

Few-body Studies at the High Intensity γ -Ray Source (HI γ S)

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Abstract. The HI γ S facility is making it possible to perform studies of few body systems at a new level of accuracy and precision. A study of the photodisintegration of the deuteron using 100% linearly polarized beams at 14 and 16 MeV has determined the splittings of the three p-wave amplitudes involved in this process for the first time. These results show that the relativistic contributions, which when included in the theory lead to a positive value of the GDH integrand above 8 MeV, are valid. The near threshold data on the photodisintegration of the deuteron provide results which are used to extract the forward spin-polarizability of the deuteron for the first time. The experimental value is in good agreement with a recent effective field theory calculation. Measurements of the absolute differential cross section of the $^3\text{He}(\gamma, n)\text{pp}$ reaction have been completed at three γ -ray energies. The measurements were made at incident γ -ray energies of 12.8, 13.5, and 14.7 MeV. It has been found that the shape of the outgoing neutron energy distribution at a given scattering angle at 12.8 MeV disagrees with current theoretical predictions. At these energies, the shape is consistent with a phase-space-only shape. At the higher energies, the measurements agree with theory.

1 Introduction

The High Intensity Gamma-ray Source (HI γ S) at Duke University is a nearly monochromatic Compton γ -ray source with a very high flux, a wide energy range, and switchable polarizations. The current HI γ S electron accelerators include a 0.18 GeV linear accelerator pre-injector, a 0.18 – 1.2 GeV booster injector, a 0.24 – 1.24 GeV storage ring, and several storage ring based Free-Electron Lasers (FELs). HI γ S utilizes intra-cavity back-scattering of FEL light in order to produce intense γ -ray beams. A recently completed upgrade which included the installation of a 1.2 GeV booster-injector and a Higher-Order Mode (HOM) damped RF cavity has made it possible to produce nearly 100% linearly and circularly polarized beams having total intensities in excess of 10^8 γ /s. Figure 1 displays the present layout of the HI γ S facility, showing the linac, the new booster synchrotron, the upgraded RF system, the 1.2 GeV storage ring, and the OK-4/OK-5 undulator system.

A review of the facility and the overall research program can be found in Reference [1]. The energies of the HI γ S beams, presently ranging from 2 to 50 MeV, will soon be extended up to 100 MeV. One of the major research programs which is being addressed with this facility is the study of few-body nuclear systems. This paper will present a brief review of recent results obtained in the case of photodisintegration of the deuteron and ^3He . These examples are intended to illustrate how the unique nearly mono-energetic beams at HI γ S are making it possible to perform studies of few-body systems at a new level of accuracy and precision.

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2 Photodisintegration of the deuteron

2.1 First observation of p-wave splitting at E_γ of 14 and 16 MeV

One of the experiments which has been performed using the HI γ S beams at relatively low energies was a study of the photodisintegration of the deuteron. This process was first discovered by Chadwick and Goldhaber [2] over 70 years ago using 2.62 MeV γ -rays from a Th C' source. A year later H.A. Bethe and R. F. Bacher published a paper [3] which proved that this process at this and higher energies was dominated by electric dipole absorption (E1), and that the outgoing nucleons were therefore emitted in a state of relative angular momentum of 1-unit, ie. a p-wave.

Since the electric dipole operator is predominantly a $\Delta S=0$ operator, and since the ground state of the deuteron is mostly $S=1$, $\ell=0$ and $J=1$, there are actually 3 p-wave amplitudes, all of which have $S=1$ but with $J=0,1$ or 2 . We have known this for more than 70 years, but we have not had any information on the splittings of these three amplitudes—until now.

The experiment which took advantage of the unique qualities of the HI γ S γ -ray beams to study this splitting employed a large neutron detector array (Blowfish) to perform a precision measurement of the $d(\gamma, n)p$ reaction at 14 and 16 MeV.[4]

The Blowfish detector array consists of 88 BC-505 liquid scintillator cells located on the surface of a 16" diameter sphere centered on the target in 8 uniformly spaced arms of equal azimuthal angles ϕ . The 11 cells in each arm are uniformly distributed between polar scattering angles θ

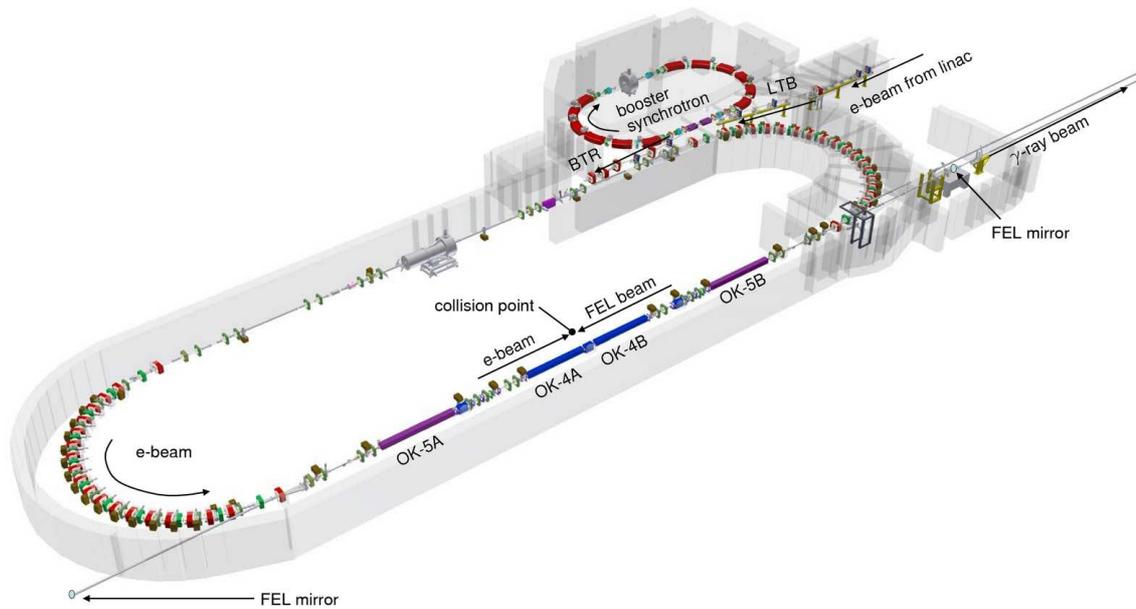


Fig. 1. The fully operational HI γ S facility in 2007. Since 2004, the following upgrade projects have been completed: (1) a newly commissioned 0.24 to 1.2 GeV top-off booster synchrotron; (2) a new 34 meter long storage ring straight section for hosting a HOM-damped RF cavity and for implementing a new injection scheme with the booster; and (3) the newly commissioned OK-5 FEL in the second 34 m long FEL straight section.

of 22.5° and 157.5° . The excellent pulse-shape discrimination (PSD) properties of BC-505 provide a strong handle on neutron/ γ identification down to about 200 keV_{ee} . This particle ID capability is enhanced and extended to lower energies by taking advantage of the pulsed nature of the HI γ S γ -ray beam to provide time-of-flight (TOF) information. An LED based gain-monitoring system is used to track the gain of each detector during runs, thus allowing an accurate tracking of the hardware thresholds for each detector. [5] The broad coverage (25% of 4π steradians) of the detector and its ability to be rotated about the beam axis makes it possible to achieve high statistical precision as well as an accurate determination of systematic effects.

The angular distribution data obtained using the Blowfish array at 14 and 16 MeV were used to construct the unpolarized differential cross section, $\sigma(\theta)$, and the linear analyzing power, $\Sigma(\theta)$, which is simply the difference between the yields in and out of the plane of polarization, divided by the sum. A Monte Carlo simulation was developed using GEANT4 to correct for effects of multiple scattering and the finite geometry of the target and detectors. The results at 14 and 16 MeV are shown in Fig. 2 where they are compared to the theoretical results of Schwamb and Arenhövel (SAPM) [6–8]. These observables can be written in terms of the amplitudes and phases of the reduced transition matrix elements (TMEs) which contribute to the reaction. Seven amplitudes were included in the fit corresponding to the $(M1)^1s_0$, $(E1)^3p_0$, 3p_1 , 3p_2 , and $(E2)^3d_1$, 3d_2 , and 3d_3 terms. The relative phases were obtained from the n-p scattering phase shifts (using the Fermi-Watson theorem). The results of this fitting procedure (see fits in Fig. 2) led to a splitting in the p-wave terms as shown in

Fig. 3 for the 16 MeV data set. The solid line at the value of 0.42 is the value which the squares of all three amplitudes would have if there were no splitting.

The fit indicated two solutions with identical χ^2 values, which are shown in Fig. 3. This is the first determination of this p-wave splitting and, as seen in Fig. 3, is in excellent agreement (especially Solution 1) with the theoretical prediction. It is interesting to notice that in the case of the 3p_2 term our Solution 1 agrees very well with the theoretical result if and only if the relativistic contributions are included.

2.2 Near-threshold measurement of differential and total cross section

A measurement of the absolute cross section for the photodisintegration of the deuteron has been carried out at the HI γ S facility. The inverse of this reaction (the radiative $n-p$ capture process) holds a key position in the Big-Bang nucleosynthesis framework in predicting the light element abundances in the universe. The cross section of the capture reaction is related to the photodisintegration cross section via the principle of detailed balance.

The experimental setup consisted of three thin (10 mm thick) Li-glass detectors placed at $\theta_{lab} = 55^\circ$, 85° , and 122° . The detectors were placed at a nominal distance of 10 in. from a thin (0.4 in.) heavy-water target. The target was oriented to minimize the scattering of neutrons in the target material, as shown in Fig. 4. The γ -ray beam was polarized in the plane of the detectors, and the typical flux on target was $\sim 10^6 \text{ } \gamma/\text{s}$. Data for time-of-flight (ToF) and pulse

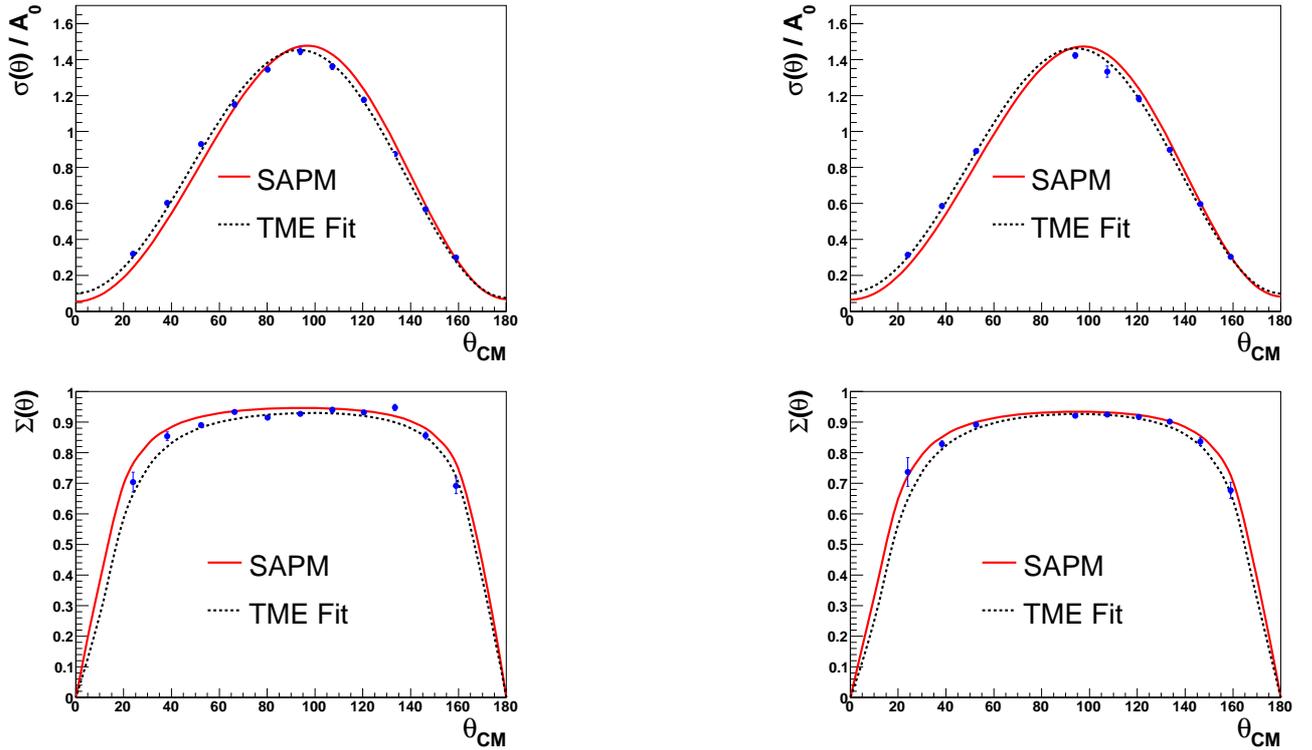


Fig. 2. Cross section and analyzing power results at 14 (left) and 16 MeV (right) along with fits from the extraction of the TME amplitudes. The solid red curves are from the calculation of SAPM and the dotted black curves represent the fit to the data. The errors on the data points are statistical only.

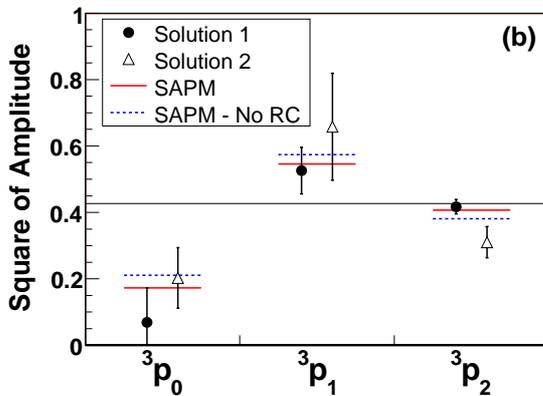


Fig. 3. The squares of the three normalized triplet E1 p-wave amplitudes at 16 MeV as extracted from the data and compared to the calculation of SAPM. The solid red line is the prediction from the full calculation and the dashed blue line is the prediction for the calculation without the inclusion of the relativistic contributions (RC). Error bars are statistical only.

height were collected at $E_\gamma = 2.44, 3.5,$ and 4.0 MeV. Total yields in each detector were obtained by integrating the ToF spectrum gated by the pulse height. Data were then corrected for Li-glass efficiencies, finite geometry corrections, and neutron multiple scattering in the target. The final values of the absolute cross section are under analysis, but should be available in the near future.

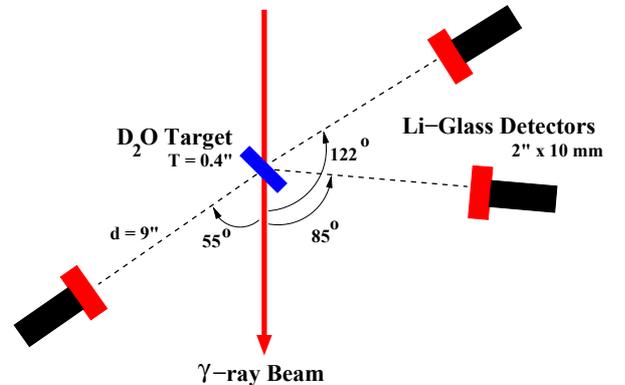


Fig. 4. (Color online) Experimental setup for measuring deuteron photodisintegration cross section and fore-aft asymmetry.

One of the motivations for this study was an earlier report [13] indicating a significant fore-aft asymmetry in the cross section at 4.0 MeV. Such an asymmetry would indicate a large E1-E2 interference, not predicted by any theory. If the cross section is expanded in terms of Legendre polynomials, then the fore-aft asymmetry (a_s) can be written as

$$a_s = \frac{0.577a_1 - 0.3849a_3}{1 - 0.392a_4} \quad (1)$$

where $a_1, a_3,$ and a_4 are coefficients of the Legendre polynomials. Restricting only to M1(s-waves), E1(p-waves),

and E2(d-waves) at these low energies, a_1 is proportional to the E1-E2 (p-d wave) interference, a_4 is proportional to the total E2 strength (small at these energies), and $a_1 = -a_3$. These relations make it possible to re-write the above equations as $a_s = 0.9622 a_1$. Therefore, a measure of the fore-aft asymmetry is indeed a measurement of the E1-E2 interference. The a_1 coefficient extracted from our data is shown in Fig.5 and compared to theory. Our result is consistent with almost no E1-E2 interference at these low energies, as predicted by the theory.

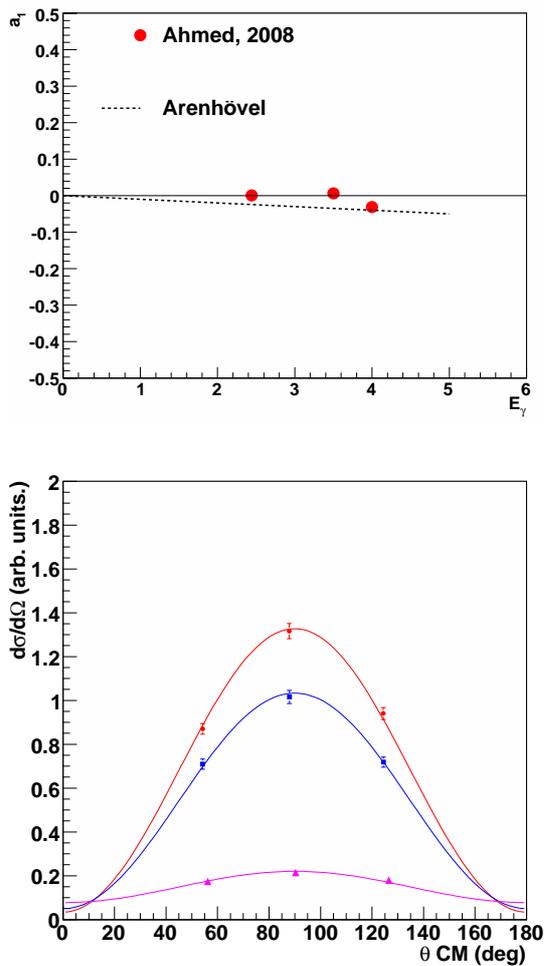


Fig. 5. (Top) The a_1 coefficient from an expansion of the cross section in Legendre polynomials as a function of energy. (Bottom) The differential cross section at 4.0 MeV (top red curve and data), 3.5 MeV (middle blue curve and data), and 2.44 MeV (bottom pink curve and data).

The angular distributions were then fitted to an unnormalized differential cross section of the form

$$\frac{d\sigma}{d\Omega} = a + b \sin^2(\theta) \quad (2)$$

where a and b are arbitrary coefficients. Since the angular coverage of this experiment was limited, the fit is unable

to constrain the behaviour of the differential cross section at extreme angles. It was however realized that other data, collected earlier to measure the photon analyzing power[9], can be used to obtain the $\theta_{CM}=0^\circ$ yield from the value of the 1S_0 M1(s-wave) transition amplitude deduced from the analyzing power data. Inclusion of these data indicates a successful reduction of the fitting errors. These data provide a measurement of the photodisintegration cross section near and at the region of Big Bang nucleosynthesis, where there is little or no world data.

3 Three-body photodisintegration of ^3He

Recently, an experiment was conducted at the HIγS facility to acquire differential and total cross section data on three-body photodisintegration of ^3He . Data were acquired at 12.8, 13.5 and 14.7 MeV. The experiment utilized a pulsed beam of γ rays collimated to a diameter of 2.54 cm. The neutrons resulting from the photon-induced three-body breakup of ^3He were detected using liquid scintillation detectors[14]. The target used in this experiment was a 400 mL cylindrical aluminum bottle filled with 168 atm of ^3He gas.

A simultaneous measurement of $d(\gamma, n)p$ was made to monitor the γ -ray beam intensity and to provide an absolute normalization for the ^3He -breakup reaction. This experimental setup consisted of two neutron scintillation detectors placed at 90° and a 47.4 mm long cylindrical heavy-water target, as shown in Fig. 6. The deuterium photodisintegration absolute cross section has been measured to better than 3% in the energy range of the current experiments[15].

The experimental data have been analyzed and normalized to the beam fluxes extracted from the $d(\gamma, n)p$ measurements. The data have also been corrected for detector efficiencies, but corrections for finite geometry and neutron multiple scattering have not been applied. The experimental data are compared with the results of two GEANT4 detector simulations which take finite geometry and neutron multiple scattering effects into account. In each of the figures, the upper, red histogram is the result of a simulation using a theoretical cross section as input. This cross section includes the contribution of the Coulomb interaction of the protons in the final state[16]. The theory predicts a peak in the neutron energy distribution at around 80% of the maximum neutron energy allowed by energy and momentum conservation. The lower, blue histogram is the simulated result when a phase-space input cross section is used.

As seen in Fig. 6, the shape of the experimental data taken at 12.8 MeV agrees well with the phase-space simulation and disagrees with the simulation using theory as an input. At the two higher energies, 13.5 MeV and 14.7 MeV, the measured results agree with the theory. These data indicate that there is a transition from a phase-space shape to a non-phase-space shape between 12.8 MeV and 13.5 MeV, a feature which is not contained in the present theoretical predictions.

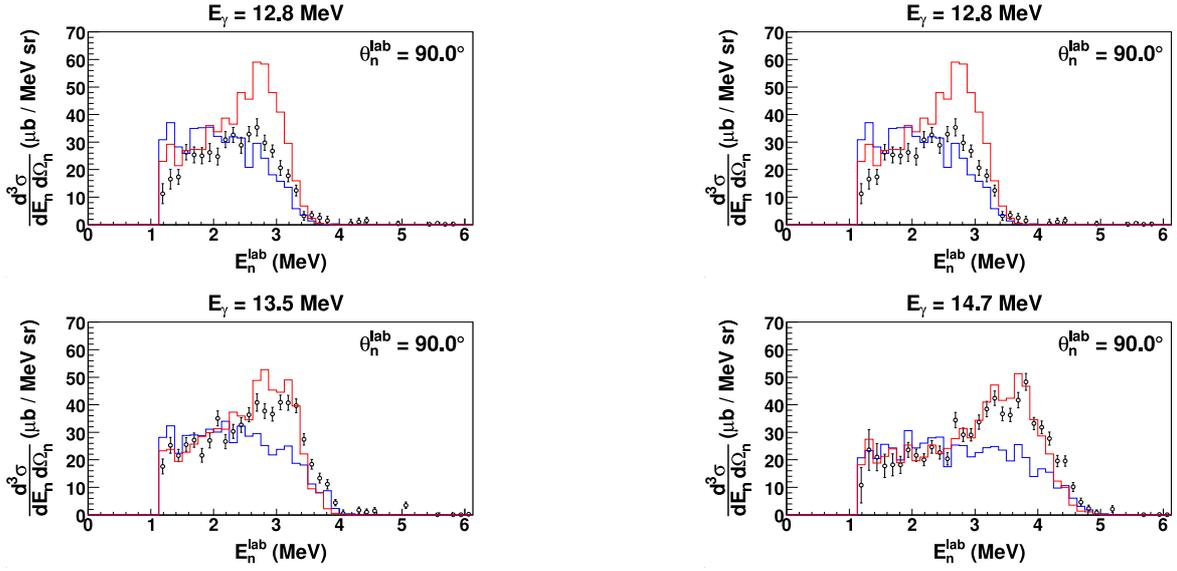


Fig. 6. (Color online) Neutron energy spectra at $E_\gamma = 12.8, 13.5, 14.7$ MeV for a detector at $\theta_n^{\text{lab}} = 90.0^\circ$. Experimental data are plotted against two sets of simulated results, one using a normalized phase-space cross section and the other using a theoretical cross section[16] as the input to the simulation. The lower, blue histogram is the phase-space-only result. The upper, red histogram is the result using the theory.

4 The Gerasimov-Drell-Hearn Sum Rule of the deuteron

The Gerasimov-Drell-Hearn (GDH) sum rule connects the helicity structure of the photo-absorption cross section to the anomalous magnetic moment of the nuclear target. It is derived using Lorentz and gauge invariance, crossing symmetry, causality and unitarity of the forward Compton scattering amplitude, and is explicitly given by

$$I_T = \int_{\omega_{\text{th}}}^{\infty} d\omega \frac{\sigma_P(\omega) - \sigma_A(\omega)}{\omega} = 4\pi^2 \alpha s_T \left(\frac{\kappa_T}{m_T} \right)^2, \quad (3)$$

where σ_P and σ_A are the cross sections for absorption of polarized photons of energy ω and helicities parallel(P) and antiparallel(A) to the target spin s_T , ω_{th} is the threshold photon energy for inelastic processes, α is the fine-structure constant, and m_T and κ_T are the target mass and anomalous magnetic moment, respectively.

The recent interest in the nucleon and deuteron GDH sum rules stems from the study of the spin dependent structure functions in deep inelastic scattering. Since the proton and neutron have relatively large anomalous moments ($\kappa_p = 1.793$ and $\kappa_n = -1.913$, respectively), the corresponding values of I obtained from Eq. (3) are large, $I_p = 204.8 \mu\text{b}$ and $I_n = 232.5 \mu\text{b}$, while the deuteron, for which $\kappa_d = -0.143$, has a comparatively small $I_d = 0.652 \mu\text{b}$. As has been previously discussed, one should expect to observe the sum of the proton and neutron strengths (and more) in the deuteron above pion threshold, indicating that a large negative contribution of about this size ($-436 \mu\text{b}$) should exist below this threshold. Indeed, Arenhövel [8] and others point out that the photodisintegration channel, which is the only photo-absorption process below the pion

threshold, gives a large negative contribution arising from the M1 transition to the resonant 1S_0 state just above the deuteron breakup threshold ($\omega - \omega_{\text{th}} < 100$ keV), since this state can only be formed if the deuteron spin and photon helicity are antiparallel. The predictions of these calculations, shown in Fig. 7, indicate a large negative contribution just above threshold followed by a positive contribution which arises from including a relativistic contribution in the theory.

The GDH Collaboration (Spokespersons: M. W. Ahmed, B. Norum, R. Pywell, and H. R. Weller) is a HI γ S Collaboration which is dedicated to measuring the integrand of this sum rule. There are two main phases to this work. The first phase has recently been completed. It consisted of using the linearly polarized gamma-ray beams and unpolarized targets to measure the near-threshold contribution to the GDH sum-rule integral of the deuteron. The results of this study have been published in Refs. [4,9].

In order to see how this is possible, we note that the difference of the parallel and antiparallel cross sections ($\sigma_P - \sigma_A$) can be expressed in terms of the Transition Matrix Elements (TMEs) of the reaction. In the region just above threshold it is known that only s-wave (M1) and p-wave (E1) TMEs contribute. We shall use the notation $^{2S+1}S_J$ and $^{2S+1}P_J$ to represent the TMEs (for outgoing neutrons having $l=0$ (s) or $l=1$ (p), respectively). In order to simplify the results, we set the 1P_1 term to zero, since it corresponds to spin-flip E1 and is expected to be negligibly small. The total photodisintegration cross section is given by:

$$\sigma_{TOT} = \frac{\pi}{6\kappa^2} \left[|^3P_0|^2 + 3|^3P_1|^2 \right]$$

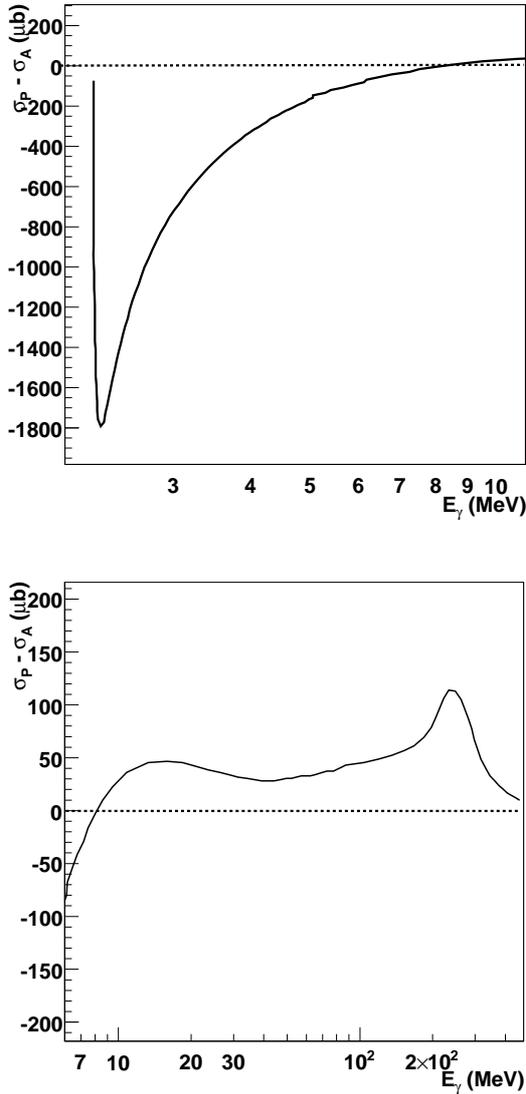


Fig. 7. The GDH sum rule integrand ($\sigma_P - \sigma_A$) for the deuteron as calculated by SAPM. The left figure shows the large negative near-threshold prediction, and the right figure shows the positive contributions which appear at higher energies when relativistic contributions are included.

$$+5|{}^3p_2|^2 + |{}^1s_0|^2 + 3|{}^3s_1|^2 \Big], \quad (4)$$

while $\sigma_P - \sigma_A$ is given by:

$$\begin{aligned} \Delta\sigma &= \sigma_P - \sigma_A \\ &= \frac{\pi}{2k^2} \left[-|{}^3p_0|^2 - \frac{3}{2}|{}^3p_1|^2 \right. \\ &\quad \left. + \frac{5}{2}|{}^3p_2|^2 - |{}^1s_0|^2 - \frac{3}{2}|{}^3s_1|^2 \right], \quad (5) \end{aligned}$$

Note that if the three p-wave E1 TMEs are identical (i.e., no j dependence), then $\sigma_P - \sigma_A$ is determined by the s-wave terms alone. Furthermore, if the $|{}^3s_1|$ term is negli-

gible, as expected, then we can write:

$$\begin{aligned} \Delta\sigma &= \frac{\pi}{2k^2} |{}^1s_0|^2 \\ &= -3\sigma(M1). \quad (6) \end{aligned}$$

Measurements of the analyzing powers at 90° were performed at HIγS and used to determine the fractional M1 contribution $S(M1)$ by assuming that the reaction proceeds via a mixture of M1(s-wave) and E1(p-wave) transitions [9]. These results were multiplied by the theoretical values of the total photodisintegration cross sections to produce the absolute M1 cross section as a function of energy which were converted to the values of $\sigma_P - \sigma_A$ using Eq. 6. Additional results can be obtained from previously measured polarized neutron capture on proton analyzing powers as well as from unpolarized photodisintegration data in which the outgoing neutron polarization was measured. In this case the analyzing power at 90° arises from the interference of the s- and p-wave amplitudes and is proportional to the phase difference between them:

$$A_y(90^\circ) = \frac{3|S||P|\sin(\phi_S - \phi_P)}{1 + \frac{9}{2}P^2}, \quad (7)$$

where

$$9|P|^2 + |S|^2 = 1.0. \quad (8)$$

Since the phase difference can be obtained using the n-p elastic scattering phase shifts, the measured values of $A_y(90^\circ)$ determine the percentage of the cross section due to s-waves (which is the M1 part). Results from these data are included in Fig. 8, along with the theoretical predictions of [8]. The basic assumption made in obtaining these

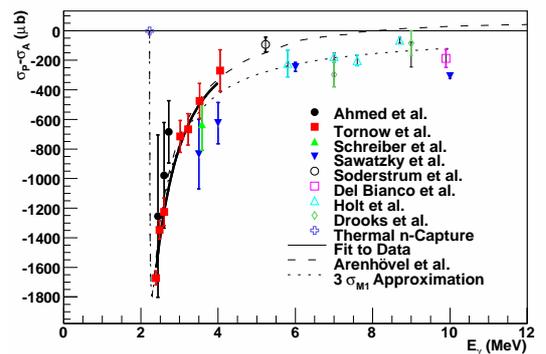


Fig. 8. Experimental data points for the GDH sum rule integrand are shown along with their statistical errors. The thick solid curve is a fit to the data while the dashed curve is a prediction from [8]. The dotted curve is the same theory assuming $\sigma_P - \sigma_A = -3\sigma(M1)$. This figure is taken from Ref. [9].

results is that the photodisintegration of the deuteron at these energies can be described using M1 (with outgoing s-waves), E1 (with outgoing p-waves) and E2 (with outgoing d-waves) TMEs. We have also assumed that there is

no splitting either in the triplet p-waves or in the triplet d-waves. This assumption means that the p- and d-wave contributions drop out of the GDH integrand.

The full theory shown in Fig. 8, when integrated from threshold up to 10 MeV using Eq. 1, predicts a value of $-634 \mu\text{b}$. The theory also predicts that the value of this integral up to pion threshold is $-520 \mu\text{b}$. A positive contribution (theoretically arising from a relativistic contribution) at energies between 10 MeV and pion-threshold is required to obtain this result. As will be seen below, the first experimental verification of the presence of this relativistic effect is contained in the p-wave splitting results described in Sec. 2.1.

As seen in Eq. 5, a knowledge of the p-wave splittings can be translated into a value of the GDH integrand. The quantity $\sigma_P - \sigma_A$ (Eq. 5) was calculated for both solutions with the results as shown in Fig. 9. Here we see that Solu-

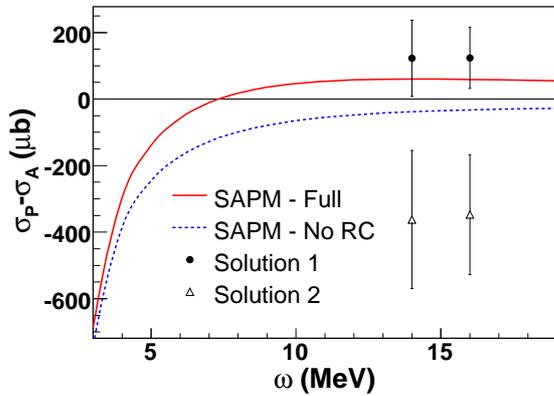


Fig. 9. $\sigma_P - \sigma_A$ for the two solutions extracted for the TME amplitudes of the reaction at each beam energy. Also shown is the full calculation of SAPM and the calculation without relativistic contributions (RC). Error bars are statistical only.

tion 1 agrees very well with the theoretical prediction especially when relativistic contributions are included in the theory. This is the first experimental evidence for a positive value of the GDH integrand in this energy region, and supports the need to include relativistic contributions in the theory. In addition to providing a value of the GDH integrand at these energies, this method of evaluating the GDH integrand provides us with a deep insight into the origin of this positive value, a result which cannot be obtained by a direct measurement. These results have been published in [4,9].

In addition to determining the GDH integral value in this near threshold energy region, these results can be used to determine the value of the so-called forward spin polarizability γ_0 , which is given by:

$$\gamma_0 = \frac{-1}{8\pi^2} \int_{\omega_{th}}^{\infty} d\omega \frac{\sigma_P(\omega) - \sigma_A(\omega)}{\omega^3}. \quad (9)$$

The result of integrating the data shown in Fig. 8 from threshold to 6 MeV is $\gamma_0 = 3.75 \pm 0.18 \text{ fm}^4$. This agrees with the result of an effective field theory calculation at

leading order, which predicts a value of 3.762 fm^4 , but is somewhat lower than the next-to-leading order result of 4.262 fm^4 [10]. Although the integral in this case is quite rapidly converging, the very limited energy range of the present experimental results is almost certainly not converged. A full report of this first determination of the forward spin-polarizability of the deuteron can be found in Ref. [9].

Although these indirect measurements have been very instructive, the continuation of this method to higher energies is not practical since the number of contributing TMEs is known to grow. The number of unknowns will therefore be too great to be able to obtain meaningful solutions. For this reason we are proposing to make direct measurements of the GDH integrand for the deuteron at energies between 5 and 100 MeV during the next three years.

The Blowfish detector, a frozen-spin polarized deuterium target [11], and circularly polarized gamma rays from the OK-5 helical undulator are required for a direct measurement of the GDH integrand. The frozen-spin target is under construction and is expected to be available in 2010[11]. The OK-5 system, which is a helical undulator based free-electron laser, has been installed and has produced circularly polarized gamma rays which were used in the first measurements of the GDH integrand for ^3He [12].

The GDH Collaboration anticipates starting this experiment in 2010. Detailed studies have indicated that a measurement of the integral over the energy range between 5 and 50 MeV to an accuracy of 5% can be achieved in less than 300 hours of running with a beam intensity of $10^7 \text{ } \gamma\text{s/s}$. Beams with energies up to 100 MeV are expected to become available in 2010. This will allow the Collaboration to extend the measurement of the GDH integrand up to 100 MeV starting in 2011. Ultimately, the Collaboration will extend these measurements up to pion threshold, but this must await the development of mirrors needed to produce beams with energies up to 145 MeV.

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