EPJ Web of Conferences **3**, 04016 (2010)
DOI:10.1051/epjconf/20100304016

The ^{2}H Electric Dipole Moment in a Separable Potential Approach

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Abstract. Measurement of the electric dipole moment (EDM) of ^{2}H or of ^{3}He may well come prior to the coveted measurement of the neutron EDM. Exact model calculations for the deuteron are feasible, and we explore here the model dependence of such deuteron EDM calculations. We investigate in a separable potential approach the relationship of the full model calculation to the plane wave approximation, correct an error in an early potential model result, and examine the tensor force aspects of the model results as well as the effect of the short range repulsion found in the realistic, contemporary potential model calculations of Liu and Timmermans. We conclude that, because one-pion exchange dominates the EDM calculation, separable potential model calculations should provide an adequate picture of the ^{2}H EDM until better than 10% measurements are achieved.

1 Introduction

With the discovery of parity (P) violation as suggested by Lee and Yang [1], Landau [2] recognized that charge conjugation along with parity (CP) invariance would imply that the electric dipole moment of the neutron should be zero. The *Standard Model* predicts values of EDMs which are smaller than are currently experimentally detectable. Thus, an unambiguous observation of a non zero EDM would imply an undiscovered source of CP violation [3, 4]. The implied new physics could arise in the strong interaction sector (*e.g.*, the θ term), or the weak interaction sector (*e.g.*, second order W boson exchange).

Both PT violating interactions as well as P conserving but T violating interactions may give rise to an EDM [3]. However, one-pion exchange contributes only to the former. We focus our attention here on the effects of the violation of PT invariance in the nuclear potential, because a method has been proposed to directly measure the EDM of a charged ion in a storage ring [5].

This prospect makes the deuteron attractive for an EDM investigation. As one will see, the contributions of the proton and neutron EDMs tend to cancel, so that the PT violating interaction should become a significant contributor to the ^{2}H EDM. Because the deuteron is well understood, reliable calculations are possible.

2 Analysis

The total one-body contribution to the deuteron EDM of the neutron and proton is the sum of their individual EDMs:

$$d_D^{(1)} = d_n + d_p \; . \tag{1}$$

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The neutron and proton EDMs can arise from a variety of sources. Liu and Timmermans [6] estimate:

$$d_D^{(1)} \sim 0.22 \times 10^{-2} \bar{G}_\pi^{(1)} + O(\bar{G}_\pi^{(0,2)}, \bar{G}_{\rho,\omega,\eta}^{(0,1)}) \; , \tag{2}$$

which has been expressed in terms of the $\bar{G}_X^{(i)}$, the product of the strong coupling constant g_{XNN} and the associated PT violating meson-nucleon coupling constant $\bar{g}_X^{(i)}$. We note that the theoretical uncertainty in the estimate for $d_D^{(1)}$ is sizable.

However, Liu and Timmermans also estimated the two-body contribution from the PT violating NN interaction to the deuteron EDM to be

$$d_D^{(2)} \sim 1.5 \times 10^{-2} \bar{G}_\pi^{(1)} + O(\bar{G}_\pi^{(0)}, \bar{G}_{\rho,\omega,\eta}^{(1)}) \; . \tag{3}$$

Therefore, for the deuteron it is clear that the nuclear physics component of $d_D^{(2)}$ dominates. Even an uncertainty of 40% in $d_D^{(1)}$ makes but a minor contribution. For this reason we investigate the model aspects of the $d_D^{(2)}$ dominant term in the ^{2}H EDM in some detail.

To aid in our discussion that follows, we separate the two-body contribution to the deuteron EDM $d_D^{(2)}$ into a plane-wave term d_{PW} and a final-state re-scattering contribution d_{MS}:

$$d_D^{(2)} = d_{PW} + d_{MS} \; . \tag{4}$$

We note that d_{PW} and d_{MS} are proportional to the coupling constant combination $g_{\pi NN}\bar{g}_{\pi NN}/16\pi$.

3 Prior Results

As background to our discussion, we recall that in 1985 Avishai [7] estimated $d_D^{(2)}$ using a separable p-wave potential of Mongan [8]. He reported a value of – 0.91 *e fm* for a physical pion mass of the exchanged meson.

Khriplovich and Korkin [9] later estimated $d_D^{(2)}$ using a zero-range approximation in the chiral limit and reported a value of – 0.96 *e fm*.

Using the contemporary potential models Av_{18}, Reid93, and Nijm II [10], Liu and Timmermans [6] estimated the value for $d_D^{(2)}$ to be – 0.73 ± 0.01 *e fm* within the range of uncertainly defined by the three potential models.

The implications of these results are several.

- If the zero range calculation is correct, then the results of Liu and Timmermans suggest that the final-state interaction reduces $d_D^{(2)}$ by less than 25%.
- Such a reduction is considerably greater than the reduction reported by Avishai. In fact, we find the Avishai result to be in error by a factor of 2; we believe that the corrected result should be – 0.48 *e* fm.
- The significant difference between the results of Avishai and Liu and Timmermans suggest that the EDM is sensitive to the potential model fit to the *NN* scattering data.

Thus, we are led to explore the sensitivity of the ^{2}H EDM to the potential model representation of the *NN* scattering data. That includes the softness of the rank-one Mongan potentials compared with the contemporary potential models emplyed by Liu and Timmermans and the sensitivity to the short range repulsion in the deuteron wave function.

4 Original Mongan $^3P_1$ Results

To explore the accuracy of the results for the deuteron EDM, we begin that process by using a set of Mongan's [8] rank-one $^3P_1$ potentials. The separable potential form factors are tabulated in Table 1 along with the range parameters β and the potential strengths λ.

Table 1. The parameters of the rank one Mongan [8] $^3P_1$ potentials. The strength of the potentials is related to Mongan's strength C_R by $\lambda = \frac{C_R}{\hbar c}$. For Case III (1969) the function $Q_1(z)$ is the Legendre function of the second kind.

Potential	form factor $g(k)$	β (F^{-1})	λ
Case I (1968)	$k/(k^2+\beta^2)$	1.574	1.9548
Case II (1968)	$k/(k^2+\beta^2)^{3/2}$	2.177	9.7532
Case II (1969)	$k/(k^2+\beta^2)^{3/2}$	2.178	10.5515
Case III (1969)	$\left[\frac{1}{k^2\pi}Q_1(1+\frac{\beta^2}{2k^2})\right]^{1/2}$	1.193	1266.9326
Case IV (1969)	$k/(k^2+\beta^2)^2$	2.661	203.3178

In Table 2 we quote the measure of the quality of the 1969 potential fits to the data as provided by Mongan, the sum of the squares of the residuals:

$$\sum R^2 = \sum_{i=1}^{50}\left[\delta^{\text{exp}}(E_i) - \delta^{\text{fit}}(E_i)\right]^2 . \qquad (5)$$

The fits are semi-quatitative at best, which is indicative of the lower precision of the experimental data in 1969.

Table 2. The residuals for the rank one Mongan [8] $^3P_1$ potentials as defined by Eq. (5)

Potential	form factor $g(k)$	$\sum R^2$
Case I (1968)	$k/(k^2+\beta^2)$	38.39
Case II (1968)	$k/(k^2+\beta^2)^{3/2}$	55.47
Case II (1969)	$k/(k^2+\beta^2)^{3/2}$	56.06
Case III (1969)	$\left[\frac{1}{k^2\pi}Q_1(1+\frac{\beta^2}{2k^2})\right]^{1/2}$	71.86
Case IV (1969)	$k/(k^2+\beta^2)^2$	63.85

For the deuteron in this study we have used the rank-one Yamaguchi–Yamaguchi [11] 4% and 7% (P_D) potentials plus the Unitary Pole Approximation (UPA) for the Reid soft core (RSC) potential [12] generated in 1973 [13].

To compare with Avishai, we look at $d_D^{(2)}$ as a function of the mass of the exchanged meson for the P_D = 4% YY deuteron in Table 3. Avishai did not specify the Mongan potential he used, but it is clear that our results are not consistent with his. The difference is of the order of a factor of 2. He used an overall strength coefficient of

$$A = g_{\pi NN}\bar{g}_{\pi NN}/8\pi \, ,$$

whereas we find the denominator should be 16π. Thus, we believe the Avishai result to be in error by a factor of 2. Therefore, we suggest that the correct result for Avishai should be – 0.46 *e* fm.

Table 3. The dependence of the deuteron EDM $d_D^{(2)}$ on the mass of the exchanged particle m in the potential V. We compare the results for three Mongan potentials with the results of Avishai [7]. In each case a Yamaguchi–Yamaguchi potential with a 4% *D*-state probability was used.

m (MeV)	100	200	300	m_π
Mongan Case I (1968)	-0.45	-0.04	0.14	-0.26
Mongan Case II(1969)	-0.71	-0.29	-0.10	-0.051
Mongan Case IV(1969)	-0.80	-0.37	-0.18	-0.060
Avishai	-1.31	-0.51	-0.13	-0.91

To compare with Khriplovich and Korkin, we look at the results in Table 4. The nominal agreement between that of Khriplovich and Korkin and the RSC (UPA) is likely fortuitous. Nevertheless, given that the result of Khriplovich and Korkin was obtained in the chiral limit of a zero range approximation, the overall agreement is reasonable. Moreover, short range repulsion would seem to account for only a 10% reduction in d_{PW}.

Table 4. Variation in the ^{2}H EDM with the strength of the tensor force and the role of short range repulsion. For the $^3P_1$ potential the Mongan Case I (1968) was used.

$^3S_1$–$^3D_1$	d_{PW}	d_{MS}	$d_D^{(2)}$
Yamaguchi 4%	-1.035	0.778	-0.257
Yamaguchi 7%	-1.083	0.800	-0.283
RSC (UPA)	-0.962	0.307	-0.655
Khriplovich & Korkin	-0.96		

5 Contemporary Mongan $^3P_1$ Results

The 1969 Mongan potentials were fit to early phase shift data. In contrast Liu and Timmermans used potentials that reproduce the 1993 Nijmegen phase shift analysis [14].

To explore the sensitivity to the $^3P_1$ phase shifts, we have refit the Mongan potentials to the 1993 Nijmegen phase shifts. The new parameters are summarized in Table 5 along with $\chi^2 = \sum_{i=1}^{11} |\delta_i^{\rm Th} - \delta_i^{\rm exp}|^2$. The fit to the phase shifts is illustrated by quoting values of δ at 1 and 100 MeV.

Table 5. Parameters for the contemporary Mongan potentials that give a best fit to the 1993 Nijmegen *np* phase shifts. Also included is the $\chi^2 = \sum_{i=1}^{11} |\delta_i^{\rm Th} - \delta_i^{\rm exp}|^2$ and shown are the fits to the phase shifts δ at 1 and 100 MeV.

Mongan′	β	C	χ^2	$\delta(1)$	$\delta(100)$
Case I	1.91	1.16	13.28	-0.04	-14.36
Case II	2.46	9.85	34.68	-0.04	-15.33
Case III	1.745	139.5	24.64	-0.09	-15.40
Case IV	2.77	168.0	86.45	-0.05	-17.20
Experiment				-0.11	-13.24

We compare in Table 6 and Table 7 results for the original Mongan potentials with results for the contemporary Mongan potentials fit to the 1993 phase shifts, for the YY deuteron models with 4% and 7% D-state probabilities.

Table 6. The deuteron EDM values for the two YY rank-one deuteron $^3S_1$–$^3D_1$ potentials and four different rank-one $^3P_1$ potentials. The $^3P_1$ potentials are the original Mongan rank-one potentials.

$^3S_1$–$^3D_1$	YY 4%		YY 7%	
	$d_{PW} = -1.035$		$d_{PW} = -1.083$	
Case	d_{MS}	$d_D^{(2)}$	d_{MS}	$d_D^{(2)}$
I	0.778	-0.257	0.800	-0.284
II	0.502	-0.533	0.524	-0.560
III	1.174	0.139	1.196	0.113
IV	0.439	-0.596	0.460	-0.623

The d_{MS} values for the contemporary Mongan potentials exhibited in Table 7 are significantly smaller, bringing $d_D^{(2)}$ into closer agreement with the result of Liu and Timmermans. The variation with P_D is comparable to that shown for the d_{PW} values.

Table 7. The deuteron EDM values for the two YY rank-one deuteron $^3S_1$–$^3D_1$ potentials and four different rank-one $^3P_1$ potentials. The $^3P_1$ potentials are of the Mongan form with parameters adjusted to fit the 1993 Nijmegen phase shifts.

$^3S_1$–$^3D_1$	YY 4%		YY 7%	
	$d_{PW} = -1.035$		$d_{PW} = -1.083$	
Case	d_{MS}	$d_D^{(2)}$	d_{MS}	$d_D^{(2)}$
I	0.512	-0.523	0.526	-0.558
II	0.356	-0.679	0.371	-0.712
III	0.748	-0.287	0.761	-0.322
IV	0.328	-0.707	0.344	-0.739

In Table 8 and Table 9 we compare results for the original Mongan potentials with results for the contemporary Mongan potentials fit to the 1993 phase shifts, for the RSC (UPA) deuteron model. Again, the results in Table 9 for the contemporary Mongan potentials are in much better agreement with that of Liu and Timmermans.

Table 8. The deuteron EDM values for the RSC (UPA) rank-one deuteron $^3S_1$–$^3D_1$ potential and four different rank-one $^3P_1$ potentials. The $^3P_1$ potentials are the original Mongan rank-one potentials.

$^3S_1$–$^3D_1$	RSC (UPA)	
	$d_{PW} = -0.962$	
Case	d_{MS}	$d_D^{(2)}$
I	0.307	-0.655
II	0.271	-0.692
III	0.430	-0.532
IV	0.264	-0.698

Table 9. The deuteron EDM values for the RSC (UPA) rank-one deuteron $^3S_1$–$^3D_1$ potential and four different rank-one $^3P_1$ potentials. The $^3P_1$ potentials are of the Mongan form with parameters adjusted to fit the 1993 Nijmegen phase shifts.

$^3S_1$–$^3D_1$	RSC (UPA)	
	$d_{PW} = -0.962$	
Case	d_{MS}	$d_D^{(2)}$
I	0.185	-0.777
II	0.181	-0.781
III	0.240	-0.722
IV	0.194	-0.768

In summary, the differences due to the improved fit to the *NN* scattering data are significant. The effect of the $^3P_1$ interaction is much reduced; d_{MS} is smaller for the contemporary Mongan potentials fit to the 1993 phase shift analysis. This suggests that the potentials fit to the more recent scattering data are weaker, affecting the EDM calculation less. Moreover, for the RSC deuteron calculations shown in Table 8, the short range repulsion in the deuteron has removed most of the $^3P_1$ model dependence. Furthermore, the separable potential model result agrees with that of Liu and Timmermans $d_D^{(2)} = -0.73 \pm 0.01$ *e fm* within about 10%.

6 Conclusions

We find Avishai's original calculation to be in error by a factor of 2; we believe that the corrected result should be – 0.46 *e* fm. This would make his estimate for the ^{2}H EDM more consistent with the plane wave result of Khriplovick and Korkin as well as the realistic potential model calculations of Liu and Timmermans.

The EDM calculation is sensitive to the quality of the fit to the $^3P_1$ *NN* scattering data. The resulting $d_D^{(2)}$ values, based upon the contemporary Mongan potentials fit to the Nijmegen 1993 phase shifts, differ strikingly from the results based upon the original Mongan potentials, especially for the YY models of the deuteron.

Fitting to the Nijmegen 1993 phase shifts reduces the contribution of d_{MS} considerably, suggesting that the contemporary Mongan $^3P_1$ potentials are significantly weaker. Thus, it may be possible to treat the $^3P_1$ interaction as a perturbation when calculating the ^{3}He EDM.

The d_{MS} term is smaller for the RSC (UPA) calculation than for the YY calculations, which leads to $d_D^{(2)}$ being closer to the Liu andTimmerman result for the realistic, contemporary potentials. Moreover, the dependence of the RSC (UPA) result on a particular $^3P_1$ potential is reduced to the point that $d_D^{(2)}$ is almost independent of the $^3P_1$ contemporary Mongan potential model used.

Finally, until the precision of ^{2}H EDM measurements is considerably enhanced (*i.e.*, reaches a level of better than 10%), it would appear that a separable potential approach to modeling the EDM should be more than adequate.

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