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Quark-Model Predictions for Axial Charges of Nucleon and N^* Resonances

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Abstract. We have investigated the axial charges of the nucleon and N^* resonances in a relativistic framework. Besides the axial charge of the nucleon, first predictions are reported for the axial charges of all well-established N^* resonances below ~1.9 GeV as produced by the relativistic constituent quark models relying on Goldstone-boson-exchange and one-gluon-exchange hyperfine interactions. The results for the axial charge of the nucleon are found close to experiment but with somewhat smaller values, similar to modern findings from quantum chromodynamics on the lattice. The predictions of the axial charges of the negative-parity $N^*(1535)$ and $N^*(1650)$ resonances also agree with what has most recently become available from lattice calculations. We discuss the roles of the axial charges of the N^* resonances for the phenomenon of chiral-symmetry restoration possibly occurring in the higher hadron spectra.

1 Introduction

The axial charge g_A of the nucleon (N) is an important quantity for the understanding of both the electroweak and strong interactions. Experimentally its value is well known, as the ratio $g_A/g_V = 1.2695 \pm 0.0029$ is measured to high precision [1]. In theory there have been many attempts to explain the size of g_A, where we mention only the studies along chiral perturbation theory [2] (see also the recent review [3]), chiral unitary approaches [4], constituent quark models [5,6], and lattice quantum chromodynamics (QCD) [7–14]; for results from the latter see also the most recent review by Renner [15]. All the theoretical predictions should evidently be consistent with the Goldberger-Treiman (GT) relation [16]

$$g_A = \frac{f_\pi g_{\pi NN}}{M_N}, \tag{1}$$

which connects the axial charge g_A with the π decay constant f_π, the πNN coupling constant $g_{\pi NN}$, and the nucleon mass M_N.

Some time ago also the axial charges of nucleon excitations have come into the focus of interest. While they are not accessible to experimental measurements, their theoretical study nevertheless appears to be relevant. Through the GT relation the N^* axial charges should give insight into the strengths of π coupling to the nucleon excitations. It has recently been suggested that chiral-symmetry restoration should be reflected by higher excitations in the hadron spectra [17,18]. In such a scenario there should appear chiral doublets of positive- and negative-parity states, and the coupling to the π should be weak. Consequently also the axial charges should become small or even vanishing with increasing excitation energy.

The axial charges of N^* resonances, however, can be "measured" within lattice QCD. First results have become available from ref. [12] for the two negative-parity nucleon excitations $N^*(1535)$ and $N^*(1650)$, with corresponding g_A values of ~0.00 and ~0.55, respectively. No results have so far been reported for positive-parity N^* resonances, but further studies along lattice QCD are under way by different groups.

We have recently started a comprehensive investigation of the axial charges of N^* resonances in the framework of the relativistic constituent quark model (RCQM). Specifically, we have extended a previous relativistic study of the nucleon axial form factors [5,6] to all of the lowest N^* excitations with $J^P = \frac{1}{2}^\pm$, $\frac{3}{2}^\pm$, and $\frac{5}{2}^\pm$. Our approach relies on solving the eigenvalue problem of a Poincaré-invariant mass operator in the framework of relativistic quantum mechanics. The axial current operator is chosen according to the spectator model (SM) [19]. For the RCQM we employed in the first instance the extended Goldstone-boson-exchange (EGBE) RCQM [20], as it is supposed to produce the most elaborate nucleon and N^* wave functions, but we also compare to predictions from the pseudoscalar Goldstone-boson-exchange (psGBE) [21,22] as well as the one-gluon-exchange (OGE) [23] RCQMs.

2 Formalism

The axial charge is defined through the value of the axial form factor $G_A(Q^2)$ at $Q^2 = 0$, where $Q^2 = -q^2$ is the four-momentum transfer. The axial form factor $G_A(Q^2)$ can be deduced from the relativistically invariant matrix element of the axial current operator $\hat{A}^\mu_a(Q^2)$, with flavor index a, sandwiched between the eigenstates of N or N^*. We denote the latter generally by $|P, J, \Sigma\rangle$, i.e. as eigenstates of the four-momentum operator $\hat{P}^\mu$, the intrinsic-spin operator $\hat{J}$ and its z-projection $\hat{\Sigma}$. Since $\hat{P}^\mu$ and the invariant mass

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operator $\hat{M}$ commute, these eigenstates can be obtained by solving the eigenvalue equation of $\hat{M}$

$$\hat{M}\,|P,J,\Sigma\rangle = M\,|P,J,\Sigma\rangle\ , \tag{2}$$

where M is the mass of N or N^*. For the various $J = \frac{1}{2}, \frac{3}{2}, \frac{5}{2}$ states considered here, the axial charges g_A are thus computed from the matrix elements of the axial current operator $\hat{A}^\mu_a$ for zero momentum transfer

$$\begin{aligned}
\left\langle P,\frac{1}{2},\Sigma'\left|\hat{A}^\mu_a\right|P,\frac{1}{2},\Sigma\right\rangle &= \bar{U}(P,\Sigma')g_A\gamma^\mu\gamma_5\frac{\tau_a}{2}U(P,\Sigma)\,,\\
\left\langle P,\frac{3}{2},\Sigma'\left|\hat{A}^\mu_a\right|P,\frac{3}{2},\Sigma\right\rangle &= \bar{U}^\nu(P,\Sigma')g_A\gamma^\mu\gamma_5\frac{\tau_a}{2}U_\nu(P,\Sigma)\,,\\
\left\langle P,\frac{5}{2},\Sigma'\left|\hat{A}^\mu_a\right|P,\frac{5}{2},\Sigma\right\rangle &= \bar{U}^{\nu\lambda}(P,\Sigma')g_A\gamma^\mu\gamma_5\frac{\tau_a}{2}U_{\nu\lambda}(P,\Sigma)\,.
\end{aligned} \tag{3}$$

Here $U(P,\Sigma)$ are the usual Dirac spinors for spin-$\frac{1}{2}$ particles, and $U^\nu(P,\Sigma)$ as well as $U^{\nu\lambda}(P,\Sigma)$ are the Rarita-Schwinger vector spinors [24] for spin-$\frac{3}{2}$ and spin-$\frac{5}{2}$ particles, respectively, where we use the same notation and normalization as specified in the appendix of ref. [25]. The γ^μ and γ_5 are the usual Dirac matrices and τ_a is the isospin matrix with Cartesian index a.

The matrix elements of $\hat{A}^\mu_a$ for any N or N^* read

$$\begin{aligned}
&\langle P,J,\Sigma'|\,\hat{A}^\mu_a(Q^2=0)\,|P,J,\Sigma\rangle = \\
&2M\sum_{\mu_i\mu'_i}\int d^3\mathbf{k}_1 d^3\mathbf{k}_2 d^3\mathbf{k}_3\frac{\delta^3(\mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3)}{2\omega_1 2\omega_2 2\omega_3}\\
&\times\Psi^\star_{PJ\Sigma'}\left(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3;\mu'_1,\mu'_2,\mu'_3\right)\\
&\times\left\langle k_1,k_2,k_3;\mu'_1,\mu'_2,\mu'_3\right|\hat{A}^\mu_a\,|k_1,k_2,k_3;\mu_1,\mu_2,\mu_3\rangle\\
&\times\Psi_{PJ\Sigma}\left(\mathbf{k}_1,\mathbf{k}_2,\mathbf{k}_3;\mu_1,\mu_2,\mu_3\right)\ .
\end{aligned} \tag{4}$$

The Ψ's are the rest-frame wave functions of the N or N^* with corresponding mass M and total angular momentum J with z-projections Σ and Σ'. Here they are represented as functions of the individual quark three-momenta $\mathbf{k}_i$, which sum up to $\mathbf{P} = \mathbf{k}_1+\mathbf{k}_2+\mathbf{k}_3 = \mathbf{0}$; $\omega_i = \sqrt{m_i^2+\mathbf{k}_i^2}$ is the energy of quark i with mass m_i, and the individual-quark spin orientations are denoted by μ_i.

The SM means that the matrix element of the axial current operator $\hat{A}^\mu_a$ between (free) three-particle states is assumed in the form

$$\begin{aligned}
&\left\langle k_1,k_2,k_3;\mu'_1,\mu'_2,\mu'_3\right|\hat{A}^\mu_a\,|k_1,k_2,k_3;\mu_1,\mu_2,\mu_3\rangle\\
&=3\,\langle k_1,\mu'_1|\,\hat{A}^\mu_{a,\mathrm{SM}}\,|k_1,\mu_1\rangle\,2\omega_2 2\omega_3\delta_{\mu_2\mu'_2}\delta_{\mu_3\mu'_3}\,.
\end{aligned} \tag{5}$$

For point-like quarks this matrix element involves the axial current operator of the active quark 1 (with quarks 2 and 3 being the spectators) in the form

$$\langle k_1,\mu'_1|\,\hat{A}^\mu_{a,\mathrm{SM}}\,|k_1,\mu_1\rangle = \bar{u}\,(k_1,\mu'_1)\,g^q_A\gamma^\mu\gamma_5\frac{\tau_a}{2}u\,(k_1,\mu_1)\ , \tag{6}$$

where $u\,(k_1,\mu_1)$ is the quark spinor and $g^q_A = 1$ the quark axial charge. A pseudovector-type current analogous to the one in Eq. (6) was recently also used in the calculation of $g_{\pi NN}$ and the strong πNN vertex form factor in ref. [26].

If we are interested only in the axial charges g_A, the expression (6) specifies to $\mu = i = 1,2,3$ and can further be evaluated to give

$$\begin{aligned}
&\bar{u}\,(k_1,\mu'_1)\,\gamma^i\gamma_5\frac{\tau_a}{2}u\,(k_1,\mu_1) = \\
&2\omega_1\chi^*_{\frac{1}{2},\mu'_1}\left\{\left[1-\frac{2}{3}\,(1-\kappa)\right]\sigma^i\right.\\
&\left.+\sqrt{\frac{5}{3}}\frac{\kappa^2}{1+\kappa}\,[\,[\mathbf{v}_1\otimes\mathbf{v}_1]_2\otimes\sigma]^i_1\right\}\frac{\tau_a}{2}\chi_{\frac{1}{2},\mu_1}\ ,
\end{aligned} \tag{7}$$

where $\kappa = 1/\sqrt{1+\mathbf{v}_1^2}$ and $\mathbf{v}_1 = \mathbf{k}_1/m_1$. Herein σ^i is the i-th component of the usual Pauli matrix σ and the symbol $[.\otimes.]^i_n$ denotes the i-th component of a tensor product $[.\otimes.]_n$ of rank n. We note that a similar formula was already published before by Dannbom et al. [27], however, restricted to the case of total orbital angular momentum $L = 0$. Our expression holds for any L, thus allowing to calculate g_A for the most general wave function of a baryon specified by J^P.

3 Results

First of all we present in Table 1 the predictions of the EGBE RCQM for the axial charges of the N ground state and the first two N^* excitations with $J^P = \frac{1}{2}^\pm$ and compare them to what is known from other approaches. Regarding the N, the g_A calculated from the EGBE RCQM is of a quality comparable to the lattice QCD results. The obtained value is slightly smaller than the experimental data of g_A=1.2695±0.0029 [1], showing a tendency of undershooting it like it is found also with modern lattice QCD results [15]. The nonrelativistic result (i.e. the one calculated with the nonrelativistic limit of the axial current operator in Eq. (6)) is much too high as is the result from the nonrelativistic $SU(6)\times O(3)$ quark model of Glozman and Nefediev [18].

Table 1. Predictions for axial charges g_A of the EGBE RCQM in comparison to available lattice QCD results [7–14], the values calculated by Glozman and Nefediev (GN) within the $SU(6)\times O(3)$ nonrelativistic quark model [18], and the nonrelativistic (NR) limit from the EGBE RCQM.

State	J^P	EGBE	Lattice QCD	GN	NR
$N(939)$	$\frac{1}{2}^+$	1.15	1.10~1.40	1.66	1.65
$N^*(1440)$	$\frac{1}{2}^+$	1.16	–	1.66	1.61
$N^*(1535)$	$\frac{1}{2}^-$	0.02	~0.00	-0.11	-0.20
$N^*(1710)$	$\frac{1}{2}^+$	0.35	–	0.33	0.42
$N^*(1650)$	$\frac{1}{2}^-$	0.51	~0.55	0.55	0.64

For the $J^P = \frac{1}{2}^-$ excitations $N^*(1535)$ and $N^*(1650)$ the predictions of the EGBE RCQM also compare well with the lattice QCD results, with a practically vanishing value for the former and a sizable g_A for the latter resonance. Again the corresponding nonrelativistic results are much different. With regard to the axial charge of $N^*(1535)$ it should be mentioned that a similarly small value was found before in a chiral unitary approach [4]. In the same context it is certainly remarkable that for $N^*(1535)$ a small g_A can obviously be obtained without advocating additional $\{QQQQ\bar{Q}\}$ components being responsible for its suppression [27].

The g_A of the positive-parity $N^*(1440)$ is practically of the same size as the one of the N, where no lattice-QCD results so far exist for this important case. For the next positive-parity excitation $N^*(1710)$ the EGBE RCQM predicts an axial charge of medium size, which is accidentally very similar to the nonrelativistic $SU(6)\times O(3)$ quark-model result.

In Tables 2 and 3 we present in a comprehensive manner the theoretical predictions of g_A for the N ground state and all positive- as well as negative-parity excitations with masses below ~1.9 GeV. The relativistic results calculated with the EGBE RCQM are compared to the ones by the other two types of RCQMs considered here, namely the psGBE as well as OGE RCQMs. At this instance, we regard the EGBE result as the most advanced one, as this particular RCQM includes all force components generated by GBE dynamics into the hyperfine interaction [20]. However, it is seen that also the psGBE RCQM, which relies only on a spin-spin hyperfine interaction, performs similarly well for all positive-parity resonances and for most of the negative-parity resonances; only for the $J^P = \frac{3}{2}^-$ states $N^*(1520)$ and $N^*(1700)$ there occur differences, which have evidently to be attributed to tensor and/or spin-orbit forces. Except for these two cases there are also no big deviations of the results with the OGE RCQM, even though the theoretical resonance masses show sometimes considerable differences.

Table 2. Mass eigenvalues and axial charges g_A of the N ground state and the positive-parity N^* resonances as predicted by the EGBE, the psGBE, and the OGE RCQMs.

		EGBE		psGBE		OGE	
State	J^p	Mass	g_A	Mass	g_A	Mass	g_A
$N^*(939)$	$\frac{1}{2}^+$	939	1.15	939	1.15	939	1.11
$N^*(1440)$	$\frac{1}{2}^+$	1464	1.16	1459	1.13	1578	1.10
$N^*(1710)$	$\frac{1}{2}^+$	1757	0.35	1776	0.37	1860	0.32
$N^*(1720)$	$\frac{3}{2}^+$	1746	0.35	1728	0.34	1858	0.25
$N^*(1680)$	$\frac{5}{2}^+$	1689	0.89	1728	0.83	1858	0.70

When we look at these results from the viewpoint of parity partners, we observe that the RCQM predictions are quite different within all positive- and negative-parity pairs up to $J^P = \frac{5}{2}^\pm$. For the first pair, the $N^*(1440)$ and $N^*(1535)$, the g_A of the former is sizable, whereas the one for the latter is almost vanishing. The relation is changed within the next pair, the $N^*(1710)$ and $N^*(1650)$, where both values

Table 3. Same as Table 2 for the negative-parity N^* resonances.

		EGBE		psGBE		OGE	
State	J^p	Mass	g_A	Mass	g_A	Mass	g_A
$N^*(1535)$	$\frac{1}{2}^-$	1498	0.02	1519	0.09	1520	0.13
$N^*(1650)$	$\frac{1}{2}^-$	1581	0.51	1647	0.46	1690	0.44
$N^*(1520)$	$\frac{3}{2}^-$	1524	-0.64	1519	-0.21	1520	-0.15
$N^*(1700)$	$\frac{3}{2}^-$	1608	-0.10	1647	-0.50	1690	-0.47
$N^*(1675)$	$\frac{5}{2}^-$	1676	0.84	1647	0.83	1690	0.80

of g_A are of comparable (medium) sizes, but the one of the negative-parity partner being bigger. Within the $J^P = \frac{3}{2}^\pm$ pair $N^*(1720)$ and $N^*(1700)$ the relation is again reversed, with the negative-parity partner exhibiting a negative value for g_A. Finally, for the $J^P = \frac{5}{2}^\pm$ pair $N^*(1680)$ and $N^*(1675)$ the axial charges are similar but have sizable magnitudes close to 1, showing no trend of decrease with growing J^P. In the present study we have not gone higher in nucleon resonances, because we cannot be sure that a RCQM with an infinitely rising confinement is appropriate for excitations beyond 2 GeV. The $J^P = \frac{3}{2}^-$ resonance $N^*(1520)$ has no parity partner. It has been claimed to be coupled strongly to the πNN channel [28]. We find a negative axial charge of considerable magnitude, at least for the EGBE RCQM. In this case the predictions of the different types of RCQMs are rather distinct, as already mentioned above in connection with the other such state $N^*(1700)$ showing a pronounced sensitivity to specific components in the hyperfine interaction.

4 Summary

We have presented first results from a study of axial charges of the N ground state and N^* resonances within the RCQM. From the comparisons that can be made so far on the one hand to experiment and on the other hand to lattice QCD data, we find quite reasonable predictions at least by the RCQMs relying on a hyperfine interaction derived from GBE. This kind of dynamics is supposed to model the spontaneous breaking of chiral symmetry in low-energy QCD. It produces a N excitation spectrum in close agreement with phenomenology. In particular, it guarantees the right level orderings of positive- and negative-parity nucleon resonances. Due to its explicit flavor dependence it is capable of describing at the same time and in a unified manner also the Λ spectrum as well as all other excitation spectra of the singlet, octet as well as decuplet states below ~1.9 GeV. Evidently, the GBE RCQM has no mechanism for chiral-symmetry restoration built in. As such it cannot be expected to yield results consistent with a scenario of chiral-symmetry restoration in higher hadron spectra. At present it is still an open problem, however, whether such a behaviour of QCD is indeed reflected in the higher N^* resonances and if yes from which excitation energy on it will appear. As the RCQM results agree with what is so far known from lattice QCD and other sources, it remains as an exciting question how these comparisons will finally turn out, once further results on axial charges of N^* resonances will become available from the other approaches.

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