^{(n-1)} \rightarrow b = \left(\frac{m_n}{m_1}\right)^{\frac{1}{n-1}}$$

Inserted in equation (10) and with $p = n^{d_f}$, we obtain

$$\left(\frac{m_n}{m_1}\right)^{\frac{n}{n-1}} - \frac{n^{d_f}}{m_1} \left(\frac{m_n}{m_1}\right)^{\frac{1}{n-1}} + \frac{n^{d_f}}{m_1} - 1 = 0 \quad (13)$$

Equations (12) and (13) allow to plot m_1 versus n when m_n and d_f are fixed. Figure 1 shows these plots when $m_n = 1$ and $d_f = 1.4$. The values obtained for the exponentiel case are larger than those for the linear case because the exponential decay being faster than the linear one, for a same fractal dimension the first layers should be denser.

The illustrations of figures 2 to 5 show examples of Sierpiński sets with vertical anisotropy satisfying the preceding conditions. They are two dimensional structures for the purpose of illustrations, but built according to the same scheme as the three dimensional ones: vertical anisotropy and random horizontal distribution. Equations

(9) and (11) remain valid in two dimensions, subdimensions however are not the same. The free parameters are n, m_1 and k for a finite segmentation step and then we will note these structures $CL(n, m_1, k)$ or $CE(n, m_1, k)$ according to the linear or exponential anisotropy. Figures 2 and 3 contain structures with linear vertical anisotropy: $CL(6, 3, 4)$, $CL(10, 4, 3)$, $CL(32, 7, 2)$ and $CL(55, 9, 2)$ and figures 4 and 5 show structures with exponentiel anisotropy: $CE(7, 4, 3)$, $CE(10, 5, 3)$, $CE(22, 8, 2)$ and $CE(49, 13, 2)$. These eight structures correspond to the large dots of figure 1. The table 2 contains the values of their subdimensions.

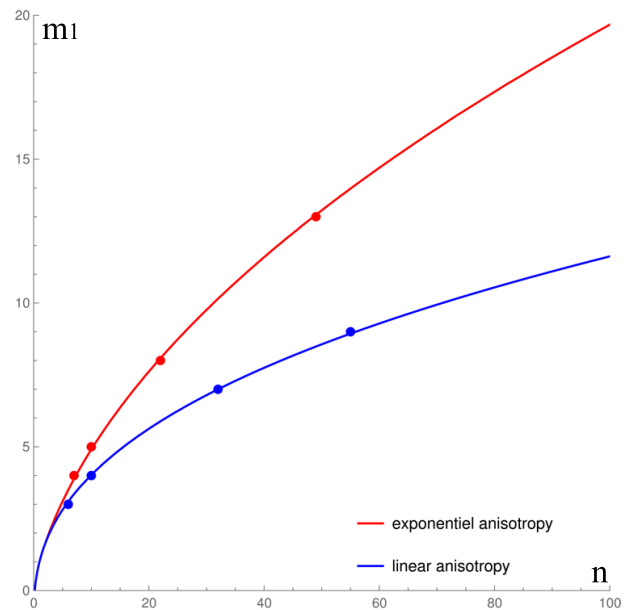


Figure 1. Plot of m_1 versus n for $d_f = 1.4$ and $m_n = 1$ in both linear and exponentiel vertical anisotropy. Large dots correspond to the structures illustrated in figures 2 to 5.

4 Conclusion

Starting from a local fractal structure of clouds and taking into account their vertical anisotropy, we derive different models of complex structures in order to describe clouds.

The structures illustrated in figures 2 to 5 seem quite diluted, this is linked to the fact that the fractal dimension

1.4 is obtained from area/perimeter measurements while our calculations are based on calculations in volume. Further measurements should clarify this point.

The study of such complex structures paves the way to a more careful local observation and understanding of clouds.

Cloud experimental observations of infrared absorption and diffraction over a large frequency spectrum can check the validity of these models and can determine effective realistic parameters.

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5 Appendix: illustrations

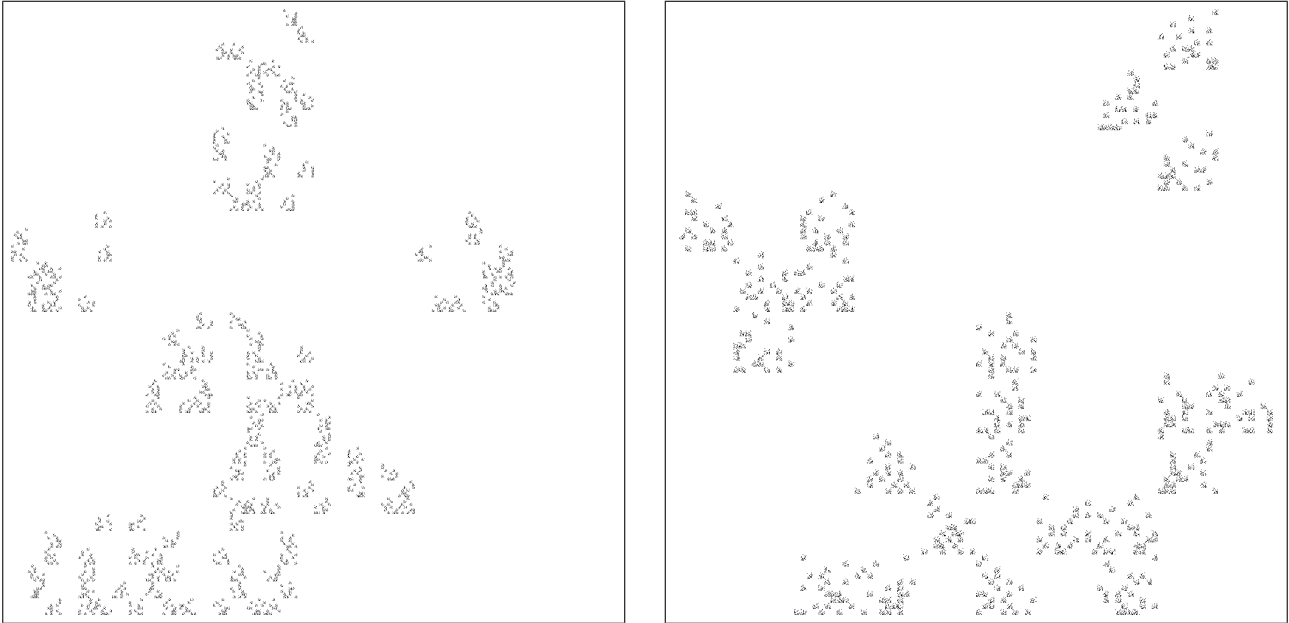


Figure 2. Random Sierpiński sets with linear vertical anisotropy: $CL(6, 3, 4)$ and $CL(10, 4, 3)$.

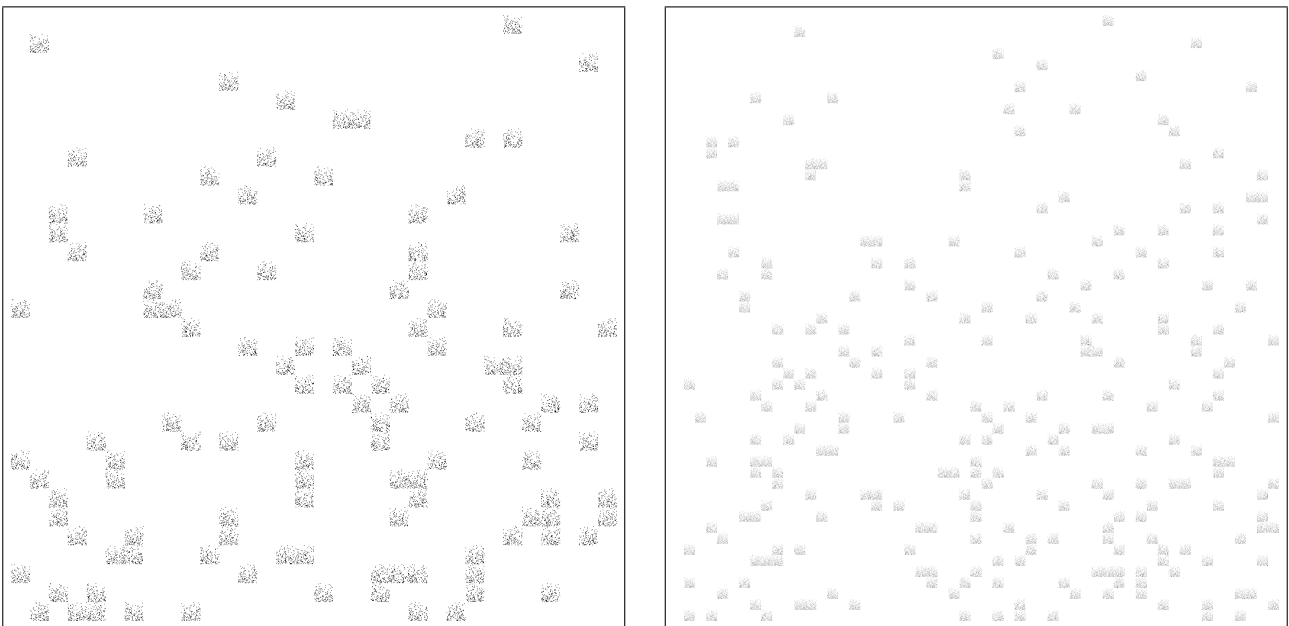


Figure 3. Random Sierpiński sets with linear vertical anisotropy: $CL(32, 7, 2)$ and $CL(55, 9, 2)$.

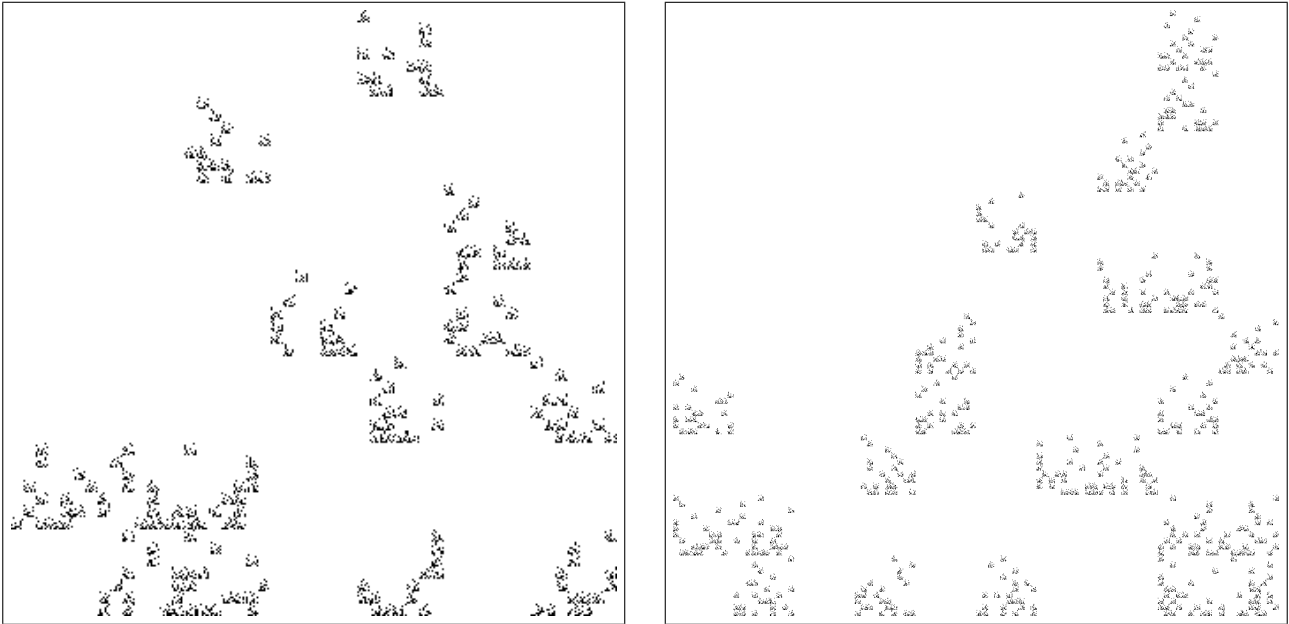


Figure 4. Random Sierpiński sets with exponential vertical anisotropy: $CE(7, 4, 3)$ and $CE(10, 5, 3)$.

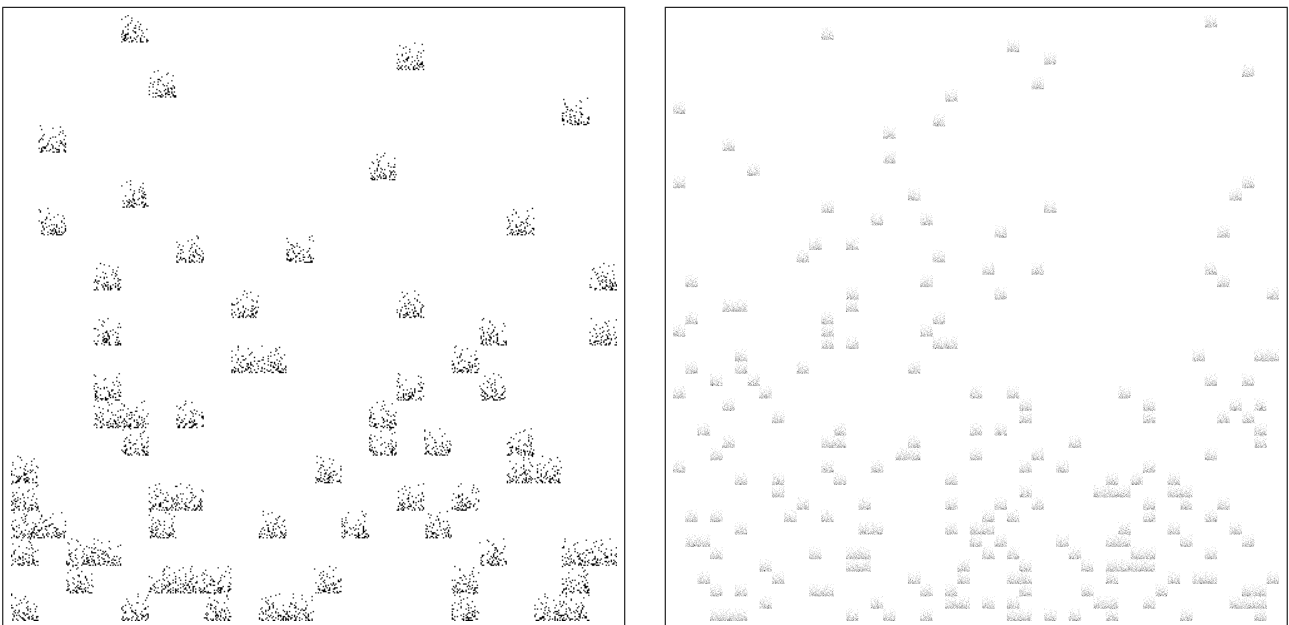


Figure 5. Random Sierpiński sets with exponential vertical anisotropy: $CE(22, 8, 2)$ and $CE(49, 13, 2)$.