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RPA correction to the optical potential

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Abstract. In studies of nucleon elastic scattering, a correction to the microscopic optical potential built from Melbourne g-matrix was found to be necessary at low nucleon incident energy [1,2]. Indeed, at energies lower than 60 MeV, the absorption generated from Melbourne g-matrix is too weak within 25%. Coupling to collective excited states of the target nucleus are not included in the g-matrix and could explain the missing absorption. We propose to calculate this correction to the optical potential using the Gogny D1S effective nucleon-nucleon interaction in the coupling to excited states of the target. We use the Random Phase Approximation (RPA) description of the excited states of the target with the same interaction.

1 Introduction

Nucleon can be used to probe the structure of nuclei through a scattering experiment. We are interested in the theoretical description of nucleon elastic scattering from a target.

The advent of new radioactive beams facilities makes nucleon scattering a key experiment. Indeed, due to their instability, those nuclei are studied in inverse kinematics, for example with a proton target [3]. For this reason it is important to have a good theoretical understanding of the nucleon scattering process in order to extract as much structure information as possible from experimental data. For this purpose, a reaction theory based on a microscopic optical potential is very well suited.

We are interested in the derivation of a microscopic model which describes the direct interaction in the elastic channel of a projectile nucleon with a target nucleus. This is a $A + 1$ body problem with A bound nucleons (target) and one nucleon in the continuum. As proposed in Feshbach theory [4], instead of treating this $A + 1$ body problem, we go back to a two-body problem using an effective potential taking into account the interaction between the nucleon and the target, the so-called optical potential. This potential is microscopic as it is built from a nucleon-nucleon interaction. Most of the time, it is non-local and depends on the incident energy of the projectile nucleon. The use of a nucleon as a probe of nuclear matter requires an antisymmetrised formalism, due to Pauli principle.

In a recent work, Dupuis *et al.* [1] study proton elastic scattering from ^{208}Pb using Melbourne g-matrix [2] as effective nucleon-nucleon interaction. They get a good agreement with experimental cross sections for high incident proton energies. Below 60 MeV, absorption is missing at back angles in center-of-mass and the experimental cross section is overestimated.

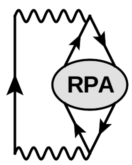

Fig. 1. Diagram of coupling to excitations of the target.

During the seventies, N. Vinh Mau *et al*. have shown that coupling to the collective states of the target nucleus in the calculation of optical potential leads to additional absorption at low incident energy [5]. This means an imaginary contribution (as well as a real contribution) that mainly contributes at low incident energy. This coupling process is described in Fig. 1. Time is oriented from the bottom to the top of the diagram. Lines with arrows are Hartree-Fock (HF) propagators determined using a two-body effective interaction. Put into words the diagram reads as follows: the incident nucleon interacts a first time with the target (wavy line), the target gets excited (excitations are described within the RPA framework), the intermediate nucleon propagates in the HF field and then interacts a second time with the nucleus that desexcites. In the following we refer to that state as "intermediate state". This diagram contributes to the elastic channel as the nucleon in the exit channel has the same nature (proton/neutron) and the same energy as in the entrance channel. In Ref. [5], N. Vinh Mau *et al.* use a simple zero-range effective interaction. They also have an incomplete description of the excited states of the target in terms of RPA. Finally they do not calculate any cross section. Bernard *et al.* [6] have also studied microscopic optical potential. They are interested in proton elastic scattering from ^{208}Pb. They use the method developed in Ref. [5] but employing a Skyrme effective interaction to generate the HF field, the RPA excited states and the

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coupling. Then they calculate a local equivalent potential to easily solve the scattering problem.

The goal of this work is to include coupling to the collective states of the target into the optical potential starting from structure calculations. This work is based mainly on the work of N. Vinh Mau *et al.* [5] and follows the work done by Dupuis *et al.* [1]. We use the Green functions formalism as in Ref. [5]. Green functions translated into diagrams enable to classify the several approximations for resummations and to identify double counting. Indeed each diagram has to be taken into account only once. We use a finite range effective interaction, the Gogny D1S interaction, to calculate the matrix elements of the optical potential. We start from the self-consistent RPA calculation with the D1S Gogny force [7] which provides the HF basis and the RPA amplitudes for all target excitations. Note that the use of RPA makes our calculation suitable mainly for double-magic target nuclei. The obtained optical potential is non-local and energy dependent. We do not plan to localise the potential as in [5] and [6] to solve the scattering problem.

First we present the formalism and the several approximations used in the calculation: HF and RPA approximations. Mass operator is determined in an approximate way and leads to the optical potential. We present some considerations about double counting due to the use of different approximations at the same time. Finally, we present some preliminary results for proton elastic scattering from ^{208}Pb.

2 Optical potential

2.1 Mass operator

As shown in Ref. [8], the optical potential is related to the mass operator, M, by a Fourier transform,

$$V_{opt}(\mathbf{r},\mathbf{r'};E) = -M(\mathbf{r},\mathbf{r'};E) = -\int_{-\infty}^{+\infty} e^{iE(t-t')} M(x,x') d(t-t'). \quad (1)$$

Mass operator, is defined by the integral equation of the one-particle Green function,

$$G_1(x,x_0) = G_1^{(0)}(x,x_0) + \int G_1^{(0)}(x,x_1) M(x_1,x_2) G_1(x_2,x_0) d^4x_1 d^4x_2, \quad (2)$$

where $x = (\mathbf{r},t)$, $G_1^{(0)}$ is the Green function for a free particle with no interaction with the nucleons of the A-body-system and G_1 is the one-body Green function that propagates the nucleon in the field of the target. Starting from an exact expression of the mass operator in terms of the one, two and three-body Green functions, N. Vinh Mau [9] has shown that if one makes the following assumptions: (i) three-body Green functions can be replaced by antisymmetrised products of one- and two-body Green functions; (ii) particle-particle propagators are treated in the ladder approximation, and particle-hole propagators in the ring (RPA) approximation; (iii) one-body Green functions can be described by HF propagators, the Fourier transform of the mass operator gives an optical potential of the form (see [5]),

$$V_{opt} = V_{BHF} + V_{RPA} - V^{(2)}, \quad (3)$$

where V_{BHF} stands for the Brueckner-Hartree-Fock (BHF) contribution to the optical potential. V_{RPA} is the RPA contribution. V_{BHF} and V_{RPA} are written in terms of diagrams in Fig.2 and Fig.3, respectively. Exchange terms are not depicted for simplicity but are included in the calculation.

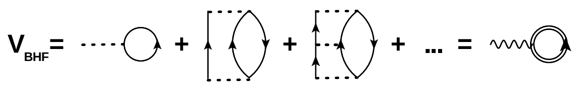

Fig. 2. Diagrammatic expression of V_{BHF}.

In Fig.2, bare two-body interaction is represented by a dashed line. The first diagram on the left hand part, is the Hartree term: the projectile interacts with the target mean-field. In the second term the projectile interacts a first time with the target creating a particle-hole pair, then interacts a second time destroying the particle-hole pair and so on for the following terms. Each direct term of the series diverges because of the hard-core of the bare two-body interaction (whereas exchange terms are finite). Nevertheless the summation of this infinite ladder series converges toward a BHF type diagram with an effective interaction, the g-matrix (in Fig. 2 with a wavy line) and a dressed BHF propagator (double line). In practice, we use the Melbourne g-matrix as effective two-body interaction [2]. It is determined in infinite nuclear matter using the Bonn-B bare interaction [10]. The Local Density Approximation (LDA) is then used to make the interaction suitable for finite nuclei. Melbourne g-matrix is used to generate V_{BHF} as done in Ref. [1]. Note that Melbourne g-matrix is complex and leads to a complex optical potential V_{BHF} with a consistent imaginary part. This is not the case in the work of Bernard *et al.* [6] where they use a Skyrme interaction to generate the HF field and as a result get a real potential.

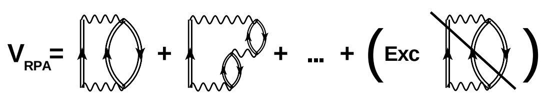

Fig. 3. Diagrammatic expression of V_{RPA}.

In Fig.3, we show the diagrammatic representation of the RPA contribution to the optical potential. It couples the projectile to the excited states of the target, through the summation of the ring diagram expansion typical from RPA. The D1S Gogny force is used in the RPA calculation and in the coupling (wavy line in Fig. 3). In a fully consistent calculation the same effective two-body interaction (wavy line in Fig 2 and 3) should be used to calculate V_{BHF} and V_{RPA}. The RPA contribution to the optical potential represented in Fig.3 reads:

$$V_{RPA}(\mathbf{r},\mathbf{r}') = \lim_{\eta\to 0^+} \sum_{N\neq 0, ijkl\lambda} \chi_{ij}^{(N)} \chi_{kl}^{(N)} \times \left(\frac{n_\lambda}{E-\epsilon_\lambda + E_N - i\eta} + \frac{1-n_\lambda}{E-\epsilon_\lambda - E_N + i\eta} \right) \times \langle \psi_0 | V | \psi_N^{(ij)} \phi_\lambda \rangle_a \langle \phi_\lambda \psi_N^{(kl)} | V | \psi_0 \rangle_a - V_{opt}^{(2)}. \quad (4)$$

ψ_0 is the ground state of the target. Index a refers to the antisymmetrisation between the particle in the intermediate state ϕ_λ (of energy ϵ_λ and occupation number n_λ) and the particle into the nucleus. The index $ij(kl)$ refers to the particle-hole (hole-particle) components of the wave function ψ_N of the excited state of the target. $\chi_{ij}^{(N)}$ are the RPA amplitudes for a particle-hole pair ij of the state ψ_N of energy E_N. E is the incident energy of the projectile nucleon. Then using the Plemelj formula where $\mathscr{P}$ stands for the principal value,

$$\frac{1}{\omega' - \omega \pm i\eta} = \mathscr{P}\frac{1}{\omega' - \omega} \mp i\pi\delta(\omega' - \omega), \quad (5)$$

one gets the real and the imaginary part of the optical potential.

The diagram expansion is useful in order to identify double counting. Indeed the second order term appears twice in the RPA term (once in the direct term and once in the exchange term as shown in Fig. 3), but it is also already taken into account inside the ladder expansion. Thus $V^{(2)}$ is equal to twice the second order term.

2.2 Schrödinger equation

The optical potential is non-local and depends on the incident energy of the projectile nucleon. It requires the resolution of the integro-differential Schrödinger equation. The partial wave development of the equation reads:

$$\left[\frac{d^2}{dr^2} + \frac{2}{r}\frac{d}{dr} + \frac{l(l+1)}{r^2} - k\right] f_{jl}(r) + \int dr' V_{opt}^{jl}(r,r') f_{jl}(r') = 0. \quad (6)$$

Thus V_{RPA} and $V^{(2)}$ need to be calculated for each partial wave (jl) and added to the optical potential V_{BHF} generated from g-matrix previously determined in [1]. We will use the DWBA98 code [11] to solve the scattering problem with a non-local and energy dependent potential.

3 Preliminary results

We consider the proton elastic scattering from ^{208}Pb. It is a double-magic nucleus. Its excited states are well reproduced by the RPA. We have done a first calculation of the imaginary part of V_{RPA} using only the central part and the density dependent part of the D1S Gogny interaction. In further calculations we will add the contribution from the spin-orbit part. As intermediate state, in a preliminary calculation, a plane wave is used. A scattering wave calculated in the BHF field will be used in the future. We look at the first 3^- states in ^{208}Pb that are low-lying and collective states.

Fig. 4 shows the diagonal ($r = r'$) part of the imaginary part of V_{RPA} when including the coupling to 3^- states of the target with excitation energy up to 15 MeV. We show the contribution for the first partial wave, $l = 0$ and $j = 1/2$. The incident energy of the proton projectile is 40 MeV. Double counting of the second order term $V^{(2)}$ has not been removed yet. $V^{(2)}$ is expected to contribute in the bulk part of the potential [5]. Moreover we see a contribution peaked at the surface of the nucleus as observed previously by N. Vinh Mau *et al*. Thus we expect a surface-peaked contribution to the imaginary part once double counting removed.

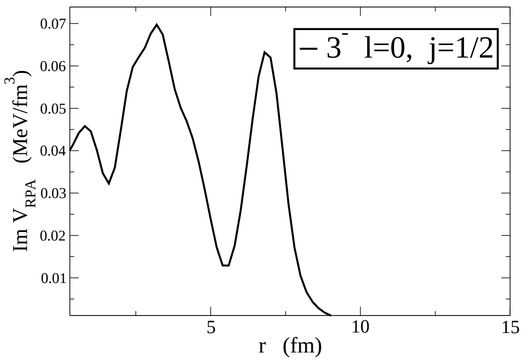

Fig. 4. Diagonal part of imaginary part of V_{RPA} (partial wave l=0 and j=1/2) for coupling to 3^- states of ^{208}Pb with E_{exc} < 15 MeV and proton incident energy E=40 MeV.

In Fig. 5, we couple only to the first 3^- state of the target with an excitation energy $E_{exc} = 3.4$ MeV. We calculate the imaginary part of V_{RPA} and sum the diagonal part over all partial waves,

$$W(r) = \sum_{lj} \frac{2j+1}{4\pi} \mathrm{Im}\{V_{RPA}^{lj}(r,r)\}. \quad (7)$$

We show the maximum of $W(r)$ for proton incident en-

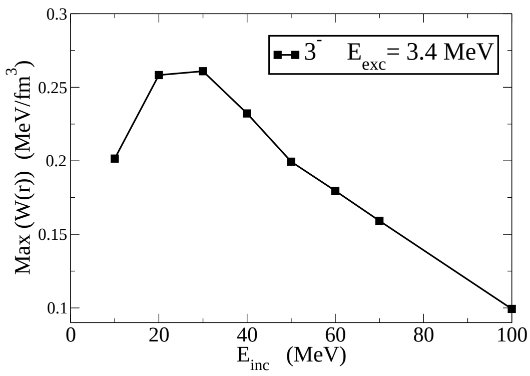

Fig. 5. Maximum of W(r) as a function of proton incident energy for the coupling to the first 3^- state of ^{208}Pb (E_{exc} = 3.4 MeV).

ergy running from 10 MeV to 100 MeV. The contribution of V_{RPA} is maximum around 30 MeV proton energy and decreases as the incident energy increases. The imaginary part of V_{RPA} acts mainly at low incident energy, where the calculation using only V_{BHF} overestimates experimental cross section. Note that these preliminary results are in qualitative agreement with the calculation done by Bernard *et al.* [6] using a Skyrme interaction.

We have also studied the non-locality of the potential. The potential expressed in function of $R = \frac{1}{2}(r + r')$ at a given $s = r - r'$ gives us information about the local shape of the potential. For a given R, we can look at the non-locality of the potential as a function of s. In Fig. 6 we show the diagonal part ($s = 0$) of the imaginary part of V_{RPA} for the coupling to the first 3^- of ^{208}Pb. The correction is surface peaked which is typical of the coupling to a collective state.

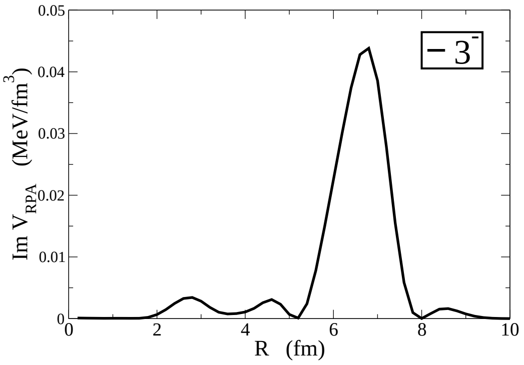

Fig. 6. Diagonal part of imaginary part of V_{RPA} (first partial wave l=0 and j=1/2) for coupling to the first 3^- state of ^{208}Pb (E_{exc}=3.4 MeV) and proton incident energy E=40 MeV.

In Fig. 7 we show the behavior of the non-locality for $R = 6.3$ fm (position of the maximum in Fig. 6). The potential has a non-locality of about 2 fm range which approximately corresponds to range obtained in previous approaches [5, 6].

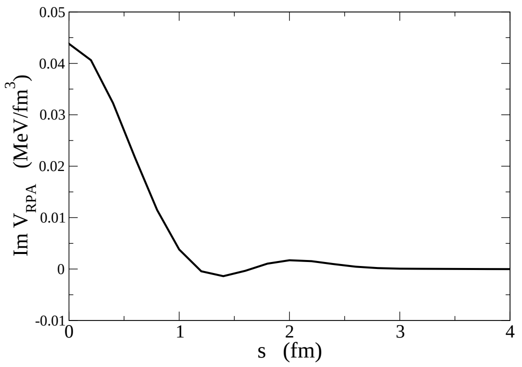

Fig. 7. Imaginary part of V_{RPA} as a function of s (first partial wave l=0 and j=1/2) for coupling to the first 3^- state of ^{208}Pb (E_{exc}=3.4 MeV) and proton incident energy E=40 MeV.

4 Outlooks

We are interested in nucleon elastic scattering theory. We have presented the calculation of the correction to the optical potential due to coupling to excited states of the target. We use the RPA formalism with Gogny force to describe the excited states of the target nucleus. The optical potential is obtained using the Green function formalism. A particular care is taken to the issue of double counting.

We present some preliminary results for the optical potential calculated using the central part and the density dependent part of the two-body interaction: the contribution to the imaginary part of the potential due to coupling to the first 3^- states of the target, the behavior of the correction with the incident energy of the projectile and its non-locality.

In the near future, we plan to continue the calculation in the following directions. The intermediate state ϕ_λ has to be determined in the BHF field of the target. Double counting has to be taken into account. Then we shall add the contribution to optical potential coming from the spin-orbit part of the two-body nucleon-nucleon effective interaction. RPA correction will be added partial wave by partial wave to the potential obtained from Melbourne g-matrix. Then we will use the DWBA98 code to solve the scattering problem with a non-local and energy dependent potential. This will hopefully cure the lack of absorption observed at low energy in nucleon elastic scattering calculations using only a g-matrix as effective interaction.

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