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Understanding the $X(3872)$ with QCD sum rules

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Abstract. The nature of the meson $X(3872)$ is still under debate. Here we assume it to be a mixture between charmonium and exotic molecular $[c\bar{q}][q\bar{c}]$ states with $J^{PC} = 1^{++}$. We find that there is only a small range for the values of the mixing angle, θ, that can provide simultaneously good agreement with the experimental value of the mass and the decay width, and this range is $5^0 \le \theta \le 13^0$. In this range we get $m_X = (3.77 \pm 0.18)$ GeV and $\Gamma(X \to J/\psi\pi^+\pi^-) = (9.3 \pm 6.9)$ MeV, which are compatible, within the errors, with the experimental values. We, therefore, conclude that the $X(3872)$ is approximately 97% a charmonium state with 3% admixture of ~88% $D^0 D^{*0}$ molecule and ~12% D^+D^{*-} molecule.

1 Introduction

Even after many years under intense investigation the $X(3872)$ still represents a puzzle and, up to now, there is no consensus about its structure. It has been first observed by the Belle collaboration in the decay $B^+ \to X(3872)K^+ \to J/\psi\pi^+\pi^-K^+$ [1]. This observation was later confirmed by CDF, D0 and BaBar [2]. The current world average mass is $m_X = (3871.4 \pm 0.6)$ MeV which is at the threshold for the production of the charmed meson pair $D^0\bar{D}^{0*}$. This state is extremely narrow, with a width smaller than 2.3 MeV at 90% confiedence level. Both Belle and Babar collaborations reported the radiative decay mode $X(3872) \to \gamma J/\psi$ [3,4], which determines $C = +$. Further studies from Belle and CDF that combine angular information and kinematic properties of the $\pi^+\pi^-$ pair, strongly favor the quantum numbers $J^{PC} = 1^{++}$ or 2^{-+} [3,5,6].

The masses of the possible charmonium states with $J^{PC} = 1^{++}$ quantum numbers can be calculated with quark models and they are [7]: $2\ ^3P_1(3990)$ and $3\ ^3P_1(4290)$, which are much bigger than the observed mass. This discrepancy was taken to be an indication that the X meson is not a conventional quark-antiquark state. Another possibility explored was to treat this state as a multiquark state, composed by c, $\bar{c}$ and a light quark antiquark pair. Another experimental finding in favor of this conjecture is the fact that the decay rates of the processes $X(3872) \to J/\psi\,\pi^+\pi^-\pi^0$ and $X(3872) \to J/\psi\pi^+\pi^-$ are comparable [3]:

$$\frac{X \to J/\psi\,\pi^+\pi^-\pi^0}{X \to J/\psi\pi^+\pi^-} = 1.0 \pm 0.4 \pm 0.3. \qquad (1)$$

This approximate equality is an evidence of strong isospin and G parity violation, which is incompatible with a $c\bar{c}$ structure for $X(3872)$. In a multiquark approach we can avoid the isospin violation problem. The next natural question is: is the X made by four quarks in a bag or by a meson-meson molecule?

Some authors [8,9] proposed that the $X(3872)$ could be a molecular $(D^{*0}\bar{D}^0 - \bar{D}^{*0}D^0)$ bound state with small binding energy. The $D^{*0}\bar{D}^0$ molecule is not an isospin eigenstate and the rate in Eq.(1) could be explained in a very natural way in this model.

Some other authors [10] suggested that $X(3872)$ is a tetraquark. They have considered diquark-antidiquark states with $J^{PC} = 1^{++}$ and symmetric spin distribution:

$$X_q = [cq]_{S=1}[\overline{cq}]_{S=0} + [cq]_{S=0}[\overline{cq}]_{S=1} \qquad (2)$$

The isospin states with $I = 0, 1$ are given by $X(I = 0) = (X_u + X_d)/\sqrt{2})$ and $X(I = 1) = (X_u - X_d)\sqrt{2}$.

In [10] the authors argue that the physical states are closer to mass eigenstates and are no longer isospin eigenstates. The most general states are then:

$$X_l = \cos\theta X_u + \sin\theta X_d, \quad X_h = \cos\theta X_d - \sin\theta X_u, \qquad (3)$$

and both can decay into 2π and 3π. Imposing the rate in Eq.(1), they obtain $\theta \sim 20^0$. Given the uncertainties inherent to hadron spectroscopy, it is interesting to confront these theoretical results with QCD sum rules (QCDSR) calculations. This was partly done in [11] where, using the same tetraquark structure proposed in ref. [10], the mass difference $M(X_h) - M(X_l)$ was computed and found to be in agreement with the BaBar measurement $(M(X_h) - M(X_l) = (3.3 \pm 0.7)$ MeV). The same calculation [11] has obtained $m_X = (3.92 \pm 0.13)$ GeV. In QCDSR we can also use a current with the features of the mesonic molecule of the type $(D^{*0}\bar{D}^0 - \bar{D}^{*0}D^0)$. With such a current the calculation reported in [12] obtained the mass $m_X = (3.87 \pm 0.07)$ GeV in a better agreement with the experimental mass. Therefore, from a QCDSR point of view, the $X(3872)$ seems to be better described with a D^*D molecular current than with

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a diquark-antidiquark current. We feel though that the subject deserves further investigation.

In this work we discuss a new possibility: the mixing between two and four-quark states. This will be implemented folowing the prescription suggested in [13] for the light sector. The mixing is done at the level of the currents and will be extended to the charm sector. In a different context (not in QCDSR), a similar mixing was suggested already some time ago by Suzuki [14]. Physically, this corresponds to a fluctuation of the $c\bar{c}$ state where a gluon is emitted and subsequently splits into a light quark-antiquark pair, which lives for some time and behaves like a molecule-like state. As it will be seen, in order to be consistent with X decay data, we must consider a second mixing between: $(c\bar{c})$ + $(D^{*0}\bar{D}^0 - \bar{D}^{*0}D^0)$ and $(c\bar{c})$ + $(D^{*+}\bar{D}^- - \bar{D}^{*-}D^+)$.

2 The choice of the current

In [14] it was shown that, because of the very loose binding of the molecule, the production rates of a pure $X(3872)$ molecule should be at least one order of magnitude smaller than what is seen experimentally. Also, the recent observation, reported by BaBar [15], of the decay $X(3872) \to \psi(2S)\gamma$ at a rate:

$$\frac{\mathcal{B}(X \to \psi(2S)\,\gamma)}{\mathcal{B}(X \to \psi\gamma)} = 3.4 \pm 1.4, \tag{4}$$

is much bigger than the molecular prediction [16], which is $\sim 4 \times 10^{-3}$. This discrepancy could be interpreted as a strong point against the molecular model and as a point in favor of a conventional charmonium interpretation, it can also be interpreted as an indication that there is a significant mixing of the $c\bar{c}$ component with the $D^0\bar{D}^{*0}$ molecule. Similar conclusion was also reached in refs. [17,18]. Therefore, we will follow ref. [13] and consider a mixed charmonium-molecular current to study the $X(3872)$ in the QCD Sum Rule framework.

The charmonium part is well represented by the conventional axial current:

$$j'^{(2)}_\mu(x) = \bar{c}_a(x)\gamma_\mu\gamma_5 c_a(x). \tag{5}$$

The $D^0\ D^{*0}$ molecule is interpolated by [19–21]:

$$j^{(4u)}_\mu(x) = \frac{1}{\sqrt{2}}\bigg[\Big(\bar{u}_a(x)\gamma_5 c_a(x)\bar{c}_b(x)\gamma_\mu u_b(x)\Big) - \Big(\bar{u}_a(x)\gamma_\mu c_a(x)\bar{c}_b(x)\gamma_5 u_b(x)\Big)\bigg], \tag{6}$$

Following [13] we define the normalized two-quark current as

$$j^{(2u)}_\mu = \frac{1}{6\sqrt{2}}\langle\bar{u}u\rangle j'^{(2)}_\mu, \tag{7}$$

and from these two currents we build the following mixed charmonium-molecular current for the $X(3872)$:

$$J^u_\mu(x) = \sin(\theta) j^{(4u)}_\mu(x) + \cos(\theta) j^{(2u)}_\mu(x). \tag{8}$$

The quark condensate $\langle\bar{u}u\rangle$ in (7) is chosen in such a way that $j^{(2u)}_\mu(x)$ and $j^{(4u)}_\mu(x)$ have the same dimension. This is, of course, arbitrary and other choices would be possible. The numerical factor $\frac{1}{6\sqrt{2}}$ is chosen so that $j^{(2u)}_\mu(x)$ and $j^{(4u)}_\mu(x)$ have the same strength (the same "modulus") and therefore the mixing proportions of the two components are entirely defined by the choice of the angle θ. Other numerical choices would, in the end, imply different angles.

3 The two point correlator

The QCD sum rules [22–24] are constructed from the two-point correlation function

$$\Pi_{\mu\nu}(q) = i\int d^4x\, e^{iq.x}\langle 0|T[J^u_\mu(x)J^{u\dagger}_\nu(0)]|0\rangle = -\Pi_1(q^2)\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) + \Pi_0(q^2)\frac{q_\mu q_\nu}{q^2}. \tag{9}$$

The sum rules approach is based on the principle of duality. It consists in the assumption that the correlation function may be described at both quark and hadron levels. At the hadronic level (the phenomenological side) the correlation function is calculated introducing hadron characteristics such as masses and coupling constants. At the quark level, the correlation function is written in in terms of quark and gluon fields and a Wilson's operator product expansion (OPE) is used to deal with the complex structure of the QCD vacuum.

The phenomenological side is treated by first parametrizing the coupling of the axial vector meson 1^{++}, X, to the current, J^u_μ, in Eq. (8) in terms of the meson-current coupling parameter λ^u:

$$\langle 0|J^u_\mu|X\rangle = \lambda^u\epsilon_\mu\,. \tag{10}$$

Inserting intermediate states for the meson X, we can write the phenomenological side of Eq. (9) as

$$\Pi^{phen}_{\mu\nu}(q) = \frac{(\lambda^u)^2}{m_X^2 - q^2}\left(-g_{\mu\nu} + \frac{q_\mu q_\nu}{m_X^2}\right) + \cdots\,, \tag{11}$$

where the Lorentz structure projects out the 1^{++} state. The dots denote higher mass axial-vector resonances. This ressonances will be dealt with through the introduction of a continuum threshold parameter s_0.

In [25] it was pointed out that a single pole ansatz can be problematic in the case of a multiquark state, and that the two-hadron reducible (2HR) contribution (or S-wave $D\bar{D}^*$ contribution, in the present case) should also be considered in the phenomenological side. However, in ref. [26] it was shown that the 2HR contribution is very small. The reason for this is the following. The 2HR contribution, in our case, can be written as [26]:

$$\Pi^{2HR}_{\mu\nu}(q) = i(\lambda_{DD^*})^2\int\frac{d^4p}{(2\pi)^4}\left(\frac{-g_{\mu\nu} + p_\mu p_\nu/m^2_{D^*}}{p^2 - m^2_{D^*}} \times \frac{1}{(p-q)^2 - m_D^2}\right), \tag{12}$$

where

$$\langle 0|j_\mu^{(4u)}|DD^*(p)\rangle = \lambda_{DD^*}\varepsilon_\mu(p). \tag{13}$$

Following ref. [26] the current two-meson coupling: λ_{DD^*}, can be written in terms of the D meson decay constant, f_D, and the coupling of the D^* meson with a 4-quark current. This last quantity should be very small, because the properties of the D^* meson, both in spectroscopy and in scattering, are very well understood if it is an ordinary quark-antiquark state. Therefore, the parameter λ_{DD^*}, should be very small, as in the case of the pentaquark [26], and the 2HR contribution can be safely neglected.

We work up to dimension 8 at the leading order in α_s. The light quark propagators are calculated in coordinate-space and then Fourier transformed to the momentum space. The charm quark part is calculated directly into the momentum space, with finite m_c, and combined with the light part. The correlator in Eq. (9) can be written as:

$$\begin{aligned}\Pi_{\mu\nu}(q) = &\left(\frac{\langle\bar{u}u\rangle}{6\sqrt{2}}\right)^2\cos^2(\theta)\,\Pi_{\mu\nu}^{(2,2)}(q) + \\ &+\frac{\langle\bar{u}u\rangle}{6\sqrt{2}}(\sin(2\theta))\,\Pi_{\mu\nu}^{(2,4)}(q) + \\ &+\sin^2(\theta)\,\Pi_{\mu\nu}^{(4,4)}(q),\end{aligned} \tag{14}$$

with:

$$\Pi_{\mu\nu}^{(i,j)}(q) = i\int d^4x\, e^{iq.x}\langle 0|T[j_\mu^{(i)}(x)j_\nu^{(j)\dagger}(0)]|0\rangle. \tag{15}$$

After making a Borel transform of both sides, and transferring the continuum contribution to the OPE side, the sum rule for the axial vector meson up to dimension-eight condensates can be written as:

$$\begin{aligned}(\lambda^u)^2e^{-m_X^2/M^2} &= \\ =\left(\frac{\langle\bar{u}u\rangle}{6\sqrt{2}}\right)^2&\cos^2(\theta)\,\Pi_1^{(2,2)}(M^2) + \\ +\frac{\langle\bar{u}u\rangle}{6\sqrt{2}}&(\sin(2\theta))\,\Pi_1^{(2,4)}(M^2) + \\ &+\sin^2(\theta)\,\Pi_1^{(4,4)}(M^2),\end{aligned} \tag{16}$$

where:

$$\Pi_1^{(2,2)}(M^2) = \int_{4m_c^2}^{s_0} ds\, e^{-s/M^2}\rho_{pert}^{(22)}(s) + \Pi_{\langle G^2\rangle}^{(22)}(M^2), \tag{17}$$

$$\Pi_1^{(2,4)}(M^2) = \int_{4m_c^2}^{s_0} ds\, e^{-s/M^2}\rho_{\langle\bar{u}u\rangle}^{(24)}(s) + \Pi_{\langle\bar{u}Gu\rangle}^{(24)}(M^2), \tag{18}$$

$$\begin{aligned}\Pi_1^{(4,4)}(M^2) = &\int_{4m_c^2}^{s_0} ds\, e^{-s/M^2}\Big[\rho_{pert}^{(44)}(s) + \rho_{\langle\bar{u}u\rangle}^{(44)}(s) + \\ &+\rho_{\langle\bar{u}u\rangle^2}^{(44)}(s) + \rho_{\langle G^2\rangle}^{(44)}(s) + \rho_{\langle\bar{u}Gu\rangle}^{(44)}(s)\Big] + \\ &+\Pi_{\langle\bar{u}u\rangle\langle\bar{u}Gu\rangle}^{(44)}(M^2),\end{aligned} \tag{19}$$

and

$$\rho_{pert}^{(22)}(s) = \frac{s}{4\pi^2}\left(1-\frac{4m_c^2}{s}\right)^{\frac{3}{2}}, \tag{20}$$

$$\begin{aligned}\Pi_{\langle G^2\rangle}^{(22)}(M^2) &= -\frac{\langle g^2G^2\rangle}{3\cdot 2^5\pi^2}\int_0^1 d\alpha\Bigg[\frac{2\alpha(1-\alpha)M^2+m_c^2}{\alpha(1-\alpha)M^2} + \\ &+2m_c^2\frac{(2\alpha-1)m_c^2+\alpha(\alpha^2-1)M^2}{M^4\alpha^3(\alpha-1)}\Bigg]e^{-\frac{m_c^2}{\alpha(1-\alpha)M^2}},\end{aligned} \tag{21}$$

$$\rho_{\langle\bar{u}u\rangle}^{(24)}(s) = -\frac{\langle\bar{u}u\rangle}{6\sqrt{2}}\rho_{pert}^{(22)}(s), \tag{22}$$

$$\Pi_{\langle G^2\rangle}^{(24)}(M^2) = -\frac{\langle\bar{u}u\rangle}{6\sqrt{2}}\Pi_{\langle G^2\rangle}^{(22)}(M^2) \tag{23}$$

$$\Pi_{\langle\bar{u}Gu\rangle}^{(24)}(M^2) = \frac{5\langle\bar{q}g_s\sigma\cdot Gq\rangle}{3\cdot 2^6\sqrt{2}\pi^2}\int_0^1 d\alpha\,\frac{m_c^2}{1-\alpha}\,e^{-\frac{m_c^2}{\alpha(1-\alpha)M^2}}, \tag{24}$$

$$\rho_{pert}^{(44)}(s) = \frac{3}{2^{12}\pi^6}\int_{\alpha_{min}}^{\alpha_{max}} d\alpha\int_{\beta_{min}}^{\beta_{max}} d\beta\,\frac{1-(\alpha+\beta)^2}{\alpha^3\beta^3}K^4(\alpha,\beta), \tag{25}$$

$$\rho_{\langle\bar{u}u\rangle}^{(44)}(s) = -\frac{3m_c\langle\bar{u}u\rangle}{2^7\pi^4}\int_{\alpha_{min}}^{\alpha_{max}} d\alpha\int_{\beta_{min}}^{\beta_{max}} d\beta\frac{(1+\alpha+\beta)}{\alpha\beta^2}K^2(\alpha,\beta), \tag{26}$$

$$\rho_{\langle\bar{u}u\rangle^2}^{(44)}(s) = \frac{m_c^2}{2^4\pi^2}\langle\bar{u}u\rangle^2\sqrt{1-\frac{4m_c^2}{s}}, \tag{27}$$

$$\begin{aligned}\rho_{\langle G^2\rangle}^{(44)}(s) = \frac{\langle g^2G^2\rangle}{2^{11}\pi^6}\int_{\alpha_{min}}^{\alpha_{max}} d\alpha\int_{\beta_{min}}^{\beta_{max}} d\beta\Bigg[m_c^2\frac{(1-(\alpha+\beta)^2)}{\alpha^3} \\ -\frac{1-2\alpha-2\beta}{\alpha\beta^2}K(\alpha,\beta)\Bigg]K(\alpha,\beta),\end{aligned} \tag{28}$$

$$\begin{aligned}\rho_{\langle\bar{u}Gu\rangle}^{(44)}(s) = -\frac{3m_c}{2^8\pi^4}\langle\bar{u}g\sigma\cdot Gu\rangle\int_{\alpha_{min}}^{\alpha_{max}} d\alpha\Bigg[\frac{2(m_c^2-\alpha(1-\alpha)s)}{1-\alpha} \\ -\int_{\beta_{min}}^{\beta_{max}} d\beta\left(1-2\frac{\alpha+\beta}{\beta}\right)\frac{K(\alpha,\beta)}{\beta}\Bigg],\end{aligned} \tag{29}$$

$$\begin{aligned}\Pi_{\langle\bar{u}u\rangle\langle\bar{u}Gu\rangle}^{(44)}(M^2) = -\frac{m_c^2\langle\bar{u}u\rangle\langle\bar{u}g_s\sigma\cdot Gu\rangle}{2^5\pi^2}\int_0^1 d\alpha\Bigg[\frac{\alpha(1-\alpha)M^2+m_c^2}{\alpha(1-\alpha)M^2} \\ -\frac{1}{1-\alpha}\Bigg]e^{-\frac{m_c^2}{\alpha(1-\alpha)M^2}}.\end{aligned} \tag{30}$$

The integration limits are:

$$\alpha_{min} = \frac{1-\sqrt{1-\frac{4m_c^2}{s}}}{2},\qquad \alpha_{max} = \frac{1+\sqrt{1-\frac{4m_c^2}{s}}}{2}$$

$$\beta_{min} = \frac{\alpha}{\alpha\frac{q^2}{m_c^2} - 1}, \qquad \beta_{max} = 1 - \alpha$$

and we define $K(\alpha,\beta) \equiv (\alpha+\beta)m_c^2 - \alpha\beta q^2$. Taking the derivative of Eq. (16) with respect to $1/M^2$ and dividing the result by Eq. (16) we can obtain the mass of m_X without worrying about the value of the meson-current coupling λ^u. The expression thus obtained is analised numerically using the following values for quark masses and QCD condensates [11]:

$$\begin{aligned} m_c(m_c) &= (1.23 \pm 0.05)\ \text{GeV}, \\ \langle\bar{u}u\rangle &- (0.23 \pm 0.03)^3\ \text{GeV}^3, \\ \langle\bar{u}g\sigma.Gu\rangle &= m_0^2\langle\bar{u}u\rangle, \\ m_0^2 &= 0.8\ \text{GeV}^2, \\ \langle g^2G^2\rangle &= 0.88\ \text{GeV}^4. \end{aligned} \quad (31)$$

In Fig. 1 we show the contributions of the terms in Eqs. (20) to (30) grouped by condensate dimensions divided by the RHS of Eq. (16). We have used $s_0^{1/2} = 4.4$ GeV and $\theta = 9°$, but the situation does not change much for other choices of these parameters. It is clear that the OPE is converging for values of $M^2 \geq 2.6$ GeV^2 and we will limit our analysis to that region. The upper limit to the value of M^2 comes by imposing that the QCD pole contribution should be bigger than the continuun contribution. The maximum value of M^2 that satisfies this condition depends on the value of s_0, being more restrictive for smaller s_0. In Fig. 2 we show a comparison between the pole and continuun contributions for the smaller s_0 we will be considering ($s_0^{1/2} = 4.4$) and $\theta = 9°$. The condition obtained from Fig. 2 is $M^2 \leq 3.2$ GeV^2, but in this case, the dependence on the choice of θ is very strong. Taking into account the variation of θ we have determined that, for $5° \leq \theta \leq 13°$, the QCDSR are valid in the region $2.6\,\text{GeV}^2 \leq M^2 \leq 3.0\,\text{GeV}^2$. In Fig. 3, we show the X meson mass in this region. We see that the results are reasonably stable as a function of M^2. From Fig. 3 we obtain $m_X = (3.80 \pm 0.08)$ GeV where the error includes the variation of both s_0 and M^2. If we also take into account the variation of θ in the region $5° \leq \theta \leq 13°$ we get $m_X = (3.77 \pm 0.18)$ GeV, which is in a good agreement with the experimental value. The value obtained for the mass grows with the value of the mixing angle θ, but for $\theta \geq 30°$ it reaches a stable value being completely determined by the molecular part of the current. Wiht Eq. (16) we can also obtain λ^u by fixing m_X equal to the experimental value ($m_X = 3.87$ GeV). Using the same region in θ, s_0 and M^2 that we have used in the mass analysis we obtain:

$$\lambda^u = (3.6 \pm 0.9).10^{-3}\ \text{GeV}^5. \quad (32)$$

4 Decay of the X(3872)

As mentioned above, the X decays into $J/\psi\,\pi^+\pi^-\pi^0$, with a strength that is compatible to that of the $J/\psi\pi^+\pi^-$ mode, as given by Eq .(1). This decay suggests an appreciable transition rate to $J/\psi\,\omega$ and establishes strong isospin violating effects. It still does not completely exclude a $c\bar{c}$ interpretation for X since the origin of the isospin and G parity non-conservation in Eq. (1) could be of dynamical origin due to $\rho^0 - \omega$ mixing [27]. However, the observation of the ratio in Eq. (1) is an important point in favor of the molecular picture proposed by Swanson [16]. In this molecular picture the $X(3872)$ is mainly a $D^0\bar{D}^{*0}$ molecule with a small but important admixture of $\rho J/\psi$ and $\omega J/\psi$ components.

Although a $D^0\bar{D}^{*0}$ molecule is not an isospin eingenstate, the ratio in Eq. (1) can not be reproduced by a pure $D^0\bar{D}^{*0}$ molecule. This can be seen through the observation that the decay width for the decay $X \to J/\psi V \to J/\psi F$ where $F = \pi^+\pi^-(\pi^+\pi^-\pi^0)$ for $V = \rho(\omega)$ is given by [10,28]

$$\frac{d\Gamma}{ds}(X \to J/\psi f) = \frac{1}{8\pi m_X^2}|\mathcal{M}|^2 B_{V\to F} \times \frac{\Gamma_V m_V}{\pi}\frac{p(s)}{(s-m_V^2)^2 + (m_V\Gamma_V)^2}, \quad (33)$$

where

$$p(s) = \frac{\sqrt{\lambda(m_X^2, m_\psi^2, s)}}{2m_X}, \quad (34)$$

with $\lambda(a,b,c) = a^2+b^2+c^2-2ab-2ac-2bc$. The invariant amplitude squared is given by:

$$|\mathcal{M}|^2 = g_{X\psi V}^2 f(m_X, m_\psi, s), \quad (35)$$

where $g_{X\psi V}$ is the coupling constant in the vertex $XJ/\psi V$ and

$$f(m_X, m_\psi, s) = \frac{1}{3}\left(4m_X^2 - \frac{m_\psi^2 + s}{2} + \frac{(m_X^2 - m_\psi^2)^2}{2s} + \frac{(m_X^2 - s)^2}{2m_\psi^2}\right)\frac{m_X^2 - m_\psi^2 + s}{2m_X^2}. \quad (36)$$

Therefore, the ratio in Eq. (1) is given by:

$$\frac{\Gamma(X \to J/\psi\,\pi^+\pi^-\pi^0)}{\Gamma(X \to J/\psi\,\pi^+\pi^-)} = \frac{g_{X\psi\omega}^2 m_\omega \Gamma_\omega B_{\omega\to\pi\pi\pi} I_\omega}{g_{X\psi\rho}^2 m_\rho \Gamma_\rho B_{\rho\to\pi\pi} I_\rho}, \quad (37)$$

where

$$I_V = \int_{(nm_\pi)^2}^{(m_X-m_\psi)^2} ds \left(f(m_X, m_\psi, s) \times \frac{p(s)}{(s-m_V^2)^2 + (m_V\Gamma_V)^2}\right). \quad (38)$$

Using $B_{\omega\to\pi\pi\pi} = 0.89$, $\Gamma_\omega = 8.49$ GeV, $m_\omega = 782.6$ MeV, $B_{\rho\to\pi\pi} = 1$, $\Gamma_\rho = 149.4$ GeV and $m_\rho = 775.5$ MeV we get

$$\frac{\Gamma(X \to J/\psi\,\pi^+\pi^-\pi^0)}{\Gamma(X \to J/\psi\,\pi^+\pi^-)} = 0.118\left(\frac{g_{X\psi\omega}}{g_{X\psi\rho}}\right)^2. \quad (39)$$

The couplings, $g_{X\psi V}$, can be evaluated through a QCDSR calculation for the vertex, $X(3872)J/\psi V$, that centers in the three-point function given by

$$\Pi_{\mu\nu\alpha}(p, p', q) = \int d^4x d^4y\, e^{ip'.x}\, e^{iq.y}\,\Pi_{\mu\nu\alpha}(x, y), \quad (40)$$

with

$$\Pi_{\mu\nu\alpha}(x,y) = \langle 0|T[j_\mu^\psi(x) j_\nu^V(y) j_\alpha^{X^\dagger}(0)]|0\rangle, \quad (41)$$

where $p = p' + q$ and the interpolating fields are given by:

$$j_\mu^\psi = \bar{c}_a \gamma_\mu c_a, \quad (42)$$

$$j_\nu^V = \frac{N_V}{2}(\bar{u}_a \gamma_\nu u_a + (-1)^{I_V} \bar{d}_a \gamma_\nu d_a), \quad (43)$$

with $N_\rho = 1$, $I_\rho = 1$, $N_\omega = 1/3$ and $I_\omega = 0$. If $X(3872)$ is a pure $D^0\bar{D}^{*0}$ molecule, j_α^X is given by Eq. (6). In this case the only difference in the OPE side of the sum rule is the factor N_V and, therefore, regardless the approximations made in the OPE side and the number of terms considered in the sum rule one has

$$\Pi^V_{\mu\nu\alpha}(p,p',q) = N_V \Pi^{OPE}_{\mu\nu\alpha}(p,p',q). \quad (44)$$

To evaluate the phenomenological side of the sum rule we insert, in Eq.(41), intermediate states for X, J/ψ and V. We get [28]:

$$\Pi^{(phen)}_{\mu\nu\alpha}(p,p',q) = \frac{i\lambda_X m_\psi f_\psi m_V f_V \, g_{X\psi V}}{(p^2 - m_X^2)(p'^2 - m_\psi^2)(q^2 - m_V^2)} \times \Bigg(- \epsilon^{\alpha\mu\nu\sigma}(p'_\sigma + q_\sigma) - \epsilon^{\alpha\mu\sigma\gamma} \frac{p'_\sigma q_\gamma q_\nu}{m_V^2} - \epsilon^{\alpha\nu\sigma\gamma} \frac{p'_\sigma q_\gamma p'_\mu}{m_\psi^2}\Bigg). \quad (45)$$

Therefore, for a given structure the sum rule is given by:

$$\frac{i\lambda_X m_\psi f_\psi m_V f_V \, g_{X\psi V}}{(p^2 - m_X^2)(p'^2 - m_\psi^2)(q^2 - m_V^2)} = N_V \Pi^{OPE}(p,p',q), \quad (46)$$

from where, considering $m_\rho \simeq m_\omega$ one gets:

$$\frac{g_{X\psi\omega} f_\omega}{g_{X\psi\rho} f_\rho} = \frac{N_\omega}{N_\rho} = \frac{1}{3}. \quad (47)$$

Using $f_\rho = 157$ MeV and $f_\omega = 46$ MeV we obtain

$$\frac{g_{X\psi\omega}}{g_{X\psi\rho}} = 1.14, \quad (48)$$

and using this result in Eq. (39) we finally get

$$\frac{\Gamma(X \to J/\psi\, \pi^+\pi^-\pi^0)}{\Gamma(X \to J/\psi\, \pi^+\pi^-)} \simeq 0.15. \quad (49)$$

It is very important to notice that this is a very general result that does not depend on any approximation in the QCDSR. This result shows that the admixture of $\rho J/\psi$ and $\omega J/\psi$ components in the molecular model of ref.[16] is indeed very important to reproduce the data in Eq. (1). It is also important to notice that, in a QCDSR calculation of the decay rate $X \to J/\psi V$, the $c\bar{c}$ admixture in the $D^0\bar{D}^{*0}$ molecule, as given by Eq. (8), does not solve the problem of geting the ratio in Eq.(1). This can be seen by using, in Eq. (41), $j_\alpha^X = J_\alpha^u$, with J_α^u given by Eq. (8). One gets:

$$\Pi_{\mu\nu\alpha}(x,y) = \frac{\langle \bar{u}u \rangle}{2\sqrt{6}} \cos(\theta) \Pi^{c\bar{c}}_{\mu\nu\alpha}(x,y) + \sin(\theta) \Pi^{mol}_{\mu\nu\alpha}(x,y), \quad (50)$$

where

$$\Pi^{c\bar{c}}_{\mu\nu\alpha}(x,y) = \langle 0|T[j_\mu^\psi(x) j_\nu^V(y) j_\alpha^{'(2)^\dagger}(0)]|0\rangle, \quad (51)$$

and

$$\Pi^{mol}_{\mu\nu\alpha}(x,y) = \langle 0|T[j_\mu^\psi(x) j_\nu^V(y) j_\alpha^{(4u)^\dagger}(0)]|0\rangle, \quad (52)$$

with $j_\alpha^{'(2)}$ and $j_\alpha^{(4u)}$ given by Eqs. (5) and (6). Using the currents in Eqs.(43) and (42) for the mesons V and J/ψ, it is easy to see that

$$\Pi^{c\bar{c}}_{\mu\nu\alpha}(x,y) = \frac{N_V}{2} \,\mathrm{Tr}\left[\gamma_\mu S^c_{ac}(x)\gamma_\alpha\gamma_5 S^c_{ca}(-x)\right] \times \times \,\mathrm{Tr}\left[\gamma_\nu S^u_{bb}(0) + (-1)^{I_V}\gamma_\nu S^d_{bb}(0)\right]. \quad (53)$$

For $V = \rho$ with $I_\rho = 1$ the result in Eq. (53) is obviously zero due to isospin conservation, in the case that the quark u and d are degenerate. However, even for $V = \omega$ ($I_\omega = 0$), the result in Eq. (53) is zero because $\mathrm{Tr}\left[\gamma_\mu S^q_{bb}(0)\right] = 0$. Therefore, in the OPE side, the three-point function is given only by the molecular part of the current in Eq (8):

$$\Pi_{\mu\nu\alpha}(x,y) = \sin(\theta) \Pi^{mol}_{\mu\nu\alpha}(x,y), \quad (54)$$

that can not reproduce the experimental observation in Eq. (1), as demonstrated above.

In the following, to be able to reproduce the data in Eq.(1), instead of the admixture of $\rho J/\psi$ and $\omega J/\psi$ components to the $D^0\bar{D}^{*0}$ molecule, as done by Swanson [16], we will consider a small admixture of D^+D^{*-} and D^-D^{*+} components. In this case, instead of Eq.(8) we have

$$j_\mu^X(x) = \cos\alpha J_\mu^u(x) + \sin\alpha J_\mu^d(x), \quad (55)$$

with $J_\mu^u(x)$ and $J_\mu^d(x)$ given by Eq.(8).

If we consider the quarks u and d to be degenerate, *i.e.*, $m_u = m_d$ and $\langle\bar{u}u\rangle = \langle\bar{d}d\rangle$, the change in Eq.(8) to Eq.(55) does not make any difference in the results in Sec. III.

By inserting j_μ^X, given by Eq. (55), in Eq. (41) and considering the quarks u and d to be degenerate, one has

$$\Pi_{\mu\nu\alpha}(p,p',q) = \sin(\theta)\frac{N_V}{2\sqrt{2}}\Big(\cos\alpha + (-1)^{I_V}\sin\alpha\Big)\Pi^{OPE}_{\mu\nu\alpha}(p,p',q), \quad (56)$$

with

$$\Pi^{OPE}_{\mu\nu\alpha}(p,p',q) = \int d^4u \int \frac{d^4k}{(2\pi)^4}\Bigg(\mathrm{Tr}\left[\gamma_\mu S^c_{a'c}(k)\gamma_5 \times \times S^q_{ab'}(-y)\gamma_\nu S^q_{b'b}(y)\gamma_\alpha S^c_{ba'}(k-p')\right] + - \,\mathrm{Tr}\left[\gamma_\mu S^c_{a'c}(k)\gamma_\alpha S^q_{ab'}(-y)\gamma_\nu S^q_{b'b}(y)\gamma_5 S^c_{ba'}(k-p')\right]\Bigg). \quad (57)$$

In the phenomenological side, considering the definition of λ^u in Eq.(10) and the definition of the current in (55), we can define

$$\lambda_X = \cos\alpha\lambda^u + \sin\alpha\lambda^d = (\cos\alpha + \sin\alpha)\lambda^q, \qquad (58)$$

where λ^q was evaluated in Sec. III, and is given in Eq. (32). Using Eq.(58) in Eq.(45), the phenomenological side of the sum rule is now given by:

$$\Pi^{(phen)}_{\mu\nu\alpha}(p,p',q) = \frac{i(\cos\alpha+\sin\alpha)\lambda^q m_\psi f_\psi m_V f_V\, g_{X\psi V}}{(p^2-m_X^2)(p'^2-m_\psi^2)(q^2-m_V^2)}$$
$$\times\Bigg(-\epsilon^{\alpha\mu\nu\sigma}(p'_\sigma+q_\sigma) - \epsilon^{\alpha\mu\sigma\gamma}\frac{p'_\sigma q_\gamma q_\nu}{m_V^2}$$
$$-\epsilon^{\alpha\nu\sigma\gamma}\frac{p'_\sigma q_\gamma p'_\mu}{m_\psi^2}\Bigg). \qquad (59)$$

Combining Eqs. (56) and (59) we get the following relation between the coupling constants:

$$\frac{g_{X\psi\omega} f_\omega}{g_{X\psi\rho} f_\rho} = \frac{N_\omega\Big(\cos\alpha+\sin\alpha\Big)}{N_\rho\Big(\cos\alpha-\sin\alpha\Big)}. \qquad (60)$$

Substituting the above result into Eq. (39) and the using the numerical values for f_ω and f_ρ we have

$$\frac{\Gamma(X\to J/\psi\,\pi^+\pi^-\pi^0)}{\Gamma(X\to J/\psi\,\pi^+\pi^-)} \simeq 0.15\left(\frac{\cos\alpha+\sin\alpha}{\cos\alpha-\sin\alpha}\right)^2. \qquad (61)$$

The above relation determines $\alpha \sim 20^0$ for reproducing the experimental result in Eq.(1). A similar relation was obtained in ref. [29] where the decay of the X into two and three pions goes through a $D\,D^*$ loop. Concerning this calculation, it would be interesting to check if the obtained results would change once charm meson form factors [30] are introduced.

Once the mixing angle α is defined, we can evaluate the decay rate itself, for any one of the decays: $X\to J/\psi\rho$ or $X\to J/\psi\omega$, since they will be the same. Therefore, we choose to work with $X\to J/\psi\omega$ since the combination $\cos\alpha+\sin\alpha$ appears in both sides of the sum rule and the result for $g_{X\psi\omega}$ is independent of α. In the OPE side we consider condensates up to dimension five , as shown in Fig. 4 and Fig. 5 . Taking the limit $p^2 = p'^2 = -P^2$ and doing a single Borel transform to $P^2 \to M^2$, we get in the structure $\epsilon^{\alpha\nu\sigma\gamma}p'_\sigma q_\gamma p'_\mu$ (the same considered in ref.[28]) ($Q^2 = -q^2$):

$$C(Q^2)\left(e^{-m_\psi^2/M^2} - e^{-m_X^2/M^2}\right) + B\,e^{-s_0/M^2} =$$
$$(Q^2+m_\omega^2)\Pi^{(OPE)}(M^2,Q^2), \qquad (62)$$

where

$$\Pi^{(OPE)}(M^2,Q^2) = \frac{\langle\bar{q}q\rangle}{6\sqrt{2}\pi^2Q^2}\Bigg[\left(\frac{m_0^2}{3Q^2}+\right.$$
$$\left.-1\right)\int_{4m_c^2}^{u_0} du\, e^{-u/M^2}\sqrt{1-4m_c^2/u}\left(\frac{1}{2}+\frac{m_c^2}{u}\right)+$$
$$-\frac{m_0^2}{16}\int_0^1 d\alpha\frac{1+3\alpha}{\alpha}\,e^{\frac{-m_c^2}{\alpha(1-\alpha)M^2}}\Bigg]. \qquad (63)$$

In Eq. (62)

$$C(Q^2) = \frac{6}{\sin(\theta)} m_\omega f_\omega \frac{f_\psi\lambda^q}{m_\psi(m_X^2-m_\psi^2)} g_{X\psi\omega}(Q^2), \qquad (64)$$

and B gives the contribution of the pole-continuum transitions [28,31,32]. s_0 and u_0 are the continuum thresholds for X and J/ψ respectively. Notice that in Eq.(63) we have introduced the form factor $g_{X\psi\omega}(Q^2)$. This is because the meson ω is off-shell in the vertex $XJ/\psi\omega$.

If we parametrize $C(Q^2)$ as a monopole:

$$C(Q^2) = \frac{c_1}{Q^2+c_2}, \qquad (65)$$

we can fit the left hand side of Eq. (62) as a function of Q^2 and M^2 to the QCDSR results in the right hand side, obtaining c_1, c_2 and B. The function $C(Q^2)$ (and consequently $g_{X\psi\omega}(Q^2)$) should not depend on M^2, so we limit our fit region to 3.0 GeV$^2 \le M^2 \le 3.5$ GeV^2 where $C(Q^2)$ is clearly stable in M^2 for all values of Q^2.

We do the fitting for $s_0^{1/2} = 4.4$ GeV as the results do not depend much on this parameter. The results are:

$$c_1 = 2.5\times10^{-2}\ \text{GeV}^7,$$
$$c_2 = 38\ \text{GeV}^2,$$
$$B = 2.9\times10^{-4}\ \text{GeV}^5. \qquad (66)$$

From Eqs. (64) and (65) we can obtain the form factor $g_{X\psi\omega}(Q^2)$. Since the coupling constant is defined as the value of the form factor at the meson pole: $Q^2 = -m_\omega^2$, to determine the coupling constant we have to extrapolate $g_{X\psi\omega}(Q^2)$ to a Q^2 region where the sum rules are no longer valid (since the QCDSR results are valid in the deep Euclidian region). Using $m_\psi = 3.1$ GeV, $m_X = 3.87$ GeV, $f_\psi = 0.405$ GeV, $\lambda^u = 3.6\times10^3$ GeV^5 from Eq. (32) and varying θ in the range $5^\circ \le \theta \le 13^\circ$, we get:

$$g_{X\psi\omega} = g_{X\psi\omega}(-m_\omega^2) = 5.4\pm2.4 \qquad (67)$$

The decay width is given by:

$$\Gamma\left(X\to J/\psi\pi^+\pi^-\pi^0\right) = g^2_{X\psi\omega}\frac{m_\omega\Gamma_\omega}{8\pi^2m_X^2}B_{\omega\to\pi\pi\pi}I_\omega, \qquad (68)$$

which, together with Eq. (67) gives us:

$$\Gamma\left(X\to J/\psi\pi^+\pi^-\pi^0\right) = (9.3\pm6.9)\ \text{MeV}. \qquad (69)$$

The result in Eq. (69) is in complete agreement with the experimental upper limit. It is important to notice that the width grows with the mixing angle θ, as can be seen from Eq. (64), while the mass grows with θ. Therefore, there is only a small range for the values of this angle that can provide simultaneously good agreement with the experimental values of the mass and the decay width, and this range is $5^\circ \le \theta \le 13^\circ$. This means that the $X(3872)$ is basically a $c\bar{c}$ state with a small, but fundamental, admixture of molecular DD^* states. By molecular states we mean an admixture between $D^0\bar{D}^{*0}$, $\bar{D}^0D^{*0}$ and D^+D^{*-}, D^-D^{*+} states, as given by Eq. (55).

5 Conclusions

We have presented a QCDSR analysis of the two-point and three-point functions of the $X(3872)$ meson, by considering a mixed charmonium-molecular current. We find that the sum rules results are compatible with experimental data. These results were obtained by considering the mixing angle in Eq. (8) in the range $5° \leq \theta \leq 13°$.

We have also studied the mixing between the $D^0\bar{D}^{*0}$, $\bar{D}^0 D^{*0}$ and D^+D^{*-}, D^-D^{*+} states by imposing the ratio in Eq. (1). In accordance with the findings in ref. [10] we found that the mixing angle in Eq. (55) is $\alpha \sim 20^0$.

With the knowledge of these two mixing angles we conclude that the $X(3872)$ is basically a $c\bar{c}$ state ($\sim$97%) with the molecular $D^0\bar{D}^{*0}$, $\bar{D}^0 D^{*0}$ ($\sim$88%) and D^+D^{*-}, D^-D^{*+} ($\sim$12%) states.

This small molecular component could, in principle, be a consequence of neglecting the two-hadron reducible contribution in the phenomenological side. However, as argued in section III, we expect the 2HR contribution to be small and the results to hold even if we had taken it into consideration.

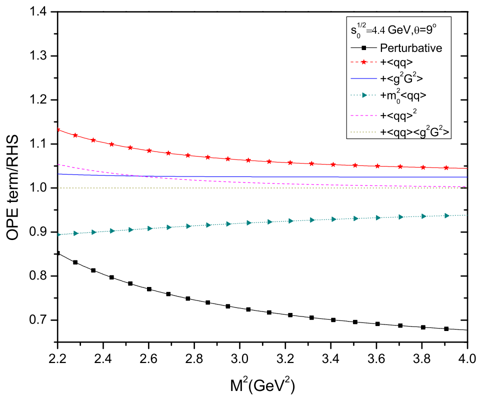

Fig. 1. Relative contributions of the terms in eqs. (20) to (30) grouped by condensate dimensions. We start with the perturbative contribution and each subsequent line represents the addition of one extra condensate in the expansion.

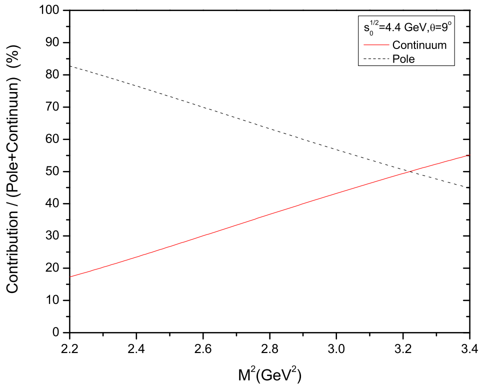

Fig. 2. The dashed line shows the relative pole contribution (the pole contribution divided by the total, pole plus continuum, contribution) and the solid line shows the relative continuum contribution.

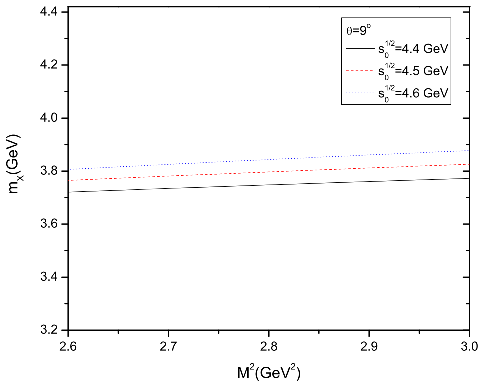

Fig. 3. The X meson mass as a function of the sum rule parameter (M^2) in the stability region for different values of the continuum threshold: $s_0^{1/2} = 4.4$ GeV (solid line), $s_0^{1/2} = 4.5$ GeV (dashed line) and $s_0^{1/2} = 4.6$ GeV (dotted line).

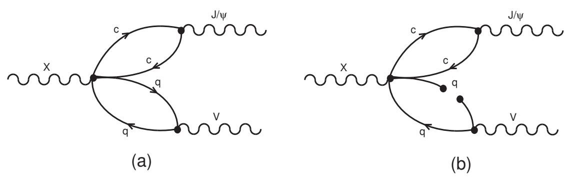

Fig. 4. Leading order diagrams which contribute to the X decay.

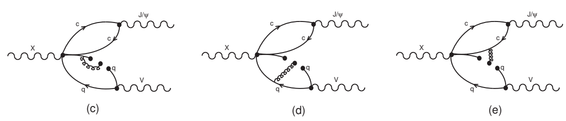

Fig. 5. Non-perturbative corrections to the leading order diagrams of the X decay.

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