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Compound Nucleus Reaction Theory for Synthesis of Super-Heavy Elements

Yasuhisa Abe[1,a], Caiwan Shen[2], David Boilley[3,4], and Bertrand G. Giraud[5]

[1] RCNP, Osaka Univ. Ibaraki, Osaka 567-0047 Japan
[2] School of Science, Huzhou Teachers College, Huzhou, Zhejiang, 313000 China
[3] Univ. Caen, Esplanade de la Paix, B.P. 5186, F-14032 Caen cedex, France
[4] GANIL, CNRS/IN2P3-CEA/DSM, BP 55027, F-14076 Caen cedex 5, France
[5] Institut Physique Théorique, Centre d'Etudes-Saclay, Gif-sur-Yvette, F-91191 France

Abstract. Present status of theory for synthesis of Super-Heavy Element (SHE) is reported, based on the compound nucleus reaction. Specific aspects are discussed in fusion and survival probabilities. They are the hindrance of fusion in the former and the fragility of the compound nucleus in the latter. Examples of applications are given.

1 Introduction

As it is well-known, residue cross sections for SHE are extremely small, and are expected to become even smaller if heavier elements are attempted [1]. Therefore, theoretical prediction of promising incident systems and optimum incident energy is being strongly desired for planning of future experiments. According to the compound nucleus reaction theory, residue cross section for SHE nuclei is given by the following formula,

$$\sigma = \frac{\pi}{k^2}\Sigma(2J+1)\cdot P^J_{fusion}\cdot P^J_{surv}, \quad (1)$$

where P^J_{fusion} and P^J_{surv} denote fusion and survival probabilities for the total spin J, respectively. The extremely small residue cross sections stem from two effects : the hindrance in fusion of incident heavy ions and the fragility of the compound nucleus. They are to be taken into account in their respective factors in the above formula [2]. Thus, in order to predict SHE productions quantitatively, we have to calculate the fusion probability as well as the survival probability accurately.

As for the former, reaction theories developed for lighter systems cannot encompass heavy systems relevant to SHE, where the fusion hindrance has been well-known to exist experimentally since many years. It is well accepted that this phenomena is due to the appearance of an additionnal inner barrier that has to be overpassed. Most of the models for the formation of the spherical compound nucleus are based on the dissipation-fluctuation dynamics for the collective degrees of freedom [3]. They turn out to provide a qualitative understanding on why fusion is hindered so strongly, and to improve quantitative predictions of residue cross sections remarkably. Present status of our understanding is reported below.

The survival probability can be calculated by the statistical theory of decay. The stability of SHE is almost completely given by the so-called shell correction energy of the compound nucleus in the ground state, which disappears as excitation energy increases. As the compound nucleus formed by nuclear reactions is excited, and thus lose at least partially its stability, depending on its excitation energy, it is very fragile. This feature is taken into account by the so-called Ignatyuk prescription that the level-density parameter a is taken to be excitation-energy dependent [4]. We have developed a new computer code KEWPIE2 [5, 6] that can calculate the survival probability as well as dynamical observables related to the fission. Examples of theoretical results are given on fission time scale [7] as well as on predictions of SHE [8].

2 Mechanism of Fusion Hindrance

2.1 Overcoming of the Coulomb barrier and the saddle point

In fusion process for synthesis of SHE, it is crucial to recognize that there is another barrier in addition to the usual Coulomb barrier. We do not discuss about the Coulomb barrier in the present paper, but mainly about the second barrier which is actually the saddle point or the ridge-line located between di-nucleus configuration and the spherical compound nucleus. After overcoming the Coulomb barrier, projectile and target contact each other to form a di-nucleus configuration, which is considered to be highly deformed as a compound nucleus. On the other hand, saddle point of SHE is located close to the spherical shape, due to the fact that the fissility parameter is close to 1.0. Therefore, the di-nucleus system formed by the projectile and the target nuclei has to overcome the saddle point in order

[a] e-mail: abey@rcnp.osaka-u.ac.jp

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to reach the spherical shape. Considering violent and dissipative nature of nuclear interactions, the incident kinetic energy is almost lost at the moment of the matter contact between the incident nuclei. This indicates that the system does not have enough kinetic energy to overcome the saddle. This is a main physical cause of the fusion hindrance.

Before proceeding to realistic calculations, it is meaningful and indispensable to understand essential points of the hindrance mechanism theoretically. With the simplifications of the inverted parabola for the saddle point shape and of a constant friction coefficient, Langevin equation is analytically solved. The probability of overcoming of the saddle, i.e. the formation probability is obtained as follows,

$$P_{form}(t; q_0, p_0) = \frac{1}{2}\mathrm{erfc}(-\frac{\langle q(t)\rangle}{\sqrt{2}\sigma_q(t)}), \qquad (2)$$

where $\langle q(t)\rangle$ and $\sigma_q^2(t)$ denote the average trajectory and its variance, respectively. They are given in terms of the physical parameters of the system, i.e., the friction coefficient, the curvature of the saddle and the initial values (q_0, p_0) [9, 10].

At time t being at the infinity, the probability is simply given by the following expression, with the assumption of the equilibrium distribution for the initial momentum p_0 in the di-nucleus configuration,

$$P_{form} = \frac{1}{2}\mathrm{erfc}(\sqrt{\frac{V}{T}}), \qquad (3)$$

where V denotes the saddle point height relative to the energy of the compound system formed by the incident channel. The formula apparently describes the features of the hindrance observed in experiments, i.e., extremely small probability as well as its very slow increase as function of the incident energy which is only reflected through the temperature T of the system [10, 11]. The formula is also used by Swiatecki et al [12] for the explanation of the measured excitation functions of so-called cold fusion path. It should be noticed that one-dimensional radial fusion is not physically realistic, as is clarified below.

Using eq. (2), we can study the time-dependence of the flux overcoming the saddle and show that the fusion occurs within a short time duration around a few to several in unit of $\hbar$/MeV [10].

2.2 Dynamics from di-nucleus to mono-nucleus

To describe the shape evolution from di-nucleus to spherical mono-nucleus configurations, at least three degrees of freedom are necessary: the radial, neck and mass-asymmetry degrees of the two-center parameterization. In addition to the radial motion discussed above, separate analyses of the other twos are also instructive for understanding the fusion dynamics, though there are coupling among them through Liquid Drop Model (LDM) potential as well as the friction tensor.

Firstly, the neck motion is analysed in terms of so-called neck correction parameter ϵ, which varies from 1.0 for the initial di-nucleus to 0.0 for the mono-nucleus with

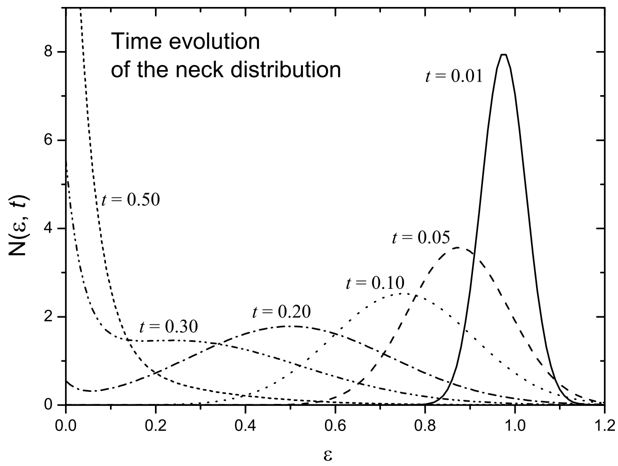

Fig. 1. Typical time evolution of neck distribution.

the full neck radius. At the contact configuration, LDM energy is almost linear in ϵ. Thus, we can solve analytically a corresponding Smoluchowski equation with constant friction coefficient. The solution describes time evolution of distribution of ϵ, starting with δ-function at 1.0, and reaching the equilibrium distribution at time enough later, as shown in Fig. 1.

It should be noticed here that the time necessary for that is one order of magnitude shorter than the time scale of the radial fusion mentioned in subsection 2.1. This indicates that the di-nucleus formed by the incident projectile and target nuclei immediately undertake denecking, i.e., filling-up of the neck crevice to form the highly deformed mono-nucleus [13].

Secondly, we make the same analysis of the mass-asymmetry degree of freedom, starting with δ-function at the incident mass-asymmetry. The results show that the mass-asymmetry distribution reaches the equilibrium in a few in unit of $\hbar$/MeV, which is similar to the time scale of the radial fusion [14]. This indicates that motion of the mass-asymmetry has to be treated at the same time with the radial one in a coupled way. Actually, this justifies the use of multi-dimensional Langevin approach [3] for predictions of SHE cross sections [15, 16].

2.3 Critical distance for denecking of di-nuclear system

Further study is made on denecking during the approaching process of collisions. In the previous subsection, at the contact configuration, the neck crevice immediately fills out to form the compound nucleus. But at what moment colliding system starts to be connected, i.e., to form a di-nucleus system? The denecking is considered to be mainly due to the surface energy. Thus, not only at the contact configuration analysed above, but also in close approach of two matter surfaces within nuclear interaction range, denecking is expected to be favoured. In order to answer the question quantitatively, ϵ dependence of LDM potential which calculates the surface energy with the finite range

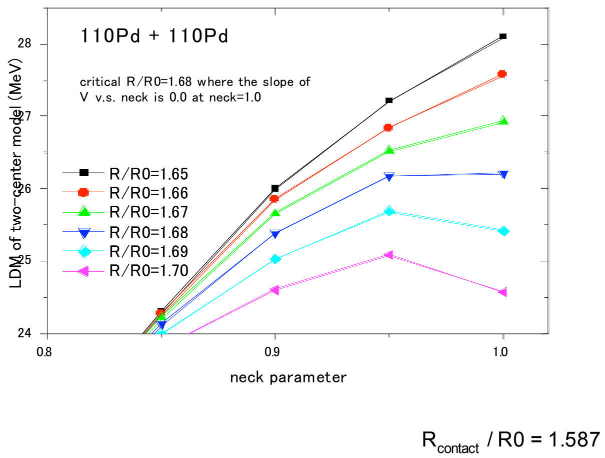

Fig. 2. Dependence of slope on distance between two incident ions.

nuclear interaction, is calculated for various relative distance. The results are shown in Fig. 2.

At larger separations, the potential slope is positive, which unfavours two incident ions to be connected. On the other hand, at shorter distances toward the contact distance, the slope is negative, which favours two ions to be connected with neck of growing radius, as is discussed in the previous subsection. The border between two regimes defines the critical distance for formation of compound nucleus with a large deformation. In other words, at this distance, shape evolution of the di-nucleus system starts toward the spherical compound nucleus. That partially explains the phenomenological introduction by Swiatecki et al [12] of an additionnal initial separation s between the two fusing nuclei. Of course, such a separation s depends on the compound nucleus formed as well as on mass-asymmetry of the incident channel, which is under investigation.

3 Survival Probability

3.1 Statistical theory of decay

In SHE production, most of the compound nuclei undergo fission, and thus the very tiny fraction that decays by particles emission has to be calculated. Therefore, Monte Carlo method is not suitable and another numerical method is used. In addition, KEWPIE2 code [5,6] can also give the time evolution of the cascade by solving Bateman type equations in order to calculate dynamical observables related to the fission time measurements of SHE [17]. Of course, it is analytically confirmed that for the case of emission of one kind of particle and fission decay, the results coincide with well-known formula at the limit of time being the infinity.

As usual, we use Weisskopf formula [22] for width of particles emission with transmission coefficients instead of the inverse cross sections and Bohr-Wheeler one for fission decay [23] with Kramers [24] and Strutinski correction [25] factors [3]. KEWPIE2 includes emissions of neutron, proton, triton, ^{3}He and alpha particles, and γ rays. The level-density formula employed is given by Bohr-Mottelson [18] with the parameter a calculated in terms of Toke-Swiatecki formula [19] with deformation dependence. Details are given in Ref. [6]. It was checked by comparisons with HIVAP [20].

3.2 Stability against fission by shell correction energy

We understand intuitively that the shell correction energy of the ground state gives rise to a stability against fission decay in addition to LDM barrier, neglecting that of the saddle point. This feature is taken into account in the calculation of the fission barrier that can be approximated by

$$B_f = B_f^{LDM} - \delta E, \tag{4}$$

where B_f^{LDM} is the barrier height of LDM, which is nearly equal to zero in SHE and δE is the shell correction energy of the compound nucleus. Correction energies are expected to be negative around the so-called SHE island, but are not yet known experimentally, of course. There are several predictions by structure models, but we usually employ those predicted by P. Moller et al. [21], as reference.

The decay is expressed in terms of Bohr-Wheeler formula given by the expression,

$$\Gamma_f^{BW} = \frac{1}{2\pi\rho_g(E^*)} \cdot \int_0^{E^*-B_f} d\epsilon\, \rho_s(E^* - B_f - \epsilon), \tag{5}$$

where ρ_g and ρ_s denote level density of the ground state and the saddle point, respectively. E^* is the excitation energy of the compound nucleus. The shell correction reflects itself in level density at low excitation. Ignatyuk et al. [4] suggested a practical parameterization of the level density parameter a,

$$a_g = a_g^{TS} \cdot [1 + f(E^*) \cdot \frac{\delta E}{E^*}]. \tag{6}$$

The superscript TS signifies the parameter by Toke and Swiatecki [19]. Here,

$$f(E^*) = 1 - \exp[-E^*/E_d], \tag{7}$$

where E_d is called shell damping energy. Assuming the parameter for the saddle point configuration a_s to be equal to a_g^{TK}, we can approximately evaluate the fission width with a simple expression,

$$\Gamma_f^{BW} \approx \exp(-B^{eff}/T), \tag{8}$$

where T denotes the temperature of the system and

$$B^{eff} = B_f + f(E^*) \cdot \delta E. \tag{9}$$

In two regimes of excitation energies, we can obtain physically reasonable behavior of the effective fission barrier B^{eff},

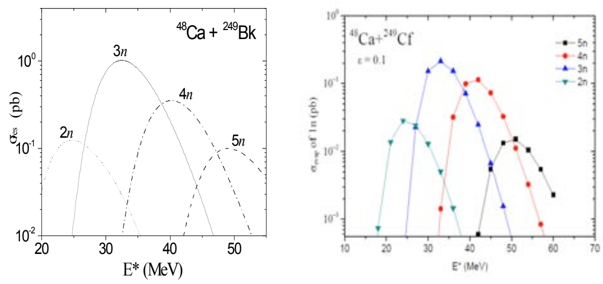

Fig. 3. Prediction of the excitation function for Z=117.

$$B^{eff} \longrightarrow B_f + E^* \cdot \frac{\delta E}{E_d} \approx B_f \quad \text{if} \quad E^* \ll E_d, \quad (10)$$

$$\longrightarrow B_f + \delta E \approx B_f^{LDM} \quad \text{if} \quad E^* \gg E_d. \quad (11)$$

At low excitation, the height is nearly equal to that of the ground state, i.e., to that stabilized by the shell correction energy, while at high excitation, it tends to that of LDM which is essentially zero. The computer codes accomodate the above features by precise numerical integration in Eq. (5), and thus are well applicable to SHE calculations.

If the fusion cross sections were known, we could extract δE from the experimental data with an accuracy of 1 MeV. The main ambiguities of the model are on the fusion part.

4 Example of Applications

According to the results given above, 2-dimensional Langevin calculations of the formation of the spherical compound nuclei are made for the new elements of $Z = 117$ and 118 with ^{48}Ca projectile plus actinide targets [8]. The neck parameter is taken to be zero which approximates the equilibrium distribution. The passing over the Coulomb barrier is calculated by the use of the empirical formula with modifications of the parameters suitable for those systems. Details are given in Refs. [8,16]. The results for $Z = 117$ are shown in Fig. 3. Experiments on those systems are strongly called for.

Another application is on the fission life time distribution over all possible compound nuclei involved in decay chain, starting from highly excited compound nucleus, which are recently measured by the crystal blocking method [17]. See Refs. [7,26].

5 Remarks

It is important to have a reliable theory for the fusion part of the reaction, in order to be able to constrain the shell correction energy from the experimental data. To achieve such a goal, one needs to reduce the number of free parameters to zero. Here, we have studied the neck degree of freedom that is sometimes used as a free parameter in some studies and the additional separation introduced by Swiatecki et al [12]. Systematic study of residue cross sections for SHE is under way. As for ambitious attempt of synthesizing the element of Z=120, we will provide reliable predictions of excitation functions for a few promising incident channels.

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