

Inclusive rare Λ_b decays to photon

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Abstract. Inclusive $H_b \rightarrow X_s \gamma$ decays, with H_b a beauty baryon, are treated exploiting an expansion up to the third order in the inverse of the b quark mass m_b and to LO in α_s , keeping the dependence on the hadron spin. Double distribution $\frac{d^2\Gamma}{dy d\cos\theta_p}$ is computed for polarized baryons, with $y = 2E_\gamma/m_b$, E_γ the photon energy and θ_p the angle between the baryon spin vector and the photon momentum in the H_b rest-frame. Modifications to the photon polarization asymmetry can probe effects of physics beyond the Standard Model. A systematic method to treat the singular terms in the photon energy spectrum obtained by the OPE is proposed.

1 Introduction

Processes induced at the quark level by the $b \rightarrow s \gamma$ transition occur at loop level in the Standard Model (SM) and are therefore sensitive to physics beyond it [1–3]. Upon integration of the heavy quanta, an effective Hamiltonian is obtained in terms of local operators and Wilson coefficients [4, 5]. Physics beyond the Standard Model can induce new operators and/or modify the Wilson coefficients [6–13].

The radiative $b \rightarrow s$ transition has been analyzed in theory [14]. As for experiment, after the first observation of the $B \rightarrow K^*(892) \gamma$ mode [15] other exclusive processes have been observed [16, 17]. For baryons, the rate and the photon polarization of $\Lambda_b \rightarrow \Lambda \gamma$ have been measured [18, 19], and an upper bound has been put to $\mathcal{B}(\Xi_b^- \rightarrow \Xi^- \gamma)$ [20].

Considering the inclusive mode $H_b \rightarrow X_s \gamma$, several analyses have been carried out for B mesons. In this paper we focus on the case of beauty baryons, in particular Λ_b [21]. Invoking quark-hadron duality, it is possible to exploit the Operator Product Expansion (OPE) and Heavy Quark Effective Theory (HQET) [22] to express inclusive decay widths as an expansion in $1/m_b$. Input quantities are the hadronic matrix elements of local operators.

Several issues in inclusive modes induced by the $b \rightarrow s \gamma$ transition need to be considered. The actual expansion parameter is the inverse of the energy released in the process. In some regions of the phase space it is not small. Signals about the reliability of the method show up as singularities in differential distributions. Singular terms appear at higher orders in the Heavy Quark Expansion (HQE) in form of the delta distribution and its derivatives. The origin of such terms can be related to the Fermi motion of the heavy quark in the decaying hadron, and can be accounted for introducing a shape function which encodes information on the distribution of the b quark residual momentum in the hadron [23–25]. In case of B , measurements have been exploited to constrain its moments [26].

Another source of uncertainty in inclusive $b \rightarrow s \gamma$ processes are the resolved photon contributions [27], that appear at $\mathcal{O}(m_b^{-1})$ [28, 29].

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The photon energy spectrum can be measured above an energy threshold. For $\bar{B}$ the HFLAV Collaboration provides $\mathcal{B}(\bar{B} \rightarrow X_s \gamma) = (3.49 \pm 0.19) \times 10^{-4}$ for $E_\gamma > 1.6 \text{ GeV}$ to be compared to the SM result: $\mathcal{B}(\bar{B} \rightarrow X_s \gamma) = (3.36 \pm 0.23) \times 10^{-4}$ for the same threshold [5].

Baryon modes, such as $\Lambda_b \rightarrow X_s \gamma$ provide the further possibility to investigate observables sensitive to the spin of the decaying hadron. Heavy baryons with a b-quark produced in Z^0 and top-quark decays are expected to have a sizable polarization, as observed at LEP [30–32]. The application of HQE to baryons requires new information, namely the operator matrix elements for specified hadron spin. These have been determined in [33]. Here we also investigate a method to treat the singular terms in the inclusive photon spectrum to reconstruct the Λ_b shape function, a method which can be systematically applied when higher order terms in the $1/m_b$ expansion are computed.

The plan of the paper is the following. Section 2 describes the $b \rightarrow s \gamma$ low-energy Hamiltonian both in SM and in extensions. In section 3 we review the application of the HQE to the inclusive $H_b \rightarrow X_s \gamma$ process with H_b a baryon, keeping the dependence on the baryon spin. The correlation between the photon and Λ_b polarizations is studied. A treatment of the singular terms is discussed in section 4. The last section summarizes.

2 Generalized $b \rightarrow s \gamma$ effective Hamiltonian

The low-energy Hamiltonian governing $b \rightarrow s \gamma$ transition can be written as

$$H_{\text{eff}}^{b \rightarrow s \gamma} = -4 \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \sum_i [C_i(\mu) O_i + C'_i(\mu) O'_i], \quad (1)$$

G_F is the Fermi constant, $V_{tb(s)}$ the CKM matrix elements.¹ Eq. (1) comprises the magnetic penguin operators $O_7 = \frac{e}{16\pi^2} [\bar{s} \sigma^{\mu\nu} (m_s P_L + m_b P_R) b] F_{\mu\nu}$ and $O_8 = \frac{g_s}{16\pi^2} [\bar{s}_\alpha \sigma^{\mu\nu} \left(\frac{\lambda^a}{2}\right)_{\alpha\beta} (m_s P_L + m_b P_R) b_\beta] G_{\mu\nu}^a$, with $P_{R,L} = \frac{1 \pm \gamma_5}{2}$ helicity projectors, α, β colour indices, λ^a the Gell-Mann matrices. $F_{\mu\nu}$ and $G_{\mu\nu}^a$ are the electromagnetic and gluonic field strengths, e and g_s the electromagnetic and strong coupling constants, m_b and m_s the b and s quark mass. Current-current operators $O_1 = (\bar{s}_\alpha \gamma^\mu P_L c_\beta) (\bar{c}_\beta \gamma_\mu P_L b_\alpha)$ and $O_2 = (\bar{s} \gamma^\mu P_L c) (\bar{c} \gamma_\mu P_L b)$ and the QCD penguin operators $O_3 = (\bar{s} \gamma^\mu P_L b) \sum_q (\bar{q} \gamma^\mu P_L q)$, $O_4 = (\bar{s}_\alpha \gamma^\mu P_L b_\beta) \sum_q (\bar{q}_\beta \gamma^\mu P_L q_\alpha)$, $O_5 = (\bar{s} \gamma^\mu P_L b) \sum_q (\bar{q} \gamma^\mu P_R q)$ and $O_6 = (\bar{s}_\alpha \gamma^\mu P_L b_\beta) \sum_q (\bar{q}_\beta \gamma^\mu P_R q_\alpha)$, where the sum runs over $q = \{u, d, s, c, b\}$, are also comprised. The remaining operators, absent in SM, are analogous to the QCD penguins but have a scalar or tensor structure [34]. The primed operators have opposite chirality with respect to the unprimed ones.

In SM the process $b \rightarrow s \gamma$ is described by photon penguin diagrams, with the photon coupled either to the intermediate fermion or to the $W^\pm$, giving rise to the magnetic operator O_7 , the only operator contributing at lowest order in QCD. The renormalization group evolution to the scale $\mu_b \simeq O(m_b)$ also involves O_8 and $O_{1,\dots,6}$. Large logarithms producing a strong enhancement of the rate are generated by the mixing of these operators into O_7 . The anomalous dimension matrix governing the mixing is regularization scheme dependent. Such dependence is taken into account by defining an effective coefficient $C_7^{\text{eff}}(\mu_b)$ which includes contributions of $O_{1,\dots,6}$ [35]. Therefore, O_7 represents the dominant contribution to $b \rightarrow s \gamma$, with the SM Wilson coefficients known at NNLO in QCD [14, 36].

In this study we describe a calculation at LO in α_s so that only the operators O_7 and O'_7 have been considered. Hence, the effective Hamiltonian at the scale μ_b can be written as

$$H_{\text{eff}}^{b \rightarrow s \gamma} = -4 \frac{G_F}{\sqrt{2}} V_{tb} V_{ts}^* \{C_7^{\text{eff}} O_7 + C_7'^{\text{eff}} O_7'\} = -4 \frac{G_F}{\sqrt{2}} \lambda_t \frac{e}{16\pi^2} \sum_{i=7,7'} C_i^{\text{eff}} J_{\mu\nu}^i F^{\mu\nu}, \quad (2)$$

¹Doubly Cabibbo suppressed terms proportional to $V_{ub} V_{us}^*$ have been neglected in (1).

where $\lambda_t = V_{tb} V_{ts}^*$, $J_{\mu\nu}^i = [\bar{s} \sigma_{\mu\nu} (m_s (1 - P_i) + m_b P_i) b]$ and $P_7 = P_R$ and $P_{7'} = P_L$.

3 Inclusive decay width

The differential inclusive decay width can be written as

$$d\Gamma = [dq] \frac{G_F^2 |\lambda_t|^2}{8 m_{H_b}} \frac{\alpha}{\pi^2} \sum_{i,j=7,7'} C_i^{\text{eff}*} C_j^{\text{eff}} W_{MN}^{ij} \mathcal{F}^{MN}, \quad (3)$$

with $[dq] = \frac{d^3 q}{(2\pi)^3 2q^0}$ and $\mathcal{F}^{MN} = -4 q^\nu q^{\nu'} g^{\mu\mu'}$. By the optical theorem, the hadronic tensor W_{MN}^{ij} is related to the discontinuity of the forward scattering amplitude depicted in Fig. 1

$$T_{MN}^{ij} = i \int d^4 x e^{-iq \cdot x} \langle H_b(p, s) | T[J_M^{i\dagger}(x) J_N^j(0)] | H_b(p, s) \rangle, \quad (4)$$

across the cut corresponding to the process $H_b(p, s) \rightarrow X_s(p_X) \gamma(q, \epsilon)$: $W_{MN}^{ij} = \frac{1}{\pi} \text{Im}[T_{MN}^{ij}]$. The range of the invariant mass n_X^2 of the states produced in B and Λ_b decays (with $p_X =$

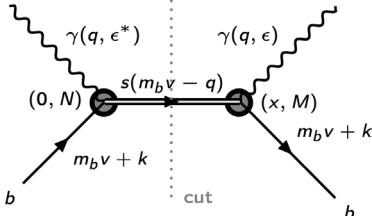


Figure 1. Discontinuity of the forward amplitude across the cut of the radiative process.

$p - q$) is $p_X^2 \in [m_{K^*}^2, m_B^2]$ and $p_X^2 \in [m_\Lambda^2, m_{\Lambda_b}^2]$, respectively. For $m_b \rightarrow \infty$, p_X^2 is almost always large enough to exploit the short-distance limit $x \rightarrow 0$ in Eq. (4), thus allowing a computation of T^{ij} and W^{ij} by an OPE with expansion parameter $1/m_b$ [37, 38], which can be constructed by expressing the hadron momentum $p = m_H v$, where the v the four-velocity, in terms of m_b and of a residual momentum k : $p = m_b v + k$. The QCD b quark field is rescaled $b(x) = e^{-im_b v \cdot x} b_v(x)$, with $b_v(x)$ and satisfies the equation of motion $b_v(x) = (P_+ + \frac{i \not{D}}{2m_b}) b_v(x)$ with $P_+ = \frac{1+\not{v}}{2}$. In terms of $b_v(x)$ Eq. (4) becomes:

$$T_{MN}^{ij} = i \int d^4 x e^{i(m_b v - q) \cdot x} \langle H_b(v, s) | T[\hat{J}_M^{i\dagger}(x) \hat{J}_N^j(0)] | H_b(v, s) \rangle. \quad (5)$$

$\hat{J}_i$ contains the field b_v . The HQE is obtained from

$$T_{MN}^{ij} = \langle H_b(v, s) | \bar{b}_v(0) \bar{\Gamma}_M^i \frac{1}{m_b \not{v} + \not{k} - q - m_s} \Gamma_N^j b_v(0) | H_b(v, s) \rangle, \quad (6)$$

with $\bar{\Gamma}_M^i = \gamma^0 \Gamma_M^{i\dagger} \gamma^0$ and $\Gamma_M^7 = \sigma_{\mu\nu} (m_b P_R + m_s P_L)$, $\Gamma_M^{7'} = \sigma_{\mu\nu} (m_b P_L + m_s P_R)$. Replacing $k \rightarrow iD$, with D the QCD covariant derivative, and considering $|k| \sim \mathcal{O}(\Lambda_{\text{QCD}})$ we have

$$T_{MN}^{ij} = \sum_{n=0}^{+\infty} \langle H_b(v, s) | \bar{b}_v(0) \bar{\Gamma}_M^i (\not{p}_s + m_s) [i \not{D} (\not{p}_s + m_s)]^n \Gamma_N^j b_v(0) | H_b(v, s) \rangle \frac{(-1)^n}{\Delta_0^{n+1}}, \quad (7)$$

where p_s is the s quark momentum and $\Delta_0 = p_s^2 - m_s^2$. Using the trace formalism [39], we can write the n -th term in the series as

$$\begin{aligned} & \langle H_b(v, s) | \bar{b}_v(0) \bar{\Gamma}_M^i (\not{p}_s + m_s) \underbrace{i \not{D} (\not{p}_s + m_s) \dots i \not{D} (\not{p}_s + m_s)}_{n \text{ times}} \Gamma_N^j b_v(0) | H_b(v, s) \rangle = \\ & = \left[\bar{\Gamma}_M^i (\not{p}_s + m_s) \prod_{k=1}^n [\gamma_{\mu_k} (\not{p}_s + m_s)] \Gamma_N^j \right]_{ab} \underbrace{\langle H_b(v, s) | \bar{b}_v(0) i D^{\mu_1} \dots i D^{\mu_n} b_v(0) | H_b(v, s) \rangle}_{(\mathcal{M}^{\mu_1 \dots \mu_n})_{ba}} \end{aligned} \quad (8)$$

with a and b Dirac indices. For any order in the expansion, all the matrix elements $(\mathcal{M}^{\mu_1 \dots \mu_n})_{ba}$ can be written in terms of nonperturbative parameters

$$O(m_b^{-n}) \dots \begin{cases} O(m_b^{-3}) \begin{cases} O(m_b^{-2}) \begin{cases} -2 M_H \hat{\mu}_\pi^2 = \langle H_b | \bar{b}_v i D^\mu i D_\mu b_v | H_b \rangle \\ 2 M_H \hat{\mu}_G^2 = \langle H_b | \bar{b}_v (-i \sigma_{\mu\nu}) i D^\mu i D^\nu b_v | H_b \rangle \\ 2 M_H \hat{\rho}_D^3 = \langle H_b | \bar{b}_v i D^\mu (i v \cdot D) i D_\mu b_v | H_b \rangle \\ 2 M_H \hat{\rho}_{LS}^3 = \langle H_b | \bar{b}_v (-i \sigma_{\mu\nu}) i D^\mu (i v \cdot D) i D^\nu b_v | H_b \rangle \end{cases} \end{cases} \end{cases} \dots \quad (9)$$

$\mathcal{M}^{\mu_1 \dots \mu_n}$ have been derived at $O(m_b^{-3})$ for a polarized baryon in [33], extending the previous results [40–44].

The double distribution, with respect to y and $\cos \theta_P$, using the methods proposed in [39, 45], comprises two terms

$$\frac{d^2 \Gamma}{dy d \cos \theta_P} = \tilde{\Gamma}_1 + \cos \theta_P \tilde{\Gamma}_2. \quad (10)$$

Integrating (10) in $\cos \theta_P$, (θ_P being the angle between the hadron spin s and the photon momentum q), one has $\tilde{\Gamma}_1 = \frac{1}{2} \frac{d\Gamma}{dy}$ and the photon energy spectrum

$$\begin{aligned} \frac{1}{\Gamma_0} \frac{d\Gamma}{dy} = & \left[1 - \frac{\hat{\mu}_\pi^2}{2m_b^2} - \frac{\hat{\mu}_G^2}{2m_b^2} \frac{3+5z}{1-z} - \frac{10\hat{\rho}_D^3}{3m_b^3} \frac{1+z}{1-z} \right] \delta(1-z-y) \\ & + \left[\frac{\hat{\mu}_\pi^2}{2m_b^2} (1-z) - \frac{\hat{\mu}_G^2}{6m_b^2} (3+5z) - \frac{4\hat{\rho}_D^3}{3m_b^3} (1+2z) + \frac{2\hat{\rho}_{LS}^3}{3m_b^3} (1+z) \right] \delta'(1-z-y) \\ & + \left[\frac{\hat{\mu}_\pi^2}{6m_b^2} (1-z)^2 - \frac{\hat{\rho}_D^3}{3m_b^3} (1-z)(1+2z) + \frac{\hat{\rho}_{LS}^3}{6m_b^3} (1-z^2) \right] \delta''(1-z-y) \\ & - \frac{\hat{\rho}_D^3}{18m_b^3} (1-z)^2 (1+z) \delta'''(1-z-y), \end{aligned} \quad (11)$$

where $z = \frac{m_\pi^2}{m_b^2}$, $\Gamma_0 = \frac{\alpha G_F^2 |V_t|^2}{32\pi^4} m_b^5 (1-z)^3 [|C_+^{\text{eff}}|^2 + |C_+'^{\text{eff}}|^2]$, $C_+^{\text{eff}} = C_7^{\text{eff}} + \sqrt{z} C_7'^{\text{eff}}$ and $C_+'^{\text{eff}} = \sqrt{z} C_7^{\text{eff}} + C_7'^{\text{eff}}$. $\tilde{\Gamma}_2$ is given by:

$$\begin{aligned} -\frac{2}{\Gamma_0} \frac{|C_+^{\text{eff}}|^2 + |C_+'^{\text{eff}}|^2}{|C_+^{\text{eff}}|^2 - |C_+'^{\text{eff}}|^2} \tilde{\Gamma}_2 = & \left[1 - \frac{13\hat{\mu}_\pi^2}{12m_b^2} - \frac{3\hat{\mu}_G^2}{4m_b^2} \frac{5+3z}{1-z} - \frac{\hat{\rho}_D^3}{6m_b^3} \frac{31+9z}{1-z} \right] \delta(1-z-y) \\ & + \left[\frac{\hat{\mu}_\pi^2}{2m_b^2} (1-z) - \frac{\hat{\mu}_G^2}{2m_b^2} (3+z) - \frac{2\hat{\rho}_D^3}{m_b^3} (1+z) \right] \delta'(1-z-y) \\ & + \left[\frac{\hat{\mu}_\pi^2}{6m_b^2} (1-z)^2 - \frac{\hat{\rho}_D^3}{3m_b^3} (1-z)(1+2z) \right] \delta''(1-z-y) \\ & - \frac{\hat{\rho}_D^3}{18m_b^3} (1-z)^2 (1+z) \delta'''(1-z-y). \end{aligned} \quad (12)$$

The angular differential distribution also comprises two terms:

$$\frac{d\Gamma(H_b \rightarrow X_s \gamma)}{d \cos \theta_P} = A + B \cos \theta_P, \quad (13)$$

with $A = \frac{1}{2} \Gamma(H_b \rightarrow X_s \gamma)$ and $B = -\frac{\Gamma_0}{2} \frac{|C_+^{\text{eff}}|^2 - |C_+'^{\text{eff}}|^2}{|C_+^{\text{eff}}|^2 + |C_+'^{\text{eff}}|^2} \left[1 - \frac{13\hat{\mu}_\pi^2}{12m_b^2} - \frac{3\hat{\mu}_G^2}{4m_b^2} \frac{5+3z}{1-z} - \frac{\hat{\rho}_D^3}{6m_b^3} \frac{31+9z}{1-z} \right]$.

Integrating in $\cos \theta_P$, the inclusive $H_b \rightarrow X_s \gamma$ width is given by

$$\Gamma(H_b \rightarrow X_s \gamma) = \Gamma_0 \left[1 - \frac{\hat{\mu}_\pi^2}{2m_b^2} - \frac{\hat{\mu}_G^2}{2m_b^2} \frac{3+5z}{1-z} - \frac{10\hat{\rho}_D^3}{3m_b^3} \frac{1+z}{1-z} \right]. \quad (14)$$

For $C'_7 \rightarrow 0$ the SM result is recovered.

Another interesting observable is the photon polarization asymmetry $A_P(\cos \theta_P)$ in $b \rightarrow s \gamma$ transition that quantifies the difference between left and right handed photon produced (the definition can be found in [21]). In SM the photon polarization asymmetry is $A_P(\cos \theta_P) \simeq -1$ for almost all $\cos \theta_P$, it increases only for $\cos \theta_P \rightarrow 1$, see Fig. 2 (left panel). Physics beyond SM can produce a deviation of the asymmetry from the SM value with the largest effect for $\cos \theta_P \simeq 1$. Considering both C_7 and C'_7 real and varying $C_7^{\text{eff}}/C_7^{\text{eff}} \in [-0.3, 0.3]$ we obtain the asymmetry in Fig. 2 (right panel) versus $C_7^{\text{eff}}/C_7^{\text{eff}}$ for selected values of $\cos \theta_P$.

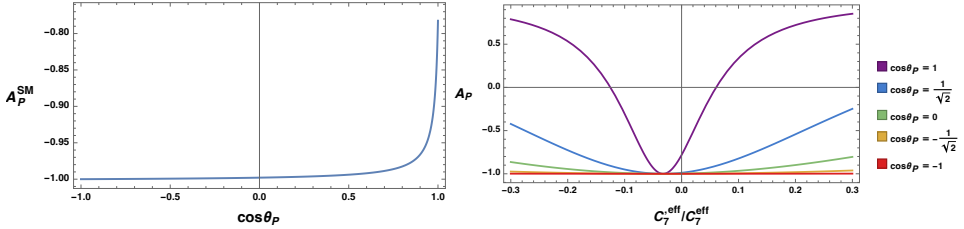


Figure 2. Photon polarization asymmetry $A_P(\cos \theta_P)$ versus $\cos \theta_P$ in SM (left) and photon polarization asymmetry $A_P(\cos \theta_P)$ for several values of $\cos \theta_P$, varying $C_7^{\text{eff}}/C_7^{\text{eff}}$ (right).

4 Treatment of the singular terms

The spectrum obtained by the short-distance OPE does not account for possible smearing close to the end point region. In this region, the Fermi motion of the b quark and its soft interactions with the light degrees of freedom in the hadron are relevant. It is possible to resum the singular terms defining the spectral function [23–25, 38, 46]

$$S_s(y) = \sum_{n=0}^{\infty} \frac{M_n}{n!} \delta^{(n)}(1 - z - y), \quad (15)$$

and introducing the quantity $f(k_+)$ as²

$$S_s(y) = \int dk_+ \delta\left(1 - y - z + \frac{k_+}{m_b}\right) [f(k_+) + O(m_b^{-1})]. \quad (16)$$

The photon energy spectrum is obtained by the convolution [24]

$$\frac{d\Gamma}{dy} = \int dk_+ f(k_+) \frac{d\Gamma^*}{dy}. \quad (17)$$

In $\frac{d\Gamma^*}{dy}$ the b quark mass m_b is replaced by $m_b^* = m_b + k_+$, an exact substitution at tree level. For $k_+ \in [-m_b, m_{H_b} - m_b]$, replacing $m_b \rightarrow m_b^*$ in the variable y , one finds ($m_s = 0$ to simplify the discussion): $y \rightarrow \frac{2E_\gamma}{(m_b + k_+)}$. Hence, for $k_+^{\text{max}} = m_{H_b} - m_b$ the maximum photon energy is $E_\gamma = \frac{m_{H_b}}{2}$ (physical end-point). The shape function provides an interpretation of the singular terms in the photon energy spectrum obtained in the previous section. Writing Eq. (11) as

$$\frac{1}{\Gamma} \frac{d\Gamma}{dy} = \sum_{n=0}^3 \frac{M_n}{n!} \delta^{(n)}(1 - z - y), \quad (18)$$

²Perturbative corrections to the shape function and its moments are discussed in [47, 48].

with Γ in Eq. (14) and the moments

$$\begin{aligned} M_0 &= 1, \quad M_1 = \frac{\hat{\mu}_\pi^2}{2m_b^2} (1-z) - \frac{\hat{\mu}_G^2}{6m_b^2} (3+5z) - \frac{4\hat{\rho}_D^3}{3m_b^3} (1+2z) + \frac{2\hat{\rho}_{LS}^3}{3m_b^3} (1+z), \quad (19) \\ M_2 &= \frac{\hat{\mu}_\pi^2}{3m_b^2} (1-z)^2 - \frac{2\hat{\rho}_D^3}{3m_b^3} (1-z)(1+2z) + \frac{\hat{\rho}_{LS}^3}{3m_b^3} (1-z^2), \quad M_3 = -\frac{\hat{\rho}_D^3}{3m_b^3} (1-z)^2 (1+z). \end{aligned}$$

Each moment M_n in Eqs. (19), has an expansion $M_n = \sum_{k=n}^{\infty} \frac{M_{n,k}}{m_b^k}$ [24]. M_n in (15) are related to the moments of the photon energy spectrum: $\langle y^k \rangle = \frac{1}{\Gamma} \int_0^{1-z} dy y^k \frac{d\Gamma}{dy}$. Using (18) we have:

$$\langle y^k \rangle = \sum_{n=0}^{\infty} \frac{M_n}{n!} \int_0^{1-z} dy y^k \delta^{(n)}(1-z-y) = \sum_{j=0}^k \binom{k}{j} (1-z)^{k-j} M_j, \quad (20)$$

$$\langle y \rangle = (1-z) + M_1, \quad \langle y^2 \rangle = (1-z)^2 + 2(1-z)M_1 + M_2, \quad \sigma_y^2 = \langle y^2 \rangle - \langle y \rangle^2. \quad (21)$$

The moments of the measured photon energy spectrum can be used to determine the HQET parameters as well as the $\cos \theta_P$ distribution (13).

Let us describe the effect of the Fermi motion. Each order in the $1/m_b$ expansion, the photon energy spectrum obtained by the OPE corresponds to a monochromatic line. At LO, the line is at $y = 1-z$, the next terms correspond to a displacement of this position. The convolution (17) with the shape function provides the smearing of the spectrum.

The shape function is a nonperturbative quantity to be determined by methods such as lattice QCD, or which must be suitably parametrized. We adopt a different point of view. Including infinite terms, the sum in Eq. (18) coincides with $S_s(y)$ in (15). In the sum, the first term corresponds to a monochromatic line at the zero of the δ function, with $\langle y \rangle = 1-z$ and $\sigma_y^2 = 0$, the LO results in Eqs. (21). We write $\delta(b-y) = \lim_{\sigma_y \rightarrow 0} \frac{1}{\sqrt{2\pi}\sigma_y} e^{-\frac{(b-y)^2}{2\sigma_y^2}}$, with $b = (1-z) = \langle y \rangle_{LO}$ and σ_y^2 at each order in $1/m_b$, starting from $1/m_b^2$. For $m_b \rightarrow \infty$ the condition $\sigma_y \rightarrow 0$ reproduces the partonic result. Our ansatz consists in substituting

$$S_s(y) = \sum_{n=0}^{\infty} \frac{M_n}{n!} \delta^{(n)}(1-z-y) \quad \rightarrow \quad S_s(y) = \sum_{n=0}^{\infty} \frac{M_n}{n!} (-1)^n \frac{d^n}{dy^n} \frac{1}{\sqrt{2\pi}\sigma_y} e^{-\frac{(b-y)^2}{2\sigma_y^2}}. \quad (22)$$

Using the representation of the Hermite polynomials $H_n(x) = (-1)^n e^{x^2} \frac{d^n}{dx^n} e^{-x^2}$, we obtain:

$$S_s(y) = \frac{1}{\sqrt{2\pi}\sigma_y} \sum_{n=0}^{\infty} \frac{M_n}{n!} \left(-\frac{1}{\sqrt{2}\sigma_y} \right)^n e^{-\frac{(b-y)^2}{2\sigma_y^2}} H_n \left(\frac{b-y}{\sqrt{2}\sigma_y} \right). \quad (23)$$

Denoting by $\langle y^k \rangle_N$ the moments computed using this expression for the spectral function,

$$\langle y^k \rangle_N = \int_0^{y_{\max}} dy y^k S_s(y), \quad \text{where} \quad \lim_{\sigma_y \rightarrow 0} \langle y^k \rangle_N = \langle y^k \rangle, \quad (24)$$

with $\langle y^k \rangle$ in (20). While b is fixed to the LO result for $\langle y \rangle$, σ_y can be determined at an arbitrary order in the $1/m_b$ expansion. At odds with most models proposed for the shape function, by construction the ansatz (23) can include all moments M_n once they are computed.

In Fig. 3 we plot the spectral function obtained at LO, $O(m_b^{-2})$ and $O(m_b^{-3})$. The maximum photon energy is larger than the partonic result $y = 1-z$. In the same figure we plot the shape function obtained from (16) at $O(m_b^{-2})$ and $O(m_b^{-3})$. As a consequence of broadening the spectrum through the substitution in (22), there is a tail exceeding the physical end-point $y_{\max} \simeq \frac{m_{\Lambda_b}}{m_b} (1-z)$, and a tail in the shape function exceeding $k_+^{\max} = m_{\Lambda_b} - m_b$. This is a spurious effect of the truncation. When higher orders in the HQE are included, the area below such tails approaches to zero.

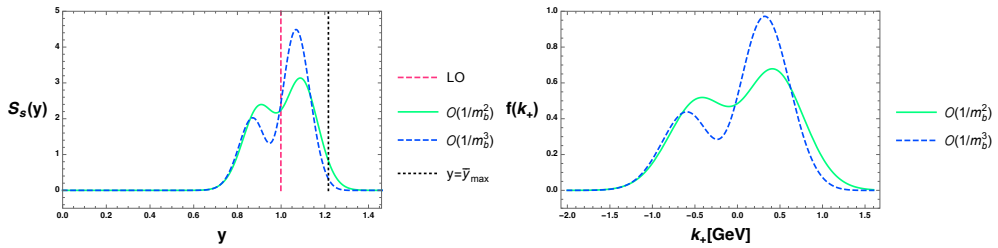


Figure 3. Function $S_s(y)$ (left) and shape function $f(k_+)$ (right) obtained using Eq. (23) up to $n = 3$.

5 Conclusions

The HQE has been exploited to compute the inclusive decay width induced by the $b \rightarrow s \gamma$ transition for a beauty baryon. The differential width in the photon energy and in $\cos \theta_P$ allows to construct new observables with respect to mesons. The calculation has been carried out at $O(m_b^{-3})$ for non-vanishing m_s , using the baryon matrix elements determined in [33]. For the singular terms appearing in the spectrum we have proposed a treatment that can be systematically improved including higher order terms in the expansion.

I thank P. Colangelo and F. De Fazio for collaboration. This study has been carried out within the INFN project (Iniziativa Specifica) SPIF.

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