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Meson-nuclear clusters in the few-body approaches

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Abstract. Bound states in the 3-body $\phi\phi N$ system are considered. Two dimensional Faddeev equations in differential form were used. Some approximations were made during the calculation. For the $\phi - \phi$ interaction a new short-range D-wave potential was constructed, which fits the parameters of the $f_2(2010)$ resonance in the elastic channel ($f_2 \longrightarrow \phi + \phi$). For the $\phi - N$ interaction attractive forces supporting the binding energy in the ϕN system equal to 9 MeV were used.

1 Introduction

In [1] we considered, on the basis of Faddeev equations, the bound states in the systems containing one and two ϕ-mesons like ϕNN and $\phi\phi N$ where N is a neutron or a proton, and the potential $V_{\phi\phi}$ acts in the s-state.

Below we present the result of calculations for the systems with two ϕ-mesons like $\phi\phi N$ obtained in the framework of the Faddeev differential equations with $V_{\phi\phi}$ acting in the d-wave state.

Attention to the ϕ-meson- nuclear systems is called by two facts: 1) quark structure of ϕ-meson wave function mainly defined by the $\overline{s}s$ - configuration of strange quarks, 2) important role of the strange $\overline{s}s$- sea quarks in the nucleon. So one can expect exchange effects between these two hadrons.

Indications of the role of $\overline{s}s$-sea quarks follow from different experiments such as πN scattering, $\overline{p}p$- annihilation, strange part of form factors of nucleons and so on.

Indeed, in the eighties (see, e.g. [2]) it already became clear that the $\overline{s}s$ content of the nucleon is closely related to the so-called σ-term in πN - scattering, and is not very small. Later estimation of the strange scalar density y in the nucleon [3] gave

$$y = \frac{2 < p|\overline{s}s|p >}{< p|\overline{u}u + \overline{d}d|p >} = 1 - \frac{\sigma_0}{\Sigma} \qquad (1)$$

the value $y \approx 0.5$ which corresponds to the size of the πN Σ- term $\Sigma = 64 \pm 8$ MeV ($\sigma_0 = 36 \pm 7$ MeV is the octet baryon mass difference). However, manifestation of strange sea quarks appears not only in the internal structure of the nucleon [4], but also by the intensity of some reactions.

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It turns out, for example, that there are a number of reactions which proceed with large violations of the OZI-rule (Okuba-Zweig-Izuka).

The brightest example of this phenomenon can be seen in the $\overline{p}p$ - annihilation processes, if one compares the ratio of processes with ϕ and ω meson production with the corresponding predictions by the OZI-rule. In some cases, a few ten times difference is reached [5]. The same type of phenomena (OZI-rule violation) can be seen in the processes of ϕ-meson-nucleon interaction which are suppressed by the OZI-rule but are strong in reality.

Analysis of the whole picture for the hidden-strangeness amplitudes shows that their values are mainly defined by J^{PC} quantum numbers which have $\overline{s}s$-pairs in the corresponding processes or systems [5].

This observation makes understandable the fact that the OZI-rule is fulfilled in some processes and why it is violated in others.

Now, if one comes to the few-particle systems which consist of nucleon(s) and a number of ϕ-mesons (1 or 2), having mainly $\overline{s}s$ quark structure, one can expect a wider range of quantum numbers carried by $\overline{s}s$-pairs and, correspondingly, more possibilities for the manifestation of strange sea quarks in nucleons.

As it was mentioned above, in our previous work [1], we started the above program considering the 3-body systems $\phi + 2N$ and $2\phi + N$, with s-wave potentials.

In this report, we consider the 3-particle system $2\phi + N$, where $V_{\phi\phi}$ acts in the d-wave.

2 Calculations

2.1 Faddeev equations

The method of treatment is the Faddeev equations in differential form.

$$\begin{cases} -\frac{\hbar^2}{2M}(\Delta_{\bar\eta_\alpha}+\Delta_{\bar\xi_\alpha})\Psi_1(\bar\eta_1,\bar\xi_1)+ \\ \quad V_{23}\Psi(\bar\eta_\alpha,\bar\xi_\alpha) = E_0\Psi_1(\bar\eta_1,\bar\xi_1) \\ -\frac{\hbar^2}{2M}(\Delta_{\bar\eta_\alpha}+\Delta_{\bar\xi_\alpha})\Psi_2(\bar\eta_2,\bar\xi_2)+ \\ \quad V_{13}\Psi(\bar\eta_\alpha,\bar\xi_\alpha) = E_0\Psi_2(\bar\eta_2,\bar\xi_2) \\ -\frac{\hbar^2}{2M}(\Delta_{\bar\eta_\alpha}+\Delta_{\bar\xi_\alpha})\Psi_3(\bar\eta_3,\bar\xi_3)+ \\ \quad V_{12}\Psi(\bar\eta_\alpha,\bar\xi_\alpha) = E_0\Psi_3(\bar\eta_3,\bar\xi_3) \end{cases} \tag{2}$$

The Jacobi coordinates are defined as usual:

$$\mathbf{r}_i - \mathbf{r}_j = \frac{\bar\eta_\alpha}{a_\alpha} \tag{3}$$

$$\frac{m_i\mathbf{r}_i + m_j\mathbf{r}_j}{m_i+m_j} - \mathbf{r}_k = \frac{\bar\xi_\alpha}{b_\alpha} \tag{4}$$

where $\mathbf{r}_i$, m_i denote the radius-vector and the mass of particle i,

$$a_\alpha = \sqrt{\frac{m_i m_j}{(m_i+m_j)M}},\ b_\alpha = \sqrt{\frac{m_k(m_i+m_j)}{M^2}},$$

$$M = m_1 + m_2 + m_3$$

and indices α take on the following values: $\alpha = 3$ for $(ij)k = (12)3$, $\alpha = 1$ for $(ij)k = (23)1$, $\alpha = 2$ for $(ij)k = (31)2$.

Partial wave decomposition of components of Ψ has the form:

$$\Psi_\alpha(\bar\eta_\alpha,\bar\xi_\alpha) = \sum_{LMl\lambda}\frac{1}{\eta_\alpha\xi_\alpha}U_\alpha^{Ll\lambda}(\eta_\alpha,\xi_\alpha)$$

$$Y_{l\lambda}^{LM}(\hat\eta_\alpha,\hat\xi_\alpha) \tag{5}$$

where $\eta_\alpha = |\bar\eta_\alpha|$, $\xi_\alpha = |\bar\xi_\alpha|$, $\hat\eta_\alpha = \bar\eta_\alpha/|\bar\eta_\alpha|$, $\hat\xi_\alpha = \bar\xi_\alpha/|\bar\xi_\alpha|$, $Y_{l\lambda}^{LM}$ are the bispherical harmonics.

Below we will consider the s-wave $\phi-N$ interaction and the d-wave $\phi-\phi$ interaction which correspond to the $f_2(2010)$ resonance in the d-wave [PDG].

So we consider the system with $L = 2$. In that case from (5), we have

$$\Psi_1(\bar\eta_1,\bar\xi_1) = \sum_{0\le\lambda\le4}\frac{1}{\eta_1\xi_1}U_1^{22\lambda}(\eta_1,\xi_1)Y_{2\lambda}^{20}(\hat\eta_1,\hat\xi_1)$$

$$\Psi_2(\bar\eta_2,\bar\xi_2) = \frac{1}{\eta_2\xi_2}U_2^{202}(\eta_2,\xi_2)Y_{02}^{20}(\hat\eta_2,\hat\xi_2) \tag{6}$$

and $\Psi_3 = \Psi_2$ due to 2 identical particles (2ϕ-mesons).

As a result, we arrive at 7 two-dimensional equations for the partial amplitudes of the Faddeev components

$$\begin{cases} \widehat{D}_1^{20}U_1^{220} = \\ \quad V_1\left(\frac{\eta_1}{a_1}\right)\sum_{\alpha'l'\lambda'}\int\frac{\eta_1\xi_1}{\eta_{\alpha'}\xi_{\alpha'}}U_{\alpha'}^{2l'\lambda'}(\eta_{\alpha'},\xi_{\alpha'}) \\ \quad Y_{l'\lambda'}^{20}(\hat\eta_{\alpha'},\hat\xi_{\alpha'})Y_{20}^{*20}(\hat\eta_1,\hat\xi_1)\,d\Omega(\hat\eta_1)\,d\Omega(\hat\xi_1) \\ \widehat{D}_1^{21}U_1^{221} = 0 \\ \widehat{D}_1^{22}U_1^{222} = \\ \quad V_1\left(\frac{\eta_1}{a_1}\right)\sum_{\alpha'l'\lambda'}\int\frac{\eta_1\xi_1}{\eta_{\alpha'}\xi_{\alpha'}}U_{\alpha'}^{2l'\lambda'}(\eta_{\alpha'},\xi_{\alpha'}) \\ \quad Y_{l'\lambda'}^{20}(\hat\eta_{\alpha'},\hat\xi_{\alpha'})Y_{20}^{*}(\hat\eta_1)Y_{20}^{*}(\hat\xi_1)\,d\Omega(\hat\eta_1)\,d\Omega(\hat\xi_1) \\ \quad (-\sqrt{2/7}) \\ \widehat{D}_1^{23}U_1^{223} = 0 \\ \widehat{D}_1^{24}U_1^{224} = \\ \quad V_1\left(\frac{\eta_1}{a_1}\right)\sum_{\alpha'l'\lambda'}\int\frac{\eta_1\xi_1}{\eta_{\alpha'}\xi_{\alpha'}}U_{\alpha'}^{2l'\lambda'}(\eta_{\alpha'},\xi_{\alpha'}) \\ \quad Y_{l'\lambda'}^{20}(\hat\eta_{\alpha'},\hat\xi_{\alpha'})Y_{20}^{*}(\hat\eta_1)Y_{40}^{*}(\hat\xi_1)\,d\Omega(\hat\eta_1)\,d\Omega(\hat\xi_1) \\ \quad (\sqrt{2/7}) \\ \widehat{D}_2^{02}U_2^{202} = \\ \quad V_2\left(\frac{\eta_2}{a_2}\right)\sum_{\alpha'l'\lambda'}\int\frac{\eta_2\xi_2}{\eta_{\alpha'}\xi_{\alpha'}}U_{\alpha'}^{2l'\lambda'}(\eta_{\alpha'},\xi_{\alpha'}) \\ \quad Y_{l'\lambda'}^{20}(\hat\eta_{\alpha'},\hat\xi_{\alpha'})Y_{02}^{*20}(\hat\eta_2,\hat\xi_2)\,d\Omega(\hat\eta_2)\,d\Omega(\hat\xi_2) \\ \widehat{D}_3^{02}U_3^{202} = \\ \quad V_3\left(\frac{\eta_3}{a_3}\right)\sum_{\alpha'l'\lambda'}\int\frac{\eta_3\xi_3}{\eta_{\alpha'}\xi_{\alpha'}}U_{\alpha'}^{2l'\lambda'}(\eta_{\alpha'},\xi_{\alpha'}) \\ \quad Y_{l'\lambda'}^{20}(\hat\eta_{\alpha'},\hat\xi_{\alpha'})Y_{02}^{*20}(\hat\eta_3,\hat\xi_3)\,d\Omega(\hat\eta_3)\,d\Omega(\hat\xi_3) \end{cases} \tag{7}$$

$$\widehat{D}_\alpha^{l\lambda} = \frac{\hbar^2}{2M}\left(\frac{\partial^2}{\partial\rho^2}+\frac{1}{\rho}\frac{\partial}{\partial\rho}+\frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi_\alpha^2}-\right.$$

$$\left.\frac{l(l+1)}{(\rho\cos\varphi_\alpha)^2}-\frac{\lambda(\lambda+1)}{(\rho\sin\varphi_\alpha)^2}\right)+E_0 \tag{8}$$

$$\rho = \sqrt{\eta_\alpha^2+\xi_\alpha^2},\quad \tan\varphi_\alpha = \xi_\alpha/\eta_\alpha$$

$$V_\alpha = V_{ij}$$

2.2 Approximations

To solve the system of Eq. (7), let us make some approximations.

Approximation 1. There are 4 equations containing operators $\widehat{D}_1^{2\lambda}$ with $\lambda > 0$. The corresponding equations include terms with more centrifugal repulsion and due to this one can neglect the corresponding components of the wave function. Thus, 3 equations are left, 2 of which are identical. As a result, we arrive at the following two two-dimensional integrodifferential equations:

$$\begin{cases} \left(\widehat{D} + \frac{\hbar^2}{2M}\frac{6}{(\rho\cos\varphi)^2} + V_1\left(\frac{\rho\cos\varphi}{a_1}\right) - E\right) \\ \quad U_1(\rho,\varphi) = -V_1\left(\frac{\rho\cos\varphi}{a_1}\right)\sum_{\alpha'\neq 1} \\ \quad \frac{2}{\sin(2\gamma_{\alpha'1})}\int_{c-}^{c+} U_{\alpha'}(\rho,\varphi')\,h_{\alpha'1}(\varphi,\varphi')\,d\varphi' \\ \left(\widehat{D} + \frac{\hbar^2}{2M}\frac{6}{(\rho\sin\varphi)^2} + V_2\left(\frac{\rho\cos\varphi}{a_2}\right) - E\right) \\ \quad U_2(\rho,\varphi) = -V_2\left(\frac{\rho\cos\varphi}{a_2}\right)\sum_{\alpha'\neq 2} \\ \quad \frac{2}{\sin(2\gamma_{\alpha'2})}\int_{c-}^{c+} U_{\alpha'}(\rho,\varphi')\,h_{\alpha'2}(\varphi,\varphi')\,d\varphi' \end{cases} \tag{9}$$

$$(U_3 \equiv U_2)$$

where $U_1 = U_1^{220}$, $U_2 = U_2^{202}$, $V_1 = V_{\phi\phi}$, $V_2 = V_{\phi N}$,

$$\widehat{D} = -\frac{\hbar^2}{2M}\left(\frac{\partial^2}{\partial\rho^2} + \frac{1}{\rho}\frac{\partial}{\partial\rho} + \frac{1}{\rho^2}\frac{\partial^2}{\partial\varphi^2}\right)$$

$$c+ = \mathrm{Min}\left\{|\varphi + \gamma_{\alpha'\alpha}|\,,\, \pi - (\varphi + \gamma_{\alpha'\alpha})\right\}$$

$$c- = |\varphi - \gamma_{\alpha'\alpha}|$$

$$\gamma_{ij} = \arcsin s_{ij}\ ,$$

$$s_{ij} = \sqrt{\frac{m_k M}{(m_i + m_k)(m_j + m_k)}}\,,$$

$$(ijk = 123,\, 231,\, 312)$$

indices correspond to 1 for ϕ-meson, 2 and 3 for nucleons. From the expression which is given in [6] one gets

$$h_{\alpha'1} = 1/8 + (3/8)\cos 2\,\theta_{\bar{\xi}_{\alpha'}}$$
$$h_{\alpha'2} = 1/8 + (3/8)\cos 2\,(\theta_{\bar{\xi}_2} - \theta_{\bar{\eta}_{\alpha'}})$$

where

$$\cos\theta_{\eta_{\alpha'}} = (-\cos\gamma_{\alpha'\alpha}\cos\varphi + \epsilon_{\alpha'\alpha}\sin\gamma_{\alpha'\alpha}\cos\theta_{\xi_\alpha}\sin\varphi)/\cos\varphi'$$
$$\cos\theta_{\xi_{\alpha'}} = -(\epsilon_{\alpha'\alpha}\sin\gamma_{\alpha'\alpha}\cos\varphi + \cos\gamma_{\alpha'\alpha}\cos\theta_{\xi_\alpha}\sin\varphi)/\sin\varphi'$$
$$\cos\theta_{\xi_\alpha} = \frac{-\cos\varphi'^2 + \cos\gamma_{\alpha'\alpha}{}^2\cos\varphi^2 + \sin\gamma_{\alpha'\alpha}{}^2\sin\varphi^2}{\epsilon_{\alpha'\alpha}\cos\gamma_{\alpha'\alpha}\sin\gamma_{\alpha'\alpha}\sin 2\varphi}$$
$$\epsilon_{\alpha'\alpha} = \begin{cases} 1 & \alpha'\alpha = (13),(32),(21) \\ -1 & \alpha'\alpha = (31),(23),(12) \end{cases}$$

Approximation 2. One may notice that $m_n \approx m_\phi$. Therefore, it seems reasonable to make another simplification and put $m_i = m = m_\phi$.

Approximation 3. Now let us reduce the system of two 2-dimensional equations (9) to the system of one-dimensional equations in the variable ρ-hyperradius of the system considered. This is possible due to the following observation. The potential $V_{\phi n}$ is shortrange, strongly attractive and acts in the s-state. The potential $V_{\phi\phi}$ contains a centrifugal barrier, so one can expect for the $\phi\phi N$ - system the equilibrium configuration when nucleon is in the centrum and ϕ-mesons are on the opposite sides. This configuration corresponds to the values of variable φ different for different Jacobi sets. The equilibrium values are $\varphi_{eq} = 0$ for $U_1(\rho,\varphi)$ and $\varphi_{eq} = \pi/3$ for $U_2(\rho,\varphi)$. Expanding $U_1(\rho,\varphi)$ and $U_2(\rho,\varphi)$ around equilibrium values and putting the expansion into the equation (9) one immediately arrives at the system of 2 one-dimensional equations on the variable ρ

$$\begin{cases} \hbar^2/(2M)\left((c_1)'' + (c_1)'/\rho - 6\,c_1/\rho^2\right) + (E- \\ \quad V_1(\rho\sqrt{6}))c_1 = -8/\sqrt{3}\,V_1(\rho\sqrt{6})\,c_2 \\ \hbar^2/(2M)\left((c_2)'' + (c_2)'/\rho - 8\,c_2/\rho^2\right) + (E+ \\ \quad (1 + 4/\sqrt{3}\,\nu_1)\,V_2(\rho\sqrt{3/2}))\,c_2 = -4/\sqrt{3} \\ \quad \nu_2\,V_2(\rho\sqrt{3/2})\,c_1 \end{cases} \tag{10}$$

where $\nu_1 = 0.110$, $\nu_2 = 0.0555$, which was solved for the eigenvalue problem. The program used in the computation was taken from [7].

2.3 Input

As an input, the following potentials were used:

$$V_{\phi N} = a\,e^{-\mu r}/r \tag{11}$$

with $a = -1.25\,\hbar c$, $\mu = (600\ \mathrm{MeV})/(\hbar c)$ [8].

$$V_{\phi\phi} = V_0\,e^{-(r/r0)^2} \tag{12}$$

where $V_0 = -93.75$ MeV, $r_0 = 1.2$ fm.

Parameters of the $V_{\phi\phi}$-potential are chosen to fit (together with the centrifugal barrier) the position and width of the $f_2(2010)$- resonance which has one mode of decay into two ϕ-mesons. The $V_{\phi N}$ - potential supported the bound state in the ϕN system with an energy around 9.5 MeV. It is worth saying that 5-body calculation [9] for the phi-N system gives the same result around 9 MeV in one of the quark models.

2.4 Results

The dependence of the energy of the three-body system $\phi\phi N$ on the depth of the ϕN potential $|a|$ is given on Figure 1. One can see that the binding in the system appears only at $a = -3.035\,\hbar c$. It is quite large with respect to the input value.

3 Conclusion

The study of ϕ-meson-nuclear systems can shed light on the structure of distributions of s and $\overline{s}$ sea quarks in the nuclei (as in the nucleons in nuclear medium) and on the possible appearance of many-body effects related to the exchange of sea s and $\overline{s}$ quarks belonging to different baryons. The study of ϕ-meson-nuclear systems can shed light on the structure of distributions

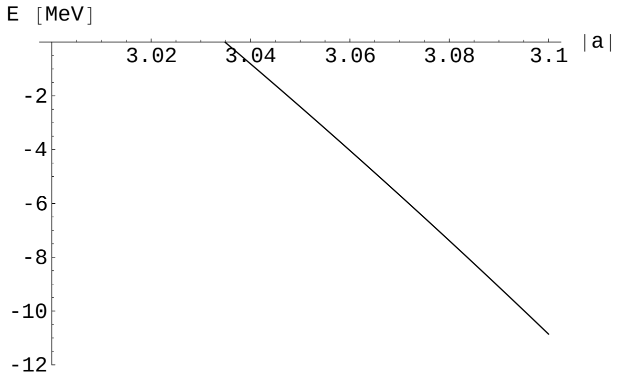

Fig. 1. The dependence of the binding energy of the $\phi\phi n$ system on the parameter $|a|$ of the $\phi - N$ interaction ($\hbar = c = 1$).

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References

1. V. B. Belyaev, W. Sandhas, and I. I. Shlyk, arXiv:0903.1703
2. R. L. Jaffe, Phys. Rev. D 21, (1980) 3215 - 3224
3. J. R. Ellis, arXive:hep-ph/0411369
4. A. W. Thomas, arXive:0907.4916
5. V. P. Nomokonov, M. G. Sapozhnikov, arXive: hep-ph/0204259
6. V. V. Pupyshev, Theor. Math. Phys. 81:1, (1989) 1072 - 1077
7. Numerical Analysis Library RCC, MSU, http://www.srcc.msu.ru/num_anal
8. H. Gao, T.-S. H. Lee, V. Marinov, Phys. Rev. C 63, (2001) 022201
9. F. Huang, Z. Y. Zhang, Y. W. Yu, arXiv: nucl-th/0601003