Photodisintegration of $^3H$ in a three dimensional Faddeev approach

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Abstract. An interaction of a photon with $^3H$ is investigated based on a three dimensional Faddeev approach. In this approach the three-nucleon Faddeev equations with two-nucleon interactions are formulated with consideration of the magnitude of the vector Jacoby momenta and the angle between them with the inclusion of the spin-isospin quantum numbers, without employing a partial wave decomposition. In this formulation the two body t-matrices and triton wave function are calculated in the three dimensional approach using AV18 potential. In the first step we use the standard single nucleon current in this article.

1 Introduction

Since the early days of the study of the nuclear physics so many efforts have been performed on 3N systems considering real or virtual photon interactions [1]-[2]. Also several studies on the behavior of 3N bound states in real or virtual photon absorption have been reported[3]-[4]. Before sixties variational approach was used for these calculations and works using this approach are still continuing. After introducing Faddeev formulation for three body systems [5]-[6], new efforts using this scheme were started. As an example one can point out the early calculations of photodisintegration [7] and photodisintegration [8] with $^3$He and $^3$H. An improvement in the photodisintegration calculation of the bound and 3N continuum with the same 3N Hamiltonian have been performed [9]. There are also other approaches to calculate electromagnetic interactions with light nuclei such as Green-function-Monte-Carlo method [10], hyperspherical harmonic expansions [11], and Lorentz integral transform (LIT) method [12]. There is a very good review of Faddeev calculations on the interaction of real or virtual photon with $^3$He [13]. In this work like previous calculations the partial wave decomposition has been used. In PW approach one should sum all PW to maximum angular momentum where the calculation is converged. The problem is that in higher energies this maximum angular momentum increases and we should solve more complicated equations. To avoid this complexity one should use vector momentum as basis states [14]. To this aim in the past decade the main steps have been taken by Ohio-Bochum collaboration (Elster, Glöckle et al.) and Bayegan et al. to implement the 3D approach in few-body bound and scattering calculations (see for examples Refs. [15]-[22]). It should be clear that the building blocks to the few-body calculations without angular momentum decomposition are two-body off-shell t-matrices, which depend on the magnitudes of the initial and final Jacoby momenta and the angle between them. Fuchruddin et al. have calculated the NN bound and scattering states in a 3D representation using the Bonn-B and the AV18 potentials [15]-[16]. Recently there has been efforts to do the same calculation using chiral potential [23], our aim in this work is to formulate photodisintegration of $^3H$ in three dimensional Faddeev approach. In the first step we ignore three body forces and we just use the single nucleon current. We will use AV18 potential and triton wave function which has been calculated in our previous work [20].

This manuscript has been organized as follow: in section 2 we explain our basic states and we evaluate all of the matrix elements in these basis. In section 3 we introduce our singularity problem and its solution. We finish in section 4 with a summary and outlook.

2 Integral equation of nuclear matrix elements without partial wave decomposition

To calculate the photodisintegration observable we first need to calculate nuclear matrix elements in the Faddeev scheme. For more details see Ref.[13].

$$ N = \frac{1}{2} (\phi_0 (1 + iG_0) P |U\rangle \) \rangle $$ (1)

$$ |U\rangle = (1 + P) J |\psi\rangle + i G_0 P |U\rangle \) \rangle $$ (2)

In above equations $t$ is NN $t$-operator which obeys Lipmann-Schwinger equations, $G_0$ is free propagator, $P$ is permutation operator, $|\psi\rangle$ is three body bound state and $|U\rangle$ is an auxillary state. Three body forces have been ignored.

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\[ |\phi_0\rangle \text{ is a subsection of the fully antisymmetric free state, } |\phi_0\rangle, \text{ in which nucleons 2 and 3 are in subsystem.} \]
\[ |\phi_0\rangle = (1 + P)|\phi_0\rangle \]  
(3)

\[ |\phi_0\rangle \text{ is also our basic state to solve the integral equation (2) and is antisymmetric under permutation of nucleons 2 and 3.} \]
\[ |\phi_0\rangle \equiv |pqqm_1m_2m_3v_1v_2v_3\rangle^a \]  
(4)

In equation (4) \( p \) and \( q \) are jacobis momenta and m’s and v’s are the spin and isospin of the individual nucleons respectively.

Orthonormality and completeness relations of these basic states can be considered as below:

\[ a(pqqm_1m_2m_3v_1v_2v_3|p'q'm'_{i_1}m'_{i_2}v'_{i_1}v'_{i_2}v'_{i_3})^a = 1 \]
\[ \left( \delta(p - p')\delta_m^m_{i_1}\delta_m^m_{i_2}\delta_v^v_{i_1}\delta_v^v_{i_2}\delta_{v'_{i_3}} \right) \]  
(5)

\[ \sum_{m_1 m_2 m_3} \int d^3p \cdot d^3q \cdot pqqm_1m_2m_3v_1v_2v_3|pqqm_1m_2m_3v_1v_2v_3\rangle^a \]  
(6)

Considering these properties we can rewrite the integral equations (1) and (2) in our basic stats.

\[ N = \frac{1}{2} \langle pqqm_1m_2m_3v_1v_2v_3|1 + ig_0 P|U \rangle \]
\[ = \frac{1}{2} \langle pqqm_1m_2m_3v_1v_2v_3|P|U \rangle \]
\[ + \frac{1}{2} \langle pqqm_1m_2m_3v_1v_2v_3|ig_0 P|U \rangle \]  
(7)

\[ \langle pqqm_1m_2m_3v_1v_2v_3|1 + PJ|\psi \rangle \]
\[ = \langle pqqm_1m_2m_3v_1v_2v_3|(1 + P)J|\psi \rangle \]
\[ + \langle pqqm_1m_2m_3v_1v_2v_3|ig_0 P|U \rangle \]  
(8)

The effect of permutation operator on our basic states can be considered as follow:

\[ \langle pqqm_1m_2m_3v_1v_2v_3|p'q'm'_{i_1}m'_{i_2}v'_{i_1}v'_{i_2}v'_{i_3} \rangle \]
\[ = \delta(p + \frac{1}{2}p' + \frac{3}{4}q' - \frac{1}{2}q') \]
\[ \times \delta_m^m_{i_1}\delta_m^m_{i_2}\delta_m^m_{i_3}\delta_v^v_{i_1}\delta_v^v_{i_2}\delta_v^v_{i_3} \]
\[ + \delta(p + \frac{1}{2}p' + \frac{3}{4}q' - \frac{1}{2}q') \]
\[ \times \delta_m^m_{i_1}\delta_m^m_{i_2}\delta_m^m_{i_3}\delta_v^v_{i_1}\delta_v^v_{i_2}\delta_v^v_{i_3} \]
\[ + \delta(p - \frac{1}{2}p' + \frac{1}{2}q') \]
\[ \times \delta_m^m_{i_1}\delta_m^m_{i_2}\delta_m^m_{i_3}\delta_v^v_{i_1}\delta_v^v_{i_2}\delta_v^v_{i_3} \]  
(9)

Where:
\[ \pi_1 = q + \frac{1}{2}q' \]
\[ \pi_2 = \frac{1}{2}q + q' \]  
(10)

Now with respect to above relation and symmetry considerations we can evaluate equations (7) and (8) as follow:

\[ \langle pqqm_1m_2m_3v_1v_2v_3|U \rangle \]
\[ = \langle pqqm_1m_2m_3v_1v_2v_3|(1 + P)J|\psi \rangle \]
\[ + \langle pqqm_1m_2m_3v_1v_2v_3|ig_0 P|U \rangle \]  
(11)

\[ \langle pqqm_1m_2m_3v_1v_2v_3|U \rangle \]
\[ = \langle pqqm_1m_2m_3v_1v_2v_3|(1 + P)J|\psi \rangle \]
\[ + \langle pqqm_1m_2m_3v_1v_2v_3|ig_0 P|U \rangle \]  
(12)

The first term in the equation(12) can be evaluated as:

\[ \langle pqqm_1m_2m_3v_1v_2v_3|(1 + P)J|\psi \rangle \]
\[ = \sum_{m',v'} \int d^3p \cdot d^3q \cdot pqqm_1m_2m_3v_1v_2v_3|p'q'm'_{i_1}m'_{i_2}v'_{i_1}v'_{i_2}v'_{i_3} \rangle \]
\[ \times \langle p'q'm'_{i_1}m'_{i_2}v'_{i_1}v'_{i_2}v'_{i_3}|(1 + P)J|\psi \rangle \]  
(13)

Now we concentrate on the elements of these equations i.e. current, two-body t-matrix and triton wave function, more precisely.

2.1 current

Considering the symmetry properties we have:

\[ a(pqqm_1m_2m_3v_1v_2v_3|(1 + P)J^SN|\psi) \]
\[ = 3\langle pqqm_1m_2m_3v_1v_2v_3|(1 + P)J^SN|\psi \rangle \]  
(14)

Matrix elements of single nucleon current can be evaluated as follow:

\[ a(pqqm_1m_2m_3v_1v_2v_3|J_{i_1}|p'q'm'_{i_1}m'_{i_2}v'_{i_1}v'_{i_2}v'_{i_3} \rangle \]
\[ = \delta(q' - q + \frac{1}{2}Q) \]
\[ \times \frac{1}{2} \delta(p - p') \delta_m^m_{i_1}\delta_m^m_{i_2}\delta_v^v_{i_1}\delta_v^v_{i_2}\delta_v^v_{i_3} \]
\[ - \delta(p + p') \delta_m^m_{i_1}\delta_m^m_{i_2}\delta_v^v_{i_1}\delta_v^v_{i_2}\delta_v^v_{i_3} \times J_{m_1 m_1'}(Q, q) \]
\[ v_1 v'_1 \]  
(15)
In above equation $Q$ is the momentum of photon. We need to rewrite the single nucleon current operator in a form which is suitable for our basic states. The current operator which we will use is:

$$J = G_E(Q)\frac{k_1 + k_1'}{2m_N} + \frac{i}{2m_N}G_M(Q)r \times (k_1 - k_1')$$ (16)

Which is summation of convection current and spin current. $G_E(Q)$ and $G_M(Q)$ are electric and magnetic form factors respectively. For the convection part we have:

$$q + Q + \frac{2}{3}K$$ (17)

As we will show we have to choose coordinate system in which the $z$ axis is along the $Q$ vector and we also need tensor component of current so the second and the third terms of the right hand side of equation (17) will vanish. Thus for the convection current we have:

$$J_{cone} = G_E(Q)\frac{q+1}{m_N}$$ (18)

And the tensor component of spin part can also be evaluated as:

$$J_{spin} = -\frac{\sqrt{2}Q}{2m_N}G_M(Q)S_z$$ (19)

### 2.2 two-body t-matrix

Two body t-matrices can be related to the one which has been calculated in the following basis[20]:

$$\langle pq|\frac{1}{2}\rangle S m_s|\frac{1}{2}\rangle T m_T|\psi\rangle = \langle pq, X_{pq}, \alpha|\psi\rangle$$ (24)

We can relate these states to our free spin and isospin states with Clebsch-Gordan coefficients.

$$g_{\gamma\alpha} = \langle \gamma|\alpha\rangle$$ (25)

Where

$$|\alpha\rangle = |\frac{1}{2}\rangle S m_s, |\frac{1}{2}\rangle T m_T$$

$$|\gamma\rangle = |m_1 m_2 m_3 y_1 y_2 y_3\rangle$$ (26)

It is very important to mention that the spin of the nucleons is quantized in the direction of the $z$ axis which in the calculation of wave function it has been chosen to be in the direction of $q$. But we have to consider the $z$ axis along the direction of incident photon $Q$. So we should first rotate the spin of the nucleons in our basis to be settled in the direction of $q$ axis. Then we should use Clebsch-Gordan coefficients to obtain the wave function in the calculated basis mentioned in the equation (24):

$$\langle pq, X_{pq}, \alpha|\psi\rangle \propto \langle pq|\frac{1}{2}\rangle S m_s|\frac{1}{2}\rangle T m_T|\psi\rangle$$ (27)
3 Singularity problem

In order to consider the singularity problem we can rewritten the equation (11) an (12) in a unified form ignoring isospin dependent which is similar to spin dependent.

\[
U_{m_1m_2m_3}(p, q, Q) = U'_{m_1m_2m_3}(p, q, Q) + \sum_{m'} \int dq'' U_{m_1m_2m_3}(\pi_2, q'', Q) \left( \frac{q''}{m} \right) \frac{d^3q''}{E - m''^2} \frac{d^3p''}{E + i\epsilon - E_d - \frac{3\pi}{4m}} 
\]

To solve this integral equation we should evaluate singularity in the denominator of the propagator which is a function of \(q''\) and angle between \(q''\) and \(q\). So instead of singular point we have a region of singularity in \(q''\) plane. There is a solution to this moving singularity in Ref.[24]. For using this method we have to put \(z\) axis along the \(q\). But because of simplification in current operator and final cross section we should choose the \(z\) axis in the direction of the momentum of the photon, \(Q\). So in order to evaluate the singularity we should use another method which is introduced in Ref.[25]. Therefore one should separate angle part of delta functions as follow:

\[
\delta(p' + \pi_1)\delta(p'' - \pi_1) = \frac{\delta(p' - \pi_1)\delta(p'' - \pi_1)}{p''^2} \frac{\delta(p' + \pi_1)\delta(p'' + \pi_1)}{p'''} 
\]

And then the integral equation can be rewrite as follow:

\[
U_{m_1m_2m_3}(p, q, Q) = U'_{m_1m_2m_3}(p, q, Q) + \sum_{m'} \int dq'' d^3p'' d^3p''' U_{m_1m_2m_3}(\pi_2, q'', Q) \left( \frac{q''}{m} \right) \frac{d^3q''}{E - m''^2} \frac{d^3p''}{E + i\epsilon - E_d - \frac{3\pi}{4m}} 
\]

After some simplification the integral equation transforms to this equation:

\[
U_{m_1m_2m_3}(p, q, Q) = U'_{m_1m_2m_3}(p, q, Q) + \sum_{m'} \frac{q}{m} \int_{0}^{\infty} dp'' \left( \frac{q''}{m} \right) \frac{d^3q''}{E - m''^2} \frac{d^3p''}{E + i\epsilon - E_d - \frac{3\pi}{4m}} 
\]

\[
\int dq'' G(q, q'', p') \int dq'' \delta(q'' - x_0) U_{m_1m_2m_3}(p''\pi_2, q'', Q) \left( \frac{q''}{m} \right) \frac{d^3q''}{E + i\epsilon - E_d - \frac{3\pi}{4m}} 
\]

In the above equation \(G\) which is always positive is defined as:

\[
G(q, q'', p') = \frac{1}{-E_d - \frac{3\pi}{4m} + \frac{1}{m}(p'^2 + \frac{1}{4}q'^2)} 
\]

\[x'' = \cos \theta''\] indicates the angle between \(q\) and \(q''\) and \(x_0\) is introduced as follow:

\[
x_0 = \frac{1}{qq''}(p'^2 - \frac{1}{4}q'^2 - q'^2) = \frac{1}{qq''}(p'^2 - \frac{1}{4}q'^2 - q'^2) 
\]

4 Summary and outlook

In this paper we have formulated the Faddeev integral equations for calculating the photodisintegration observable of triton in three dimensional approach. To this aim we introduced our basic states which contains jacobis momenta in vector forms as well as individual spin and isospin of each nucleon. So we have avoided to decompose angle states in terms of angular momentum states (partial wave approach) which is traditionally used to solve this kind of equations. The final integral equations are less complicated than the PW ones and are unique in number of the equations in all energies. We have also explained about overcoming of the moving singularity in our work.

The calculation of this observable using the AV18 potential is underway and the results will be published soon.

Adding two and three body currents as well as three body forces in our calculations are other future major works. The same calculation for radiative capture is also under consideration.

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References

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