A summary and outlook for SOTANCP5

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Abstract. The present contribution provides an overview of the SOTANCP5 conference and the progress that was reported in state-of-the-art experiment and theoretical cluster studies. An exciting array of topics ranging from correlations, halo nuclei, clustering in reactions and nuclear astrophysics, alpha-gas states, ab intio theory, alpha-particle like cluster structures, the role of the continuum and the latest developments in experimental facilities. These contributions are brought together in the following summary. Some additional thoughts and perspectives are also provided.

1 SOTANCP5

The attraction of clustering is the simplification of the complex many-body nuclear system. This underlying complexity stems from an interaction that arises from the as yet to be understood strong force of QCD (the nuclear strong force is considered to be a perturbation), a many-body system in which 3- and *N*-body forces are important and a mix of quantum particles which skip between their particle and wave-like characteristics. Models attempt to capture this essence, but a theory has been elusive.

Even in complexity, patterns or trends emerge and the nucleus has often been studied in terms of these emergent properties; clustering is one. The appearance of clusters is not purely down to the statistical coincidence that a group of nucleons have coincident spatial coordinates or momenta, but may be traced to the fundamental properties of the strong force and the mean-field. As such clustering shines a light on the deeper nature of nuclear matter. These clusters also have a profound affect on the nature of nuclear reactions, and via a combination of reactions and structure influence the synthesis of nuclei in stellar processes. See [1] for a recent review.

SOTANCP5 explored these topics and demonstrated that though much has been understood and the broad emergent properties are grasped, there is much more work to be done in terms of *the detail* and disentangling the sometimes subjective from a profound understanding which would yield a theory of nuclear clustering.

The present contribution takes a look at some of the topics explored through the conference and shares some thinking which may help shape priority areas for study before SOTANCP6, to be held in Japan in 2028.

2 Aspects of Clustering

Clustering arises due to a series of effects which may be traced to the details of nucleon-nucleon correlations, symmetries within the mean-field and the inherent link to geometric arrangements of clusters and also the energetics associated with the relationship between the binding energy of the clusters and excitation energy of the composite system. Increasingly, the role of the continuum above the cluster decay threshold is seen to play an important role. Progress has been made both through state-of-the-art developments in both experiment and theory. The following subsections illustrate key topics covered in the conference.

2.1 Correlations, weak binding and halo-nuclei

The conference heard about the nature of halo nuclei as a manifestation of clustering, whereby there is a decoupling of a core, or a cluster, from the extended distributions of the halo nucleons. Such systems allow an understanding of how the halo nucleons, e.g. neutrons, are correlated outside the confines of normal nuclear matter and, in turn, how the associated interactions then impact on the binding and nuclear radii.

The advances presented illustrated that new facilities such as RIKEN and FRIB are allowing increasingly exotic nuclei to be probed in detail, e.g. ^{8}He, ^{29}F, ^{21}N, ^{24}O and ^{18}B (Kanungo and Marques). They are allowing the search for a deeper understanding of the role of core excitations, e.g. in ^{11}Li (Yang) and hence the degree to which the clusters are inert. Exotic multi-neutron correlations were described the nuclei beyond the dripline (Tao), ^{9}H and ^{10}He, the resonances in ^{8}He (Yang) and knockout reactions from Borromean nuclei, e.g. ^{9}Be, who owe their existence to correlations and the intricacies of the strong nuclear-force (Casal). The experimental progress is impressive, but needs linking to the details of the strong-force.

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At the neutron-dripline it has long been predicted that clusterisation should play an important role, driven by the importance of the n-p interaction, and the optimisation of the systems binding energy, leading to the emergence of cores of less exotic nuclear matter surrounded by exchanged neutrons. Halos might be seen as a manifestation of this, but multi-cluster systems remain to be experimentally demonstrated.

Transfer reactions provide an alternative way of probing correlations and the influence of n-p pairing and the dependence on isospin of the pair. Measurements from GANIL were presented (Assié) to better understand the role of the angular momentum associated with single-particle orbitals in n-p pairing. This is an interesting topic. The role of collective angular momentum and high spin in the promotion of clustering was also explored (Afanasjev), where the rearrangement of single-particle orbitals in the cranked relativistic mean field produces new shell-structures and associated densities which reveal cluster symmetries. This is a reminder of the fact that there is often a coincidence between the clustering that emerges as a result of the nucleon-nucleon correlations, manifest in the high-stability of the alpha-particle and symmetries that appear in the mean-field and the role that angular momentum plays in shaping nuclear structure and the underpinning correlations.

The extension of the finite to infinite nuclear matter has been a topic of considerable interest given the relationship to stellar environments found in the likes of neutron stars (Burrello), with now the introduction of realistic nuclear forces and a framework that generates many-body interactions (Yamada).

2.2 Clusters, Reactions and Collective Excitations

There is, of course, an intimate relationship between the structure of nuclei and the reactions they involve. In the simplest case (Kanungo), interaction cross sections reveal the effective size of a nucleus, knockout reactions (Casal) may be used to interrogate momenta correlations and spatial correlations and inelastic scattering and associated electromagnetic transition rates probe collective properties. For example, the enhanced dipole excitation of ^{10}Be excited states was argued to be a signature of the 4-body molecular structure associated with the cluster states close to the alpha-decay threshold (Chen). The dipole excitation spectra of ^{8}He and ^{8}C were analysed using the complex scaling method (Kato) with a clever approach involving rotating the complex plane. The geometric structures predicted by the Algebraic Cluster Model were found to describe well the inelastic scattering of ^{12}C and ^{16}O (Fortunato). An incredible result given the rather classically motivated approach.

States in ^{16}O populated through the ^{3}He+^{13}C entrance channel (Redigolo) have been analysed to give indications of the spins of states at high excitation, 24 MeV. Such measurements reveal how unusual entrance channels can sometimes provide new insights. Another example of this is the measurement of the ^{10}B+^{10}B reaction (Jelavić) to populate cluster states in Be, B and C isotopes, and similarly states in ^{11,12,13}C using a ^{14}N beam and ^{10}B target (Palada), again providing new information regarding states above the cluster decay threshold. Some important measurements of the excited states of ^{12}Be, decaying to ^{6}He+^{6}He and ^{4}He+^{8}He, and decays of ^{13}B to alpha+^{9}Li measured at TRIUMF were shared (Soić) and some preliminary measurements of the ^{9}Li+^{11}B reaction designed to probe states in ^{16}C again at TRIUMF (Sigmund). Such measurements will continue to enhance our understanding of the details of the spectroscopy above decay thresholds.

Measurements of transfer reactions involving clustered nuclei, e.g. ^{6}Li, were presented (Lukyanov). The effect of nuclear structure on the reaction dynamics was explored in detail for halo nuclei (Di Pietro), where soft dipole modes are important. Here, there is typically a low decay threshold and coupling to the continuum is important. Clear signatures of the halo structure and collective modes was demonstrated, for example, in the suppression of the Coulomb halo in elastic scattering processes.

It has often been observed that a unified approach to the calculation of structure and reactions is something of a holy grail, yet to be achieved, but it is evident that the richness of the structural properties of clustered nuclei form an ideal testing ground. It is clear that the unified approach is important in really understanding the correlations in weakly bound, halo, systems.

The programme provided a series of examples of the power of the measurement of gamma-rays and electromagnetic transition rates as a probe of clustering. The γ-decay transitions ^{10}B populated in the ^{10}B(p,p′γ)^{10}B* reaction by using an array of large volume LaBr3:Ce, provided a strong test of no-core shell model calculations with the N^{3}LO interactions as a probe of molecular analogue states (from ^{10}Be) in ^{10}B (Kuşoğlu). Electromagnetic measurements of exotic neon isotopes using GRETINA, TRIPLEX plunger device and S800 Spectrometer was shown to reveal large E1 strength in ^{27}Ne isotope and enhanced E2 strength in ^{29}Ne isotope (Iwasaki) which need to be better understood.

2.3 Alpha-Particle Gas States

The conference remembered the contributions of Peter Schuck to the ideas of associated with the formation of alpha-particle condensation and the link to the Hoyle-state in ^{12}C, and the THSR wave-function. The THSR was represented by the “R” (Roepke) at the present meeting, with a comparison of with the Thomas-Fermi and shell-model approximation for ^{20}Ne. There was a good discussion of the difference between quarteting and alpha-clustering. This was complimented by some orthogonality condition model, OCM, calculations (Funaki) for both ^{16}O and ^{20}Ne and beyond (Zhou).

The predictions of these approaches are clear in that there continues to be predictions for a special class of states in which the overlap of the wavefunction with a collection of alpha-particle bosons in a relative 0*s*-state is significant.

Algebraic cluster model, ACM, results were also presented (Gai). These provide a different perspective to that of the alpha-gas as the geometry of the structures

play an essential role and the dynamical symmetries generate a unique spectroscopic fingerprint. It was also demonstrated (Fortunato, mentioned already) that the rather classical approach of using superimposed Gaussians to represent cluster structures and then the transition densities for inelastic scattering, that a surprisingly good reproduction of cross section data for ^{12}C and ^{16}O is produced, indicating the importance of the geometric arrangements for the ground and excited states.

The connection between clustering and the shell model has is a popular topic of discussion. The conference heard about cluster and shell coexistence and the U(3)×U(3)>U(3) algebraic structures (Cseh). This formalism provides an understanding of how the Pauli Exclusion Principle plays a role, as per the shell-model, in defining the allowed configurations in the Algebraic Cluster Model. Over the years, there has been much discussion as to the role of anti-symmetrisation, which small separations of the alpha-particles impacts the fermions within. Above the *N*alpha-decay threshold these alpha-particle gas states are weakly bound and have a large spatial extent and thus the gas-like models may provide a realistic representation.

Most cluster structures are two-body in nature, e.g. ^{8}Be (alpha+alpha) or ^{20}Ne (alpha+^{16}O), however the Hoyle-state is 3-body. This additional degree of freedom allows the development of gas-like states, where there is no fixed geometric arrangement. How to deal with such states remains an area of important theoretical focus and, in particular, the role of the continuum needs to be resolved, and the amplification of clustering which appears at threshold (Ploszajczak).

Though the theoretical picture has clarity, the experimental situation, beyond ^{12}C, remains less clear. The work at RCNP to find such states in ^{20}Ne and ^{24}Mg was presented (Kawabata). This shows some emergent evidence for states which may represent analogues of the Hoyle-state, but more remains to be achieved. The increasing excitation energy of *N*alpha-states with mass presents a real challenge. The density of states in which such alpha-gas states are embedded is increasing and hence some fragmentation of strength might be expected, as per the fragmentation of giant resonances. The question as to if the ACM presents the correct description of states in ^{12}C and ^{16}O for the *N*alpha states, as opposed to the gas-like description, still needs to be resolved. The identification of a 3_2^- state associated with the Hoyle-state would seem to be important.

Search for alpha-gas analogues was also explored, e.g. in the case of ^{13}N (Bishop) where the decay to 3alpha+p was observed, in which the proton is added to the 3alpha structure. Adding two neutrons to create ^{14}C creates another analogue (Wei), which may perhaps be related to a gas state or a chain-like structures (Wei, Chen, Di Pietro). Similarly, the disruption of the alpha-cluster provides another series of analogues. The notion of particle-holes was originally conceived by Hafstad and Teller [2], and was explored in the context of the ACM (Gai). The excitation of the alpha-particle is also possible, e.g. alpha-alpha* in ^{8}Be. The fingerprint for such states was explored (Smith). These are indeed interesting areas for future study.

2.4 Clusters in Nucleosynthesis

The coincidence of cluster structures with the cluster-decay threshold (the Ikeda picture) means that for reactions involving alpha-capture processes are strongly influenced, and enhanced by, cluster states. The most celebrated case occurs for the triple alpha-process where the ground-state of ^{8}Be and the Hoyle-state in ^{12}C plays a role. Here nuclear clustering reaches to the stars and to the existence of organic life.

Though the territory of the Hoyle-state is well trodden, there remain some puzzles. In particular the determination of its radiative decay branching ratio has provided some discussion where a recent measurement was in divergence from previous determinations. Results of a high precision measurement were presented (Dell'Aquila) confirming that all was well with the earlier, and established, measurements.

Extrapolating from the affect that the 7.65 MeV resonance has on the formation of ^{12}C in stellar processes, resonances in the Gamow window of ^{24}Mg related to ^{12}C+^{12}C fusion and clustering are also expected to be significant. Indeed, a number of earlier measurements strongly indicate that to be the case. The spin and parity of the 7.65 MeV state is 0^+, which produces a reduced barrier (Coulomb only) for the reaction. Finding 0^+ states in the ^{12}C+^{12}C Gamow window is another holy grail. Measurements of alpha inelastic scattering from ^{24}Mg at iThemba using the K600 spectrometer at 0 degrees were presented (Jenkins) revealing a series of such states. The next step is to understand their structure and the overlap with the ^{12}C+^{12}C channel. State-of-the-art calculations addressing the origin of these states using a microscopic approach were presented (Taniguchi). Work still needs to be done to unify experiment and theory.

Cluster models have historically been used to predict astrophysical reaction rates. An example was presented (Laird) of the comparison between Generator Coordinate Method and Hauser Feshbach calculations which produced orders of magnitude difference for the $\sigma(\alpha,n)/\sigma(\alpha,\gamma)$ cross section for the ^{17}O$(\alpha,\gamma)^{21}$Ne reaction. The challenges in the measurement of the properties of resonances relevant for the capture process were highlighted.

Many-channel, microscopic, calculations, incorporating three-body cluster structures in ^{9}Be and ^{9}B were presented (Lashko) with relevance to the cosmological lithium problem. This again demonstrates the challenge and necessity of unifying structure and reactions to deepen the understanding of reaction rates in cosmological and stellar processes.

The Trojan horse method relies strongly on the clustered nature of the probe nucleus, whereby one cluster is smuggled into a reaction by another. This method has long been used to explore astrophysical reaction processes at energies below the barrier, where cross sections are small, but the phase space factor in the Trojan Horse Method compensates. Measurements of the ^{19}F(p,α), using a deuteron probe (Vukman) and an application of the method for describing nuclear fusion data were presented (Stajkowski).

Neutron-rich nuclei play a role in extreme stellar environments and the progress which has been made, for example in understanding the existence of a low-lying resonance, or not, in the 4n system, tetraneutron, (Yang) allows this learning to be translated to environments such as neutron stars and the equation of state (EOS) and core collapse supernovae (Pais), linking the exotic in nuclear to the exotic in astrophysics.

2.5 Heavy Nuclear Systems

The topic of cluster correlations within heavy nuclei has been a topic of long-standing interest to the cluster community ranging from preformation associated with alpha-decay, heavy-cluster decay (famously ^{14}C) through to the influence of clustering in the ^{208}Pb+alpha system ^{212}Po and as probed through electric dipole transitions. However, the recent measurements of alpha-knockout in Sn isotopes and understanding the spectroscopic-strength variation across the isotopes has rekindled the discussion and the role of theory in describing the appearance in the dilute environment of the nuclear surface (Typel). The concept of a "local alpha strength function" was introduced (Nakatsukasa) to quantify the probability of finding the alpha-particle across the nuclear volume. In a similar spirit, via the energy density functional framework, a "purity function" (Khan) was introduced. The EDF methodology was also used to examine the case for simultaneous two-alpha decay from heavy systems (Khan), with some of the first experimental studies associated with the implantation of $^{220\text{-}222}$Ra and $^{216\text{-}218}$Rn in a thin carbon foil surrounded by silicon detectors to measure the back-to-back decays (Heitz). Understanding the 2alpha decay process relies on mapping the evolution of the system as the alpha-particles are emitted and time-dependent calculations were shown (Khan). This is similar to understanding the fission process. A cluster approach to the fission process, examining the change in the isotopic behaviour around N=152 neutron shell between No and Rf and heavier nuclei, was presented (Rogov).
The understanding of fission involves the details of how nucleons are exchanged between the fragments in the final stages of separation. The inverse of this is what transpires in multinucleon transfer in the collisions between nuclei and indeed the role of pairing and correlations in this process. Measurements of multinucleon transfer in the ^{206}Pb+^{118}Sn system were described (Diklić).

2.6 Experimental Facilities

A feature of the conference was how ingenuity and advances in technology/instrumentation were at the heart of driving experimental sensitivity. Such advances are essential in being able to keep pace with advances in state-of-the-art theory. The conference heard about the TEXas Active Target (TexAT) TPC (Bishop), the use of the ISOLDE Solenoidal Spectrometer (ISS) for measurements of d(^{7}Be, p)^{8}Be (Smith) and a combination of approaches using SOLARIS, a next-generation solenoidal spectrometer, with the two modes of operation; Si-array and Active Target mode used to explore the structure of states in ^{14}C, ^{10}Be, ^{12}Be, ^{16}O and ^{11}B (Ayyad, Beceiro-Novo).

Neutron measurement techniques with SAMURAI+NEBULA setup at RIKEN now is providing insights into neutron-rich nuclei (Marques) and the multi-detector facility of CHIMERA plus FARCOS (Risitano) are now exploring exotic nuclei such as ^{10}Be. Calorimetric techniques for detection of multi-neutron events were also presented (Lihtar). The integration of the AGATA array with the PRISMA spectrometer and the NEDA neutron array is clearly a powerful combination at Legnaro, which promises to deliver much in terms of future insights into hard to measure aspects of nuclear structure (Goasduff). The contributions from new experimental facilities in China, such as Lanzhou, are now providing impressive, and important, contributions to the field. Similarly it was good to see scientific contributions from new facilities such as the Astana cyclotron in Kazakhstan (Nauruzbayev).

2.7 State-of-the-art theory

There is a strong, and long, history of cluster studies being driven by advances in theory. At the outset some of the areas of progress since SOTANCP4 were reviewed (Kimura) alongside experiment (Rogachev). Developments such as Antisymmetrized Molecular Dynamics and Chiral Effective Field Theory type interactions performed for nuclei on the lattice have demonstrated that clustering appears not only in cluster models where it is imposed, but in ab initio type approaches and hence is a fundamental property of nuclear matter. There continues to be an important role for structure theory in fully understanding the nature of the underlying correlations and the structures they produce.

The present meeting saw some impressive illustrations of state-of-the-art theory including the work on alpha-gas states, nuclear astrophysical processes, but also detailed calculations of beta-decay using a 3-body formalism applied to ^{11}Li (Garrido) and the symmetry adapted no core shell model, where states are classified in terms of their shape rather than particle-hole character, (Launey) applied to ^{8}Be ^{15,16}O and astrophysical processes. These latter studies appear to indicate the presence of a low-lying 0^{+} state in ^{8}Be, yet to be observed experimentally.

There is future scope for a joint state-of-the-art experiment + theory programme to use precision measurements to drive a detailed understanding of the emergence of nuclear structure from nucleon-nucleon forces and correlations.

3 A Perspective

The following contribution is a personal perspective regarding the areas which might form future topics of focus. This starts with some general observations.

It is increasingly clear that there are two regions where geometric clustering plays a role and these are well illustrated by the version of the Ikeda diagram below (Fig. 1).

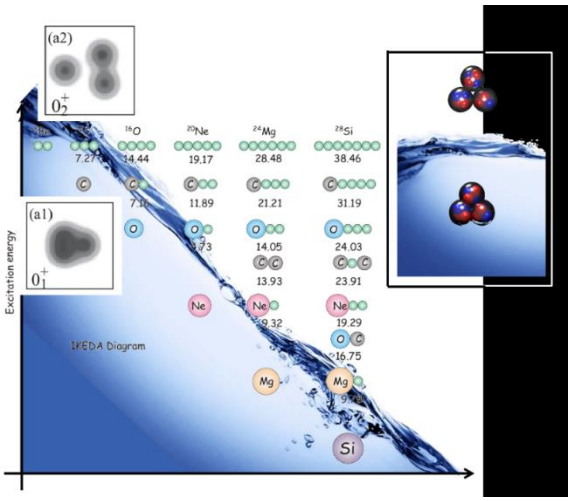

Fig. 1. Cluster structures as depicted by the Ikeda picture. There are two regions of interest; below the water-line and above the water-line.

Water is used to illustrate these two regions following the study of water clusters [3], where within the water bulk geometric clusters of water molecules have a transient existence driven by hydrogen bond and van der Waals type interactions. Above the water surface there are droplets, or vapour, in which the clusterisation is more obvious.

In nuclei, the below the water-line is defined by the first particle decay threshold, which in $N = Z$, alpha-conjugate systems is typically the alpha-decay threshold as depicted in Fig. 1.

3.1 Below the decay threshold

Below the threshold, particles are confined within the potential, and to a certain extent above the top of the potential by the Coulomb and Centrifugal barriers. Here, the nucleons follow solutions of the Schrödinger equation for the associated potential which if approximated by the harmonic oscillator (Fig. 2) show symmetries and a sequence of magic numbers which may be explained in a rather simple way (Fig. 3) as a set of repeated set of degeneracies represented by those of the spherical system [4]. These symmetries are also imprinted on the wave-functions and associated densities linked to the standing waves inside the potential [5]. This would lead to the two alpha structure of ^{8}Be, the triangular 3alpha structure of ^{12}C and the tetrahedral configuration of the ^{16}O ground-state.

The present conference has focussed a great deal on these types of structures, for example via the Algebraic Cluster Model. The lower density labelled 0_1^+ in Fig. 1 corresponds to Antisymmetrized Molecular Dynamics calculations for the ^{12}C ground-state, which again reveals the 3alpha, triangular, structure. These low-lying states reveal cluster symmetries. Using frameworks such as the multi-centre shell model, or multi-centre

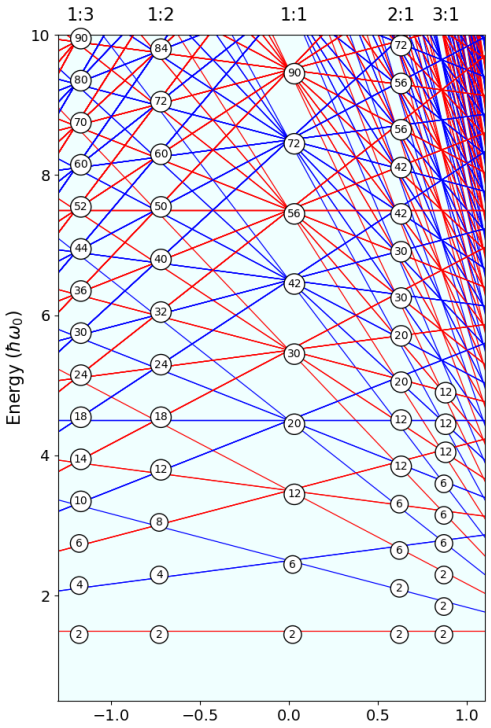

Fig. 2. Energy levels of the deformed harmonic oscillator. The numbers correspond to the degeneracies of the levels at crossing points, which occur for integer axial deformations.

oscillator it is possible to mathematically separate the mean-field configurations into their *N*alpha component parts [5]. It is reasonable to refer to these structures as cluster-structures as they reveal the cluster components, but here the Pauli Exclusion Principle and antisymmetrization plays an important role.

3.2 Above the decay threshold

Many calculations attempting to describe nuclear states above the alpha-particle decay threshold employ the bound-state approximation, in that the wavefunctions describing the system are approximated assuming that the states lie below the top of the potential. This has the benefit that it retains the exponential decaying asymptotic behaviour and avoids needing to consider the role of the continuum.

As discussed in the present conference, the continuum can have a profound impact on the behaviour of systems close to the decay threshold, sometimes referred to as super-radiance [6]. The effect of the continuum on unbound-states/resonances results from the coupling between the two, where the structures associated with the continuum wavefunctions, and the changing phase across the resonance, impact. Moreover, the exponential asymptotics are no-longer present. The lack of confinement within the potential also allows systems to expand and the alpha-gas picture may become more relevant.

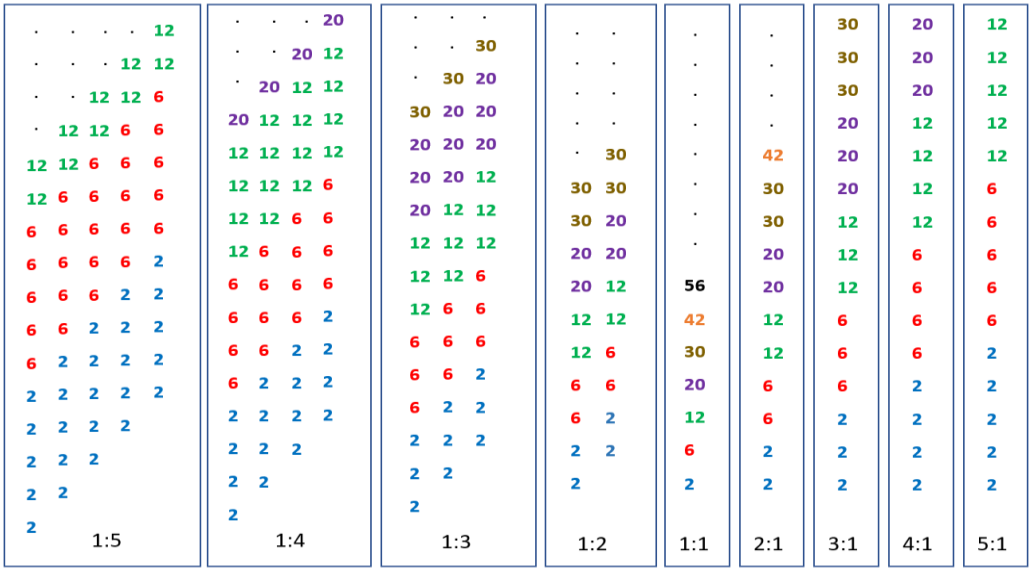

Fig. 3. The symmetries of the deformed harmonic oscillator[7]. The spherical pattern of degeneracies (how many protons or neutron may be placed in orbits) 2, 6, 12, 20, 42….. are seen to repeat at integer deformations of the HO potential.

As yet there is not a proper, holistic, treatment of the development of cluster structures in this regime and appears to be an important frontier which would explore the cross over between the quantum-like geometric behaviour below the decay threshold to one where the clusters may behave more classically where the continuum creates the transition to an open quantum system.

3.3 Topics that emerged from the present conference

There was a huge range of topics explored in the present conference and attendees will have their own perspectives on areas of focus for future research. These might include the following specific, and more general questions

- What is the role of the continuum in the theoretical description of clustering above the decay threshold? Are 3-body (and higher number of clusters) more classical than their mean-field counterparts.
- Is there an experimentally as yet to be measured second low-lying 0^+ state in ^{8}Be?
- Are there analogue cluster states involving excitations of the alpha-cluster in ^{8}Be?
- Can the 0^+ states in ^{24}Mg in the Gamow window be further characterised?
- What is the role of the Pauli Exclusion Principle in the description of states in ^{12}C and ^{16}O and why does the algebraic cluster model and quasi-classical approaches work so well?
- Can ground-state multi-cluster systems be found at the nuclear drip-lines?
- Can a comprehensive understanding of neutron correlations be developed for infinite and finite, neutron-rich, matter?
- Can simultaneous 2alpha decay be found experimentally?
- Do molecular chain states exist in ^{14}C?

It will be interesting to see the progress between now and the next conference.

The author would like to acknowledge the kind hospitality of the conference organisers and recognise the exciting scientific programme generated, whilst wishing the organisers of SOTANCP6 best of luck for the next conference in Japan.

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