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# Microscopic optical potentials for composite particle scattering at low energies

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**Abstract.** The evaluation of microscopic alpha-nucleus optical potentials within the nuclear structure approach is revisited. This approach seems best suited at low energies relevant for nucleosynthesis, but no further developments were performed for more than two decades. Here, a report on the first steps of a revival of the nuclear structure approach, i.e. numerical implementation of the formulae for the imaginary part, is given.

## 1 Introduction

Protons and alpha-particles constitute most of the baryonic mass in the universe. Thus nuclear reactions involving these particles play an important role in the formation of elements, the so-called nucleosynthesis [1]. The basic reaction chains and cycles in the burning phases of stars as well as the various processes leading to heavier nuclei than iron have been identified, and a remarkable progress towards a quantitative understanding of the nucleosynthesis has been made. However, there are still important open questions: frequently either cross sections are strongly suppressed or the target nucleus is not accessible to experiment. Consequently one must rely on theoretical predictions for many processes. A similar problem arises also in the field of nuclear technology, where (n,$\alpha$) and (n,p)-reactions lead to embrittlement of structure materials and thus to their aging. Hence reliable estimates are directly related to safety issues.

Optical potentials are basic ingredients of almost all reaction calculations. Usually these are determined by adjusting a parameterized potential to fit experimental elastic cross section data. In the absence of data either they are deduced via extrapolation or by microscopic means. In the seventies and eighties of the last century considerable effort was devoted to the formulation of nucleon-nucleus optical potentials. Especially the nuclear matter approach [2] and its refinements provide a fair description of elastic scattering at energies above about 50 MeV. At lower energies the so-called nuclear structure approach, originally derived by Vinh Mau and Bouyssy [3], has led to promising results. The approach takes into account the specific structure of the target nucleus and describes within refined formulations [4,5] about 70% of the total absorption by the inclusion of collective states in magic nuclei. Despite the promising results the approach was not further developed and at present a quantitatively reliable nucleon-nucleus optical potential for energies below about 50 MeV is still not available. A formulation similar to the nuclear structure approach was also successfully applied to alpha-nucleus scattering [6–8], but similarly to the nucleon-nucleus optical potentials no refinements were reported for more than two decades. In addition the corresponding numerical tools are not available anymore.

It is our main goal to formulate the $\alpha$-nucleus optical potentials at low energies in an improved quantitative and theoretical sound way. Therefore, as a first step we recovered the available formalism [7] for the imaginary part and implemented it numerically by up-to-date tools. Although the formalism is available in the literature [5, 7] some details concerning the applied definitions had to be studied. In addition a scattering code that accounts for the full non-locality of the optical potential has been constructed, which is essential for calculations in composite particle scattering. As a first example the procedure was successfully applied to the evaluation of the imaginary part of the $\alpha-^{16}$O optical potential.

According to the goal of this contribution we outline in chapter 2 the formalism of the nuclear structure approach of Dermawan, Osterfeld and Madsen [7] together with some details on the involved quantities. In chapter 3 we discuss the numerical implementation and show first results of the calculation of the imaginary part of the $\alpha-^{16}$O optical potential. Finally, in chapter 4 the results are summarized and an outlook to further research is given.

## 2 Nuclear Structure Approach

The optical potential plays a key role in reaction theory as a basic input. Therefore, in the seventies and eighties of the last century several microscopic approaches were developed (e.g. [9–11]) which provided either the first order term or parts of the contributions to the imaginary potential in second order perturbation theory. On the other hand a formulation of the optical potential using the concept of nuclear matter has been given (e.g. [2,12,13]). This so-called nuclear matter approach proved to be very success-

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ful for nucleon-nucleus scattering beyond about 50 MeV. This is particularly true for the recent advances in this approach [14].

The nucleon-nucleus optical potential is an effective single particle potential for a scattered (unbound) nucleon. Using Green function techniques, Bell and Squires [15] identified the optical potential as the mass operator of the one-particle Green function. Hence the evaluation of the nucleon-nucleus optical potential is directly related to the evaluation of the mass operator for finite nuclei. A comprehensive microscopic evaluation of the mass operator for finite nuclei within the Green function method was given by Vinh Mau and Bouyssy [3] who restricted their evaluations to correlations between a pair of particles and assumed the validity of the radom phase approximation (RPA). Thus this so-called nuclear structure approach accounts explicitly for the collective excitations of the nucleus which yield important contributions to the optical potential at low energies. The nuclear structure approach was then further elaborated by Osterfeld et al. [5]. The derivation in Ref. [3] shows that the nuclear matter approach [13] is equivalent to the leading term of the low-density expansion of the mass operator in nuclear matter.

The situation of microscopic optical potentials for composite projectiles, e.g. d, t, $^3$He, $\alpha$, is less satisfactory. For $\alpha$-particles, which are tightly bound and can be considered in first approximation as *elementary* objects, a microscopic approach was first suggested by Vinh Mau [6] and further developed by Dermawan et al. [7]. This approach was combined in Ref. [8] with the fish-bone model [16] in order to account for the Pauli principle at least in the first order term. For other projectiles the available models are even more phenomenological because of the additional complication of breakup.

Here, we concentrate on $\alpha$-nucleus optical potentials at low energies, which are of interest for nuclear astrophysics. Following the method of Ref. [7] we evaluate the imaginary part of the optical potential starting with the expression of Feshbach [9] for the imaginary part of non-identical particles,

$$W(\mathbf{r},\mathbf{r}') = \mathrm{Im}\sum_{M\neq 0}\langle\Psi_0|V|\Psi_M\rangle_{\mathbf{r}} g_M(\mathbf{r},\mathbf{r}')\langle\Psi_M|V|\Psi_0\rangle_{\mathbf{r}'}\,, \quad (1)$$

where $\Psi_M$ are the intermediate states of the target, $V = \sum_i V^i_{\alpha N}$ with $V_{\alpha N}$ denoting the $\alpha$-N (target-nucleon) interaction and $g_M(\mathbf{r},\mathbf{r}')$ is the Green function describing the propagation in the intermediate state with the energy $E - E_M$. Restricting the expression of Ref. [5] to $J_A = 0^+$, Dermawan et al. [7] obtained a rather simple expression for the imaginary $\alpha$-nucleus optical potential,

$$W(\mathbf{r},\mathbf{r}') = \mathrm{Im}\left[\frac{1}{16\pi^2}\sum_M\sum_{\ell,\ell_c,L}\delta_{L,J_M}\frac{\hat{\ell}^2\hat{\ell}_c^2}{\hat{L}^2}(\ell 0\ell_c 0|L0)^2 F^D_{J_M,L0L}(r)g_{\ell_c}(r,r')F^D_{J_M,L0L}(r')P_\ell(\cos\theta)\right], \quad (2)$$

where $\hat{\ell} = \sqrt{2\ell+1}$ and $g_{\ell_c}(r,r')$ is the radial part of the intermediate Green function at the energy $E - E_M$. The transition form factor $F^D$ represents the coupling matrix element of (1) and is defined by

$$F^D_{J_M,L0L}(r) = \int_0^\infty ds s^2 \rho^{J_M}_{L0L}(s) v_L(r,s)\,. \quad (3)$$

The nuclear transition density $\rho^{J_M}_{LSJ}(r)$ provides the nuclear structure information and is according to Ref. [7] given by

$$\rho^{J_M}_{LSJ}(r) = \frac{1}{\sqrt{4\pi}}\sum_{n_1,\ell_1,\mathrm{j}_1}\sum_{n_2,\ell_2,\mathrm{j}_2}\left[X^{J_M}_{j_1 j_2} + Y^{J_M}_{j_1 j_2}\right]\frac{\sqrt{3}}{2}\hat{S}\,\hat{j}_1\hat{j}_2\hat{L}\hat{J}\hat{\ell}_1 (\ell_1 0 L 0|\ell_2 0)\begin{Bmatrix}\ell_2 & \frac{1}{2} & j_2\\ \ell_1 & \frac{1}{2} & j_1\\ L & S & J\end{Bmatrix} R_{n_2\ell_2 j_2}(r)R_{n_1\ell_1 j_1}(r)\,, \quad (4)$$

where the $X_{j_1 j_2}$ and $Y_{j_1 j_2}$ are the RPA particle-hole amplitudes and $R_{n_i\ell_i j_i}(r)$ are the radial single-particle wave functions.

In order to evaluate the coupling matrix elements in Eq. (1) a mutlipole expansion of the $\alpha$-N interaction was performed in [7] following the procedure in Ref. [17]. Thus the expansion coefficient $v_L(r,s)$ in Eq. (3) is given by

$$v_L(r,s) = 2\pi\int_{-1}^{1} d(\cos\theta)\, V_{\alpha N}(r')P_L(\cos\theta)\,, \quad (5)$$

with $r' = \sqrt{r^2 + s^2 - 2rs\,(\cos\theta)}$.

The nuclear structure information enters via RPA particle-hole amplitudes. These collective states will account for a major part of the imaginary optical potential.

## 3 Implementation

We implemented the formalism outlined in Eqs. (1) to (5) into a numerical code and evaluated as a first example the imaginary part of the optical potential for $\alpha$-$^{16}$O scattering at an incident $\alpha$-energy of 30 MeV. The structure of the $^{16}$O target nucleus was described by the RPA using the particle-hole amplitudes $X^{J_M}_{j_1 j_2}$ and $Y^{J_M}_{j_1 j_2}$ of Ref. [18], which was based on a configuration space consisting of two subshells above and below the Fermi surface and included also the continuum states in a discretized way. For the RPA-eigenstates a set of harmonic oscillator states were used. The potential was given by a Woods-Saxon potential, which has been adjusted to fit experimental data.

For the $\alpha$-N interaction $V_{\alpha N}(r)$ in equation (5), a phenomenological potential of Gaussian shape was chosen

$$V_{\alpha N}(r) = V_1 \mathrm{e}^{-\beta r^2} + V_2 \mathrm{e}^{-\gamma r^2}. \quad (6)$$

The parameters of $V_{\alpha N}(r)$ at an incident energy of 30 MeV are given in Table 1.

The radial Green function $g_{\ell_c}(r,r')$ in Eq. (2) at an intermediate energy $E_{int} = E - E_M$ has to satisfy the following equation

$$\left(-\frac{\hbar^2}{2\mu}\left(\frac{1}{r}\frac{d^2}{dr^2}r - \frac{\ell_c(\ell_c+1)}{r^2}\right) + V_{DT}(r) - E_{int}\right)g_{\ell_c}(r,r') = \frac{1}{rr'}\delta(r-r'), \quad (7)$$

**Table 1.** Effective $\alpha$-nucleon interaction parameter for an incident energy of 30 MeV.

| $V_1$ | $\beta$ | $V_2$ | $\gamma$ |
|---|---|---|---|
| -310.1 | 0.422 | 319.2 | 0.505 |

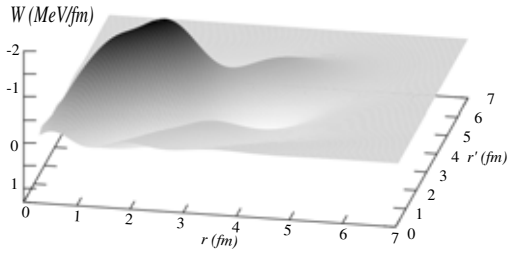


**Fig. 1.** Imaginary optical potential for $\alpha-^{16}$O-scattering at 30 MeV

where $\mu$ denotes the reduced mass of the $\alpha$-particle and $V_{DT}$ the first order $\alpha$-$^{16}$O optical potential.

The Green function and the transition matrix elements for the ground state as well as for the excited RPA-states were calculated using a direct potential $V_{DT}$ obtained by folding $V_{\alpha N}$ into the density of the target. With this the imaginary optical potential for an incident energy of 30 MeV was calculated. The outcome can be seen in Fig. 1. The overall shape of the potential is qualitatively similar to the results given by [7]. In particular, the peak of the imaginary optical potential is situated below the nuclear radius $R^{^{16}O}_{nucl} \approx 2.4$ fm of $^{16}$O, which is also the case for the imaginary optical potential of [7]. Osterfeld et al. [5] applied the nuclear structure approach to the scattering of neutrons on $^{40}$Ca with a nuclear radius of $R^{^{40}Ca}_{nucl} \approx 4.0$ fm. Of course, there exist more open channels for $^{40}$Ca than for $^{16}$O, which results in a quantitatively smaller optical potential for $^{16}$O. Figs. 2 to 5 show how the different states contribute to the imaginary optical potential at 30 MeV. The main contribution comes from the $1^-$ states and a non-negligible part from the $3^-$ states.

In [7] the evaluated imaginary optical potential $W$ is compared to a phenomenological, local one. In general, the evaluated $W$ is smaller than the phenomenological, local imaginary potential (e.g. [7]) and consequently, it can account only for about 70% of the observed absorption.

The $\alpha$-nucleus optical potential is expected to be highly non-local. Consequently the determination of the scattering wave function requires the solution of the corresponding integrodifferential Schrödinger equation. We have constructed the required numerical tools and calculated the $\alpha$-$^{16}$O differential cross section. However, the results are not exciting because at this stage, we did not take care of a proper direct potential $V_{DT}$ which accounts for the Pauli-principle between the incident $\alpha$-particle and the target nucleus. Therefore, the obtained cross sections cannot reproduce the experimental ones and are not worth to be displayed. Hence, in the next step the use of an improved direct potential, e.g. within a cluster approach (see e.g. [8]), will be essential.

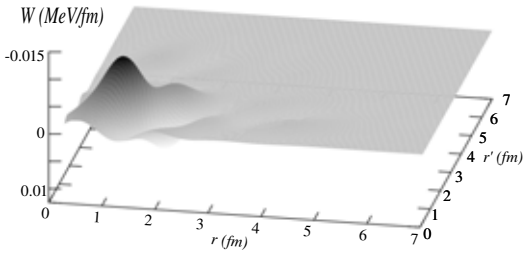


**Fig. 2.** Contributions from the $0^+$ states to the $\alpha-^{16}$O optical potential

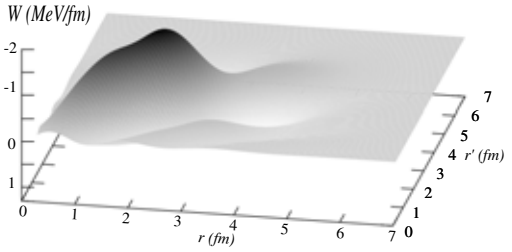


**Fig. 3.** Contributions from the $1^-$ states to the $\alpha-^{16}$O optical potential

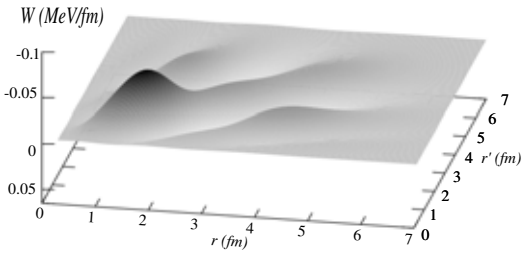


**Fig. 4.** Contributions from the $2^+$ states to the $\alpha-^{16}$O optical potential

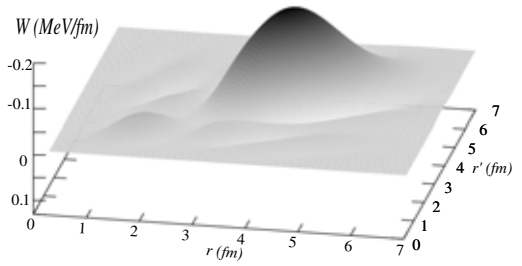


**Fig. 5.** Contributions from the $3^-$ states to the $\alpha-^{16}$O optical potential

## 4 Summary and outlook

We have rederived and checked in detail the formalism of Dermawan et. al. [7]. By using up-to-date programming techniques we set up a code, which allowed us to reproduce qualitatively the results of Ref. [7]. In particular the obtained imaginary potential at the same incident energy shows similar radial characteristics. Especially the dominating peak lies slightly below the nuclear radius of $^{16}$O in agreement with [7].

The calculations presented here are only a first step for a reliable microscopic $\alpha$-nucleus optical potential. In order to achieve a proper description of the differential elastic cross section a consistent determination of the direct (first order potential) part is required, where the combination with a Cluster approach using the techniques of the RGM or the GCM are most promising [19,20]. In addition the real part of the polarization term should also be included in future calculations. Most probably these calculations will suffer from convergence problems. Therefore, it will be important to combine these calculations with the dispersion relation approach [21] to guarantee consistency.

A further limitation of this model is its restriction to magic nuclei due to the fact that the RPA is only valid for magic or near-magic nuclei. For the extension of the model to non-magic nuclei we will also use QRPA states as well as other explicitly given intermediate channels.

Summarising we have made a first successful step to revive the nuclear structure approach and to establish the required programs. Thus we have set the basis for future improvements as discussed above.

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