

Impact of longitudinally and transversely polarized beams on probing CP -violating HZZ interactions at the ILC

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Abstract. We study possible CP-violation effects of the Higgs to Z-boson coupling at a future e^+e^- collider, e.g. the International Linear Collider (ILC). We find that the azimuthal angular distribution of the muon pair, produced by $e^+e^- \rightarrow HZ \rightarrow H\mu^+\mu^-$, can be sensitive to such a CP-violation effect when the initial beams are transversely polarized. Based on this angular distribution, we construct a CP sensitive asymmetry and obtain this asymmetry by Whizard simulation. By comparing the SM prediction with 2σ range of this asymmetry, we estimate the limit of the CP-odd coupling in the HZZ interaction.

1 Introduction

In order to explain the matter anti-matter asymmetry in the universe [1], baryogenesis has been proposed [2], which requires CP-violation. In particular, a possible CP-violation source can be introduced in the Higgs sector, which is generated by the Two-Higgs-Doublet Model (2HDM) with spontaneous CP violation [3]. In this model, the CP-even 125 GeV Higgs boson gets a CP-odd admixture and the Higgs to fermions couplings are CP-violating. So far, such a baryogenesis and CP-violation test has been exploited by [4] via $Ht\bar{t}$ interaction at LHC, and via $H\tau\tau$ at Higgs factories [5]. On the other hand, the CP-violating Higgs to fermions couplings can contribute a CP-violating Higgs to gauge bosons couplings at one-loop level. Therefore, the HVV interaction can be possibly CP-violating as well.

Currently, the LHC experiments, ATLAS [6] and CMS [7, 8] perform the measurement for the HVV coupling via $H \rightarrow 4\ell$ decay, and yield a limit on CP-odd HVV coupling, which can be interpreted as $(\tilde{c}_{ZZ})_{\text{CMS}} \sim [-1.63, 4.66]$ and $(\tilde{c}_{ZZ})_{\text{ATLAS}} \sim [-1.2, 1.75]$. On the other hand, one can probe directly the CP-violating HVV couplings at e^+e^- colliders (CEPC [9] and ILC), since a CP-violating HVV coupling can lead to deviations in the Higgs strahlung process at e^+e^- colliders. In addition, initial polarization of both beams can be applied at the ILC [10], where particularly the transverse polarization can be used to construct new CP sensitive observables [10, 11].

In this work, which is based on [12], we will focus on the Higgs strahlung process at the ILC with a center of mass energy of 250 GeV and apply transversely polarized electron-positron beams. In such a case, we construct CP sensitive observables to probe the CP-odd

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HZZ coupling. Eventually, we try to determine the limit on the CP-odd HZZ coupling and compare with other studies.

2 Theoretical framework

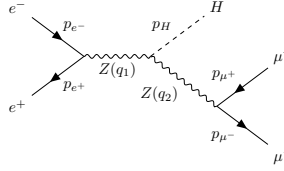
The Higgs to Z boson interaction can be expressed by the following effective lagrangian [13]:

$$\mathcal{L}_{\text{EFF}} = c_{\text{SM}} Z_\mu Z^\mu H - \frac{c_{HZZ}}{v} Z_{\mu\nu} Z^{\mu\nu} H - \frac{\tilde{c}_{HZZ}}{v} Z_{\mu\nu} \tilde{Z}^{\mu\nu} H, \quad (1)$$

where $\tilde{c}_{HZZ}$ denotes the loop-induced CP-odd coupling, and can be parameterised by the mixing angles, so that $\tilde{c}_{HZZ} \propto \sin \xi_{CP}$. Concerning the polarization of the initial electron and positron beams, one defines a projection operator, which is given by:

$$\frac{1}{2}(1 - \mathbf{P} \cdot \boldsymbol{\sigma}) = \frac{1}{2}(\delta_{\lambda\lambda'} - P^a \sigma^a_{\lambda\lambda'}) = \frac{1}{2} \begin{pmatrix} 1 - P_3 & P_1 - iP_2 \\ P_1 + iP_2 & 1 + P_3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 - f \cos \theta_P & f \sin \theta_P e^{-i\phi_P} \\ f \sin \theta_P e^{i\phi_P} & 1 + f \cos \theta_P \end{pmatrix}, \quad (2)$$

where f is the polarization fraction. The Higgs strahlung $e^+ e^- \rightarrow ZH$ is the dominant Higgs production process for an $e^+ e^-$ collider operating at $\sqrt{s} = 250$ GeV. By taking the polarization of the initial beams into account, the spin-density matrix of the Higgs strahlung $\rho_{\lambda_r \lambda_u}$ can be derived, see the following diagram,



where the λ_r, λ_u are the spin indices of the initial electron and positron. Eventually, we can obtain the spin-density matrix by applying the Bouchiat-Michel formula [14, 15], and the spin density matrix is given by

$$\begin{aligned} \rho^{ii'}(e^+ e^- \rightarrow ZH) &= \frac{1}{2}(\delta_{\lambda_r \lambda'_r} + P_-^m \sigma^m_{\lambda_r \lambda'_r}) \frac{1}{2}(\delta_{\lambda_u \lambda'_u} + P_+^n \sigma^n_{\lambda_u \lambda'_u}) M^i_{\lambda_r \lambda_u} M^{*i'}_{\lambda'_r \lambda'_u} \\ &= (1 - P_-^3 P_+^3) A^{ii'} + (P_-^3 - P_+^3) B^{ii'} + \sum_{mn}^{1,2} P_-^m P_+^n C_{mn}^{ii'}. \end{aligned} \quad (3)$$

Furthermore, we introduced the CP-odd operator for the HZZ interaction of Eq. (1), and obtain the following form of the total amplitude squared:

$$\begin{aligned} |\mathcal{M}|^2 &= (1 - P_-^3 P_+^3)(\cos^2 \xi_{CP} \mathcal{A}_{\text{CP-even}} + \sin 2\xi_{CP} \mathcal{A}_{\text{CP-odd}} + \sin^2 \xi_{CP} \tilde{\mathcal{A}}_{\text{CP-even}}) \\ &+ (P_-^3 - P_+^3)(\cos^2 \xi_{CP} \mathcal{B}_{\text{CP-even}} + \sin 2\xi_{CP} \mathcal{B}_{\text{CP-odd}} + \sin^2 \xi_{CP} \tilde{\mathcal{B}}_{\text{CP-even}}) \\ &+ \sum_{mn}^{1,2} P_-^m P_+^n (\cos^2 \xi_{CP} C_{\text{CP-even}}^{mn} + \sin 2\xi_{CP} C_{\text{CP-odd}}^{mn} + \sin^2 \xi_{CP} \tilde{C}_{\text{CP-even}}^{mn}). \end{aligned} \quad (4)$$

Note, that the CP-invariant parts with $\cos^2 \xi_{CP}$ and $\sin^2 \xi_{CP}$ are both CP-conserving, while the CP-mixing terms with $\sin 2\xi_{CP}$ violate the CP symmetry. Particularly, the CP-violating terms $\mathcal{A}_{\text{CP-odd}}$, $\mathcal{B}_{\text{CP-odd}}$ and $C_{\text{CP-odd}}^{mn}$ can be extracted by the triple-products, which are given by

$$\mathcal{A}_{\text{CP-odd}}, \mathcal{B}_{\text{CP-odd}} \propto \epsilon_{\mu\nu\alpha\beta} [p_{e^-}^\mu p_{e^+}^\nu p_{\mu^+}^\alpha p_{\mu^-}^\beta] \propto (\vec{p}_{\mu^+} \times \vec{p}_{\mu^-}) \cdot \vec{p}_{e^-}, \quad (5)$$

$$C_{\text{CP-odd}}^{mn} \propto \epsilon_{\mu\nu\rho\sigma} [(p_{e^-} + p_{e^+})^\mu p_{\mu^+}^\nu p_{\mu^-}^\rho s_{e^-}^\sigma] \propto (\vec{p}_{\mu^+} \times \vec{p}_{\mu^-}) \cdot \vec{s}_{e^-}. \quad (6)$$

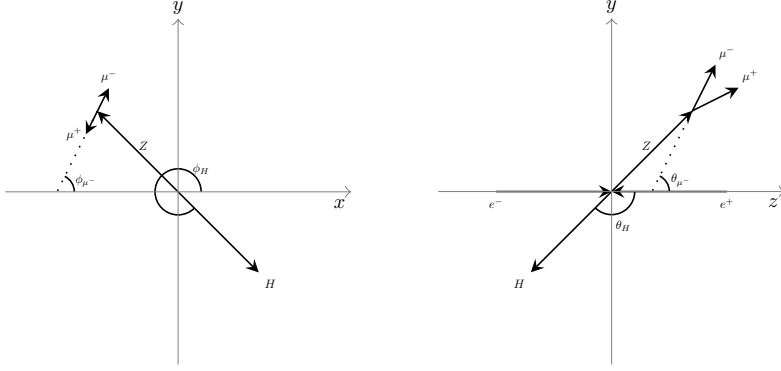


Figure 1. The coordinate system in the center of mass frame for the $e^+e^- \rightarrow H\mu^+\mu^-$ process, the left plot is the $x-y$ plane and the right plot is the $y-z$ plane. The direction of the electron beam is defined as the z -axis direction $\vec{n}_z = \vec{p}_{e^-}/|\vec{p}_{e^-}|$, while we choose the direction of the electron polarization as the y -axis $\vec{n}_y = \vec{s}_{e^-}/|\vec{s}_{e^-}|$. Thus, the x -axis can be defined by the cross product $\vec{n}_x = (\vec{s}_{e^-} \times \vec{p}_{e^-})/|\vec{s}_{e^-} \times \vec{p}_{e^-}|$

where $\vec{s}_{e^-}$ is the spin vector of electron.

The triple-product is the azimuthal-angle difference between the $\mu^+\mu^-$ plane and the spin of the electron. Therefore, defining the orientation of the azimuthal plane by fixing the direction of electron transverse polarization, the $C_{mm}^{\text{CP-mix}}$ depends directly on the azimuthal angle of the final state μ^- (see Figs. 1). Regarding the triple products in the CP-odd amplitude terms, one can define two CP-odd observables

$$O_{CP}^T = \cos\theta_H \sin 2\phi_{\mu^-}, \quad O_{CP}^{UL} = \cos\theta_{\mu^-} \sin(\phi_{\mu^-} - \phi_H). \quad (7)$$

Hence, we can define the asymmetries by the CP-odd observables:

$$\mathcal{A}_{CP} = \frac{1}{\sigma_{\text{tot}}} \int \text{sgn}(O_{CP}) d\sigma = \frac{N(O_{CP} < 0) - N(O_{CP} > 0)}{N(O_{CP} < 0) + N(O_{CP} > 0)}, \quad (8)$$

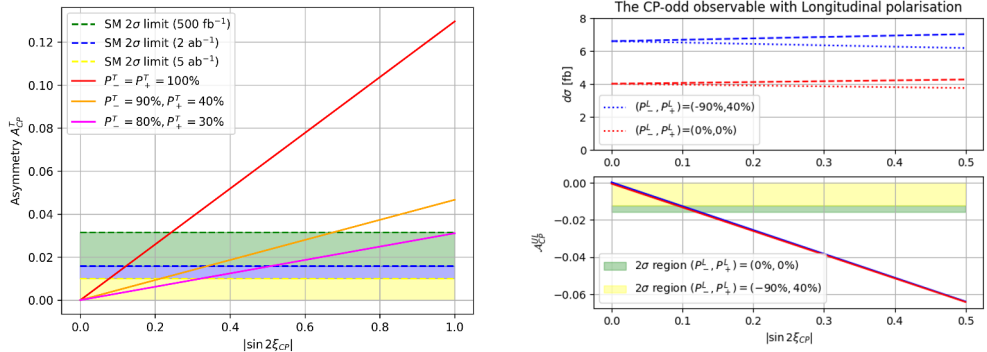
where N denotes the corresponding number of events. Since the SM is CP conserving for the neutral current, the SM background for this asymmetry is negligible. However, the number of events fluctuates statistically leading to the uncertainty of this asymmetry, which is based on a binomial distribution and given by $\Delta\mathcal{A}_{CP} = \sqrt{\frac{1-\mathcal{A}_{CP}^2}{N_{\text{tot}}}}$. For the estimation, we assume that the efficiency is close to 100% and all Higgs decays are included.

3 Determination of CP-violation at ILC

In order to study the ability of measuring CP-properties, we assume that the total cross-section is fixed to the SM value, while the CP properties are varied by the CP-mixing angle. In such a scenario, the dependence of the asymmetries $\mathcal{A}_{CP}^T$ and $\mathcal{A}_{CP}^{UL}$ are presented in Fig. 2, and both asymmetries linearly depend on $\sin 2\xi_{CP}$. For the left panel, the asymmetry $\mathcal{A}_{CP}^T$, constructed by transverse polarization, can be enhanced by a larger polarization degree. Hence, the larger transverse polarization degree would be helpful for the determination of the CP properties via $\mathcal{A}_{CP}^T$. In contrast, the right panel demonstrates that the size of $\mathcal{A}_{CP}^T$ is basically independent of the degree of polarization. However, the longitudinal polarization can enhance the total

cross-section and suppress the statistical uncertainty. Consequently, the precision of $\sin 2\xi_{CP}$ can be improved as well by increasing the degree of longitudinal polarization.

By comparing the asymmetry values with the SM 2σ upper bounds, one can determine the limit of the CP-mixing angle $\sin 2\xi_{CP}$ and obtain the numbers in Tab. 1. In the left panel, the value of $\mathcal{A}_{CP}^T$ is enhanced by larger degree of transverse polarization, while the right panel shows that the value of $\mathcal{A}_{CP}^{UL}$ is independent on degree of longitudinal polarization. However, the upper plot of the right panel demonstrate that the cross-section can be enhanced by the longitudinal polarization. Therefore, the precision of using $\mathcal{A}_{CP}^{UL}$ with longitudinally polarized beams can be roughly at 0.1, which is much better than using $\mathcal{A}_{CP}^T$ alone (around 0.3 for $(P_-^T, P_+^T) = (90\%, 40\%)$), since the $\mathcal{A}_{CP}^T$ depends highly on the degree of polarization, and the positron beams can not be strongly polarized at 250 GeV. However, one can actually measure both $\mathcal{A}_{CP}^T$ and $\mathcal{A}_{CP}^{UL}$ simultaneously, when the initial beams are transversely polarized. Therefore, we can combine the two observables for this case and the sensitivity to the CP-mixing angle can be improved as well. The combined results are also presented in Tab. 1, which shows that the combined observables can lead to a similar precision as using longitudinal polarization.



(a) The asymmetry $\mathcal{A}^T$ constructed by O_{CP}^T . The red solid line corresponds to the completely polarized beams $(P_-^T, P_+^T) = (100\%, 100\%)$, while the orange line and magenta line demonstrate the asymmetries with $(P_-^T, P_+^T) = (90\%, 40\%)$ polarized beams and $(P_-^T, P_+^T) = (80\%, 30\%)$ polarized beams, respectively. The blue and green region are the region below the 2σ limit of the SM CP-conserving case for 500 fb^{-1} and 2000 fb^{-1} , respectively.

(b) The upper plot is the cross section in the signal regions with different signs of O_{CP}^{UL} . The lower plot is the asymmetry $\mathcal{A}^{UL}$ constructed by O_{CP}^{UL} . The red lines in both plots are for unpolarized case, and the blue lines are for $(P_-^L, P_+^L) = (-90\%, 40\%)$ longitudinal polarization. In the lower plot, the yellow and green regions are the 2σ regions of SM CP-conserving cases with unpolarized beams and $(P_-^L, P_+^L) = (-90\%, 40\%)$ longitudinal polarization beams, respectively.

Figure 2. The analytical results of the asymmetries from Eq. (8) with the statistical uncertainties of the asymmetries, where the total cross section is fixed for varying $|\sin 2\xi_{CP}|$.

Furthermore, we can explore another scenario, which fixes the SM contribution and the total cross-section is varied by varying the CP-odd coupling. In this scenario, we can compare the ability of determining the CP-odd coupling with the other experiments and analysis. In Tab. 2, we present the CP-odd coupling determination limits, where the number of ILC 250 GeV using polarization is given by the case with transverse polarization, since the combined results using transverse polarization have roughly the same sensitivity to the CP-violation as the results using only longitudinal polarization with the same degree of polarization. The

results of f_{CP}^{HZZ} and $\tilde{c}_{ZZ}$ can be matched to the $\tilde{c}_{HZZ}$ interpretation by Eqs. (9) and (10)

$$f_{CP}^{HZZ} = 1/\left(1 + \frac{1}{[\tilde{c}_{HZZ}]^2 \times 0.153}\right) \text{sgn}(\tilde{c}_{HZZ}). \quad (9)$$

$$\tilde{c}_{HZZ} = \frac{m_Z^2}{v^2} \tilde{c}_{ZZ}. \quad (10)$$

We find that the result using polarization improves on the sensitivity to the CP-odd coupling compared with the unpolarized study at CEPC.

(P_-, P_+)	$\mathcal{L}$ [ab ⁻¹]	sin $2\xi_{CP}$ limit	
Observables		$\mathcal{A}_{CP}^T$	Combine $\mathcal{A}_{CP}^T$ & $\mathcal{A}_{CP}^{UL}$
Transverse polarization			
(80%, 30%)	2.0	[-0.50, 0.53]	[-0.113, 0.125]
(80%, 30%)	5.0	[-0.36, 0.36]	[-0.068, 0.079]
(90%, 40%)	2.0	[-0.33, 0.34]	[-0.118, 0.110]
(90%, 40%)	5.0	[-0.23, 0.22]	[-0.066, 0.077]
(100%, 100%)	5.0	[-0.082, 0.069]	[-0.056, 0.051]
Longitudinal polarization			
(-80%, 30%)	2.0		[-0.119, 0.082]
(-80%, 30%)	5.0		[-0.066, 0.063]
(-90%, 40%)	2.0		[-0.085, 0.106]
(-90%, 40%)	5.0		[-0.059, 0.062]
(-100%, 100%)	5.0		[-0.047, 0.053]

Table 1. The summary table for 2σ limit of the CP-mixing angle sin $2\xi_{CP}$ with center-of-mass energy 250 GeV. The column of $\mathcal{A}_{CP}^T$ shows, the results when only using the observable $\mathcal{A}_{CP}^T$ with transverse polarization, while the column of $\mathcal{A}_{CP}^{UL}$ corresponds to the results using longitudinal polarization. Note that the column of “Combine $\mathcal{A}_{CP}^T$ & $\mathcal{A}_{CP}^{UL}$ ” still uses the experimental set up of transverse polarization but measures the two observables.

Experiments Processes $\sqrt{s}$ [GeV] Luminosity [fb ⁻¹] ($ P_- , P_+ $)	ATLAS[6] $H \rightarrow 4\ell$ 13000 139	CMS[16] $H \rightarrow 4\ell$ 13000 137	HL-LHC[17] $H \rightarrow 4\ell$ 14000 3000	CEPC[9] HZ 240 5600	CLIC[18] W-fusion 3000 5000	ILC [19] Z-fusion 1000 8000	ILC $HZ, Z \rightarrow \mu^+\mu^-$ 250 5000 (90%, 40%)
$\tilde{c}_{HZZ} (\times 10^{-2})$							
68% C.L. (1 σ)limit	[-5.1, 16.6]	[-7.2, 15.2]	[-4.5, 4.5]	[-0.8, 0.8]	[-1.6, 1.6]	[-0.4, 1.6]	[-0.4, 0.7]
$\tilde{c}_{HZZ} (\times 10^{-2})$							
95% C.L. (2 σ)limit	[-16.4, 24.0]	[-22.4, 63.9]	[-9.1, 9.1]	[-1.5, 1.5]	[-3.3, 3.3]	[-1.4, 2.7]	[-1.1, 1.0]
$f_{CP}^{HZZ} (\times 10^{-3})$							
68% C.L. (1 σ)limit	[-40, 420]	[-80, 350]	[-30, 30]	[-1.04, 1.04]	[-4.1, 4.1]	[-0.26, 4.1]	[-0.26, 0.67]
$f_{CP}^{HZZ} (\times 10^{-5})$							
95% C.L. (2 σ)limit	[-410, 870]	[-760, 5880]	[-127, 127]	[-3.92, 3.92]	[-16.66, 16.66]	[-3.14, 10.81]	[-1.85, 1.53]
$\tilde{c}_{ZZ}$							
68% C.L. (1 σ)limit	[-0.37, 1.21]	[-0.53, 1.10]	[-0.33, 0.33]	[-0.06, 0.06]	[-0.12, 0.12]	[-0.03, 0.12]	[-0.03, 0.05]
$\tilde{c}_{ZZ}$							
95% C.L. (2 σ)limit	[-1.2, 1.75]	[-1.63, 4.66]	[-0.66, 0.66]	[-0.12, 0.12]	[-0.24, 0.24]	[-0.10, 0.19]	[-0.08, 0.07]

Table 2. Summary of the limits of $\tilde{c}_{HZZ}$, f_{CP}^{HZZ} and $\tilde{c}_{ZZ}$ at 68% and 95% C.L., where the results are obtained from both current LHC measurements and future colliders analysis, including HL-LHC, CEPC, ILC and CLIC.

4 Conclusion

In this work, we investigate the CP-violation effects in the process $e^+e^- \rightarrow HZ \rightarrow H\mu^+\mu^-$ with transversely polarized initial beams. Based on the azimuthal angular distribution of

the final state muons, we constructed two CP sensitive observables and exploited their dependence on the CP-violation parameters. We can measure the two observables simultaneously when the initial beams are only transversely polarized, and the two observables can be combined to improve the sensitivity to CP properties. On the other hand, if the beams are longitudinally polarized, the statistical uncertainty can be reduced by enhancing the total cross-section and the sensitivity can be improved as well. Eventually we determine the CP-odd coupling $|\tilde{c}_{HZZ}| \lesssim -0.01$ at 95% C.L. for $(P_{e^-}^T, P_{e^+}^T) = (90\%, 40\%)$ polarization and 5000 fb^{-1} at ILC 250 GeV.

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