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Few-Body Reactions in Nuclear Astrophysics: application to ^{6}He and ^{9}Be production

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Abstract. In this work we obtain the astrophysical reaction and production rates for the two-particle radiative capture processes $\alpha + n + n \rightarrow {}^6\mathrm{He} + \gamma$ and $\alpha + \alpha + n \rightarrow {}^9\mathrm{Be} + \gamma$. The hyperspherical adiabatic expansion method is used. The four-body recombination reactions $\alpha + \alpha + n + n \rightarrow {}^6\mathrm{He} + \alpha$, $\alpha + n + n + n \rightarrow {}^6\mathrm{He} + n$, $\alpha + \alpha + n + n \rightarrow {}^9\mathrm{Be} + n$ and $\alpha + \alpha + \alpha + n \rightarrow {}^9\mathrm{Be} + \alpha$ are also investigated.

1 Introduction

Once the hydrogen fuel in a star is exhausted, the production of energy and the temperature drop. The gravitational collapse increase the temperature and the helium becomes the new source of energy. Due to the lack of neutrons, the A=5 and A=8 instability gaps have to be bridged by the triple-alpha reaction $\alpha + \alpha + \alpha \rightarrow {}^{12}\mathrm{C} + \gamma$.

Nevertheless, different scenarios are also possible such as the so called “hot bubbles”. In these environments the rapid neutron capture nucleosynthesis can happen [1,2].

Among these processes, two very relevant ones are the formation of ^{9}Be through the reaction $\alpha + \alpha + n \rightarrow {}^9\mathrm{Be} + \gamma$ [3,4] and the capture of two neutrons by an α-particle leading to $^6\mathrm{He}+\gamma$ [4,5]. These two reactions can be followed by the capture of another α-particle leading to either $^{12}\mathrm{C}+n$ or $^9\mathrm{Be}+n$. Since the triple alpha reaction is too slow at the temperature-density conditions in the hot bubble, the reactions mentioned above play a crucial role.

Together with the radiative capture reactions there are also other reactions that in a high temperature neutron rich environment could also play a relevant role. In particular the four-body recombination processes $\alpha + \alpha + n + n \rightarrow {}^6\mathrm{He} + \alpha$, $\alpha + n + n + n \rightarrow {}^6\mathrm{He} + n$, $\alpha + \alpha + n + n \rightarrow {}^9\mathrm{Be} + n$ and $\alpha + \alpha + \alpha + n \rightarrow {}^9\mathrm{Be} + \alpha$.

2 Theoretical description: Production and reaction rates

Let us assume some stellar environment characterized by a temperature T, a mass density ρ and the mass abundances X_i of their different constituents. Let us assume also a nuclear reaction involving N of the elements contained in the star, which transform them into whatever M final products. The *Production Rate* for such reaction ($A_1+A_2+\cdots+A_N \rightarrow B_1+B_2+\cdots+B_M$) is defined as the number of reactions taking place per unit time and unit volume of the star. It gives then the velocity at which the products of the reaction are created.

The production rate P obviously depends on the center of mass kinetic energy E of the N-particle system involved in the reaction. We shall denote the production rate at a given energy as $P(E)$.

The total production rate is then given by:

$$P = \int dE P(E) B(E, T), \tag{1}$$

where $B(E, T)$ is the probability of finding the N particles with a center of mass kinetic energy E. This probability depends on the temperature of the star, and it is given by the Maxwell-Boltzmann distribution, which for N particles takes the form:

$$B(E, T) = \frac{1}{\Gamma(\frac{3N-3}{2})} \frac{1}{k_B T} \left(\frac{E}{k_B T}\right)^{\frac{3N-5}{2}} e^{-\frac{E}{k_B T}}, \tag{2}$$

where k_B is the Boltzmann constant. From the exponential in Eq.(2) it is clear that for a given temperature T only energies satisfying $E \lesssim k_B T$ (or at most a few times $k_B T$) are relevant. Therefore, for a temperature of 10^{10} K (or 10 GK), since $k_B T \approx 1$ MeV, only energies not bigger than a few MeV’s play a role. The typical temperature in the center of a standard star, like the sun, is of about 10^{-2} GK, while the one in a hot bubble could reach up to 10 GK [6]. Therefore, in a nuclear energy scale, we are in any case dealing with very low (sometimes extremely low) energy reactions.

The production rate $P(E)$ is given by some function $R(E)$ of the kinetic energy multiplied by the density n_i of each of the elements i entering in the reaction. This density can be written as $n_i = \rho N_A X_i / A_i$, with N_A being the Avogadro number, A_i the mass number of element i, and X_i the mass abundance of such element ($\sum_i X_i = 1$). Therefore:

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$$P(E) = n_1 \cdot n_2 \cdots n_N R(E) = \\ = (\rho N_A)^N \left(\frac{X_1}{A_1}\right) \cdot \left(\frac{X_2}{A_2}\right) \cdots \left(\frac{X_N}{A_N}\right) R(E). \tag{3}$$

$R(E)$ is the transition probability of the reaction with initial energy E, which is given by:

$$R(E) = \frac{2\pi}{\hbar} \frac{1}{g_1 \cdot g_2 \cdots g_N} \times \\ \times \int \delta(E - E_f) \left|\langle \Psi_i | W | \Psi_f \rangle\right|^2 \frac{d^3 \boldsymbol{p}_1}{(2\pi)^3} \cdots \frac{d^3 \boldsymbol{p}_{M-1}}{(2\pi)^3}, \tag{4}$$

where Ψ_i and Ψ_f are the center of mass wave functions of the initial N-body and final M-body states, W is the interaction responsible for the reaction, $\boldsymbol{p}_1, \cdots, \boldsymbol{p}_{M-1}$ are the $M-1$ relative momenta of the M products of the reaction (the remaining momentum is the M-body center of mass momentum which is integrated away), and g_i $(i = 1, \cdots, N)$ is the degeneracy of each of particle states in the incident channel.

Inserting (3) into (1) we can then write the total production rate as:

$$P = (\rho N_A)^N \left(\frac{X_1}{A_1}\right) \cdot \left(\frac{X_2}{A_2}\right) \cdots \left(\frac{X_N}{A_N}\right) \langle R(E) \rangle, \tag{5}$$

where

$$\langle R(E) \rangle = \int dE R(E) B(E, T), \tag{6}$$

which is the so called *Reaction Rate*. In any case, to avoid confusion between $R(E)$ and $\langle R(E) \rangle$, we shall refer to the first of them as the reaction rate (or reaction rate at a given energy), and to the second one as the energy weighted reaction rate.

As one could intuitively think, reactions involving more particles have a smaller reaction rate. The larger the number of particles involved, the smaller the probability to have all them close enough to react. As an example, one could consider the two-neutron radiative capture process $\alpha + n + n \to {}^6\text{He} + \gamma$ and the four-body recombination reaction $\alpha + n + n + n \to {}^6\text{He} + n$. They are both a source of ^{6}He. However, as seen in Eq.(5), the production rate is proportional to ρ^N. Therefore, for a sufficiently large star density ρ, a higher number of particles N in the initial state could compensate the decrease in the reaction rate, and then become a relevant process.

Note that the matrix element in the integrand of Eq.(4) is the same for a given process and the inverse, where the initial and final states are exchanged. The only difference between the reaction rates of both processes appears in the integration momenta, since the final states are in principle different in one case and the other. Therefore, it is always possible to relate the reaction rates for a given process and the inverse. In particular, when only two particles are involved in the initial state, Eq.(4) is simply the two-body cross section multiplied by the relative velocity between both particles, and $\langle R(E) \rangle$ for such process is usually denoted by $\langle v\sigma \rangle$. Thus, the reaction rates for processes leading to two particles can be written in terms of the two-body cross section for the inverse reaction. This is particularly relevant, since such two-body cross sections are often measurable experimentally.

Note also that the energy in Eq.(6) is the kinetic energy in the center of mass of the initial system, which is different for a given reaction and the inverse. However, the center of mass kinetic energies of both processes are related through the Q-value of the reaction. In the reaction $A_1 + A_2 + \cdots + A_N \to B_1 + B_2 + \cdots + B_M$ the initial kinetic energy E_N and the kinetic energy E_M (initial energy for the inverse process) are related by $E_N = E_M - Q$ where Q is the Q-value of the direct reaction.

2.1 Theory: Two-particle Radiative Capture

Let us consider here the radiative capture reaction $a + b + c \to A + \gamma$, where A is a bound three-body state having a, b, and c as constituents.

As discussed above, the reaction rate $\langle R_{abc}(E) \rangle$ for this process can be related to $\langle R_{A\gamma}(E_\gamma) \equiv \langle v\sigma_\gamma(E_\gamma) \rangle$, where $E = E_\gamma + B$, B is the binding energy of the three-body system A, and $\sigma_\gamma(E_\gamma)$ is the photodissociation cross section of A. This relation is given by:

$$R_{abc}(E) = \frac{\hbar^3}{c^2} \frac{8\pi}{(\mu_x \mu_y)^{3/2}} \left(\frac{E_\gamma}{E}\right)^2 \frac{2g_A}{g_a g_b g_c} \sigma_\gamma(E_\gamma) \tag{7}$$

where g_i $(i = a, b, c, A)$ is the degeneracy of particle i, and μ_x and μ_y are the reduced masses of the systems connected by the usual $\boldsymbol{x}$ and $\boldsymbol{y}$ Jacobi coordinates used to describe the three body system made by particles a, b, and c.

With the help of Eqs. (2) to (6) and using Eq.(7) we can then write the production rate for the $a + b + c \to A + \gamma$ reaction as:

$$P_{abc}(\rho, T) = n_a n_b n_c \frac{\hbar^3}{c^2} \frac{8\pi}{(\mu_x \mu_y)^{3/2}} \frac{g_A}{g_a g_b g_c} e^{-\frac{B}{k_B T}} \times \\ \times \frac{1}{(k_B T)^3} \int_{|B|}^{\infty} E_\gamma^2 \sigma_\gamma(E_\gamma) e^{-\frac{E_\gamma}{k_B T}} dE_\gamma, \tag{8}$$

which, since $n_i = \rho N_A X_i / A_i$, depends on the mass density and the temperature.

The photodissociation cross section for the inverse process $A + \gamma \to a + b + c$ can be expanded into electric and magnetic multipoles. In particular, the electric multipole contribution of order λ has the form:

$$\sigma_\gamma^{(\lambda)}(E_\gamma) = \frac{(2\pi)^3(\lambda + 1)}{\lambda[(2\lambda + 1)!!]^2} \left(\frac{E_\gamma}{\hbar c}\right)^{2\lambda - 1} \frac{d\mathcal{B}}{dE}, \tag{9}$$

where the strength function $\mathcal{B}$ for a transition between a three-body state $|n_0 J_0 M_0\rangle$ and the excited state $\{|nJM\rangle\}$ is given by:

$$\mathcal{B}(E\lambda, n_0 J_0 \to nJ) = \sum_{\mu M} |\langle nJM | O_\mu^\lambda | n_0 J_0 M_0 \rangle|^2, \tag{10}$$

where J_0, J and M_0, M are the total angular momenta and their projections of the initial and final states, and all the other needed quantum numbers are collected into n_0 and n.

Finally, the electric multipole operator involved in the previous expression is given by:

$$O^\lambda_\mu = \sum_{i=1}^{3} z_i |\boldsymbol{r}_i - \boldsymbol{R}|^\lambda Y_{\lambda,\mu}(\Omega_{y_i}) \ , \tag{11}$$

where i runs over the three particles, and where we neglect the contributions coming from the intrinsic transitions in each of the three constituents [7].

2.2 Theory: Four-Body Recombination

We now consider the process $a + b + c + d \rightarrow A + d$ where A is a bound three-body state having a, b, and c as constituents. This is the so called four-body recombination process where particle d is just a spectator taking the excess of energy released when a, b, and c create the bound state A.

As previously discussed, since we only have two particles in the final state, the reaction rate $\langle R_{abcd}(E)\rangle$ can be related to $\langle R_{Ad}(T'_z)\rangle$ ($\equiv \langle v_{Ad}\sigma_{Ad}(T'_z)\rangle$), where T'_z is the final relative two-body kinetic energy, $E = T_{z'} + B$, with B being the binding energy of the three-body system A, and $\sigma_{Ad}(T'_z)$ is the cross section for the breakup of A after collision with d. This relation is given by:

$$R_{abcd}(E) = (2\pi)^3 \sqrt{2}\hbar^6 \frac{g_A}{g_a g_b g_c} \frac{\mu'_z}{(\mu_x \mu_y \mu_z)^{3/2}} \frac{\Gamma(\frac{9}{2})}{\Gamma(\frac{3}{2})} \frac{T'_z}{E^{\frac{7}{2}}} \sigma_{Ad}(T'_z) \ , \tag{12}$$

where g_i ($i = a, b, c, A$) is the degeneracy of particle i, and μ_x, μ_y, and μ_z are the reduced masses of the systems connected by the usual $\boldsymbol{x}$, $\boldsymbol{y}$, and $\boldsymbol{z}$ Jacobi coordinates used to describe the four body system made by particles a, b, c, and d.

Inserting now Eqs.(2) and (12) into Eq.(6) we then easily get:

$$\langle R_{abcd}(E)\rangle = \frac{4(2\pi)^{\frac{5}{2}}\hbar^6 \mu'_z}{(\mu_x \mu_y \mu_z)^{3/2}} \frac{g_A}{g_a g_b g_c} e^{-\frac{B}{k_B T}} \times \times \frac{1}{(k_B T)^{\frac{9}{2}}} \int_{|B|}^{\infty} T'_z \sigma_{Ad}(T'_z) e^{-\frac{T'_z}{k_B T}} dT'_z \ . \tag{13}$$

From this expression it is evident the connection between the reaction rate and the observable σ_{Ad}, which in principle can be measured experimentally. However, to compute it numerically, one can directly do it using Eqs.(4) to (6), which require only calculation of the matrix element $\langle \Psi_i | W | \Psi_f \rangle$.

The assumptions made are two: First, the four-body wave functions are approximated as a three-body wave function plus a spectator particle in a relative plane wave; and second, the matrix element involved in the calculation is treated as the sum of three matrix elements corresponding to the interaction between the spectator particle and each of the constituents in the three-body system.

3 Numerical results: Reaction and Production rates

3.1 Two-particle Radiative Capture

Their reaction rates can be obtained from the photodissociation cross sections of the inverse processes.

For the ^{6}He, the dipole term dominates by roughly four orders of magnitude and the quadrupole result agrees with previous estimates by Görres et al. [3] and Fowler et al. [8]. For the dipole contribution our reaction rate is about one order of magnitude higher than Görres et al. [3], Efros et al. [5] and Barlett et al. [4]. The reason is that in these calculations a fully sequential capture process is assumed.

For the $\alpha + \alpha + n \rightarrow {}^9$Be $+ \gamma$ radiative process the dominant contribution comes from the $1/2^+$ states, which produce a peak below 1 GK related to the low-lying resonant state in the photodissociation cross section. Only for temperatures beyond 5 GK the $5/2^+$ contributions begins to dominate. For temperatures beyond 1 GK our calculation agrees with the one in [9].

The computed reaction rate for the production of ^{9}Be is at least one order of magnitude above the one for the production of ^{6}He. Only for temperatures around 5 GK they become comparable.

The production of ^{9}Be is dominating in all the cases for low temperatures. For higher temperatures the reaction rate for production of ^{9}Be is comparable to the one obtained for the production of ^{6}He, and the dominance of one process or another depends on the abundance of neutrons and alphas.

3.2 Four-Body Recombination

Processes having a neutron as spectator clearly dominate. The difference is of roughly two orders of magnitude. This is due to the different nuclear interaction between the spectator and the three constituents in ^{6}He or ^{9}Be, and not to the additional Coulomb repulsion felt by the spectator.

When comparing the rates for production of ^{6}He and ^{9}Be we find that the reaction $\alpha + n + n + n \rightarrow {}^6$He $+ n$ dominates for all temperatures, and the process $\alpha + \alpha + n + n \rightarrow {}^9$Be $+ n$ shows a reaction rate comparable to the one obtained for production of ^{6}He having an α-particle as spectator.

The largest production rate is the one corresponding to the $\alpha + n + n + n \rightarrow {}^6He+n$ reaction. An increase in the mass density by a certain factor, enhances by the same factor the relative four-body recombination production rate. Therefore, for temperatures as the ones estimated for a hot bubble (units of GK), the four-body recombination reactions could easily be very relevant. On the other hand, for typical temperatures in the interior of a standard star of about 10^{-2} GK the electromagnetic processes clearly dominate.

4 Summary and conclusions

Temperature and mass density are two crucial star properties which determine the production rate of a given reaction. As typical temperatures are small, only very low relative energies are relevant.

For a proper description of the radiative capture reactions we must include all the possible mechanisms and usually those of them involving less particles are more important. However, if the star density is large enough the latter could be relevant.

For temperatures as the ones estimated for a “hot bubble”, the four-body recombination reactions could be very relevant. On the other hand, for temperatures of about 10^{-2} GK the electromagnetic processes clearly dominate, and only mass densities many orders of magnitude larger could make the four-body processes to dominate.

References

1. B.S. Meyer, G.J. Mathews, W.M. Howard, S.E. Woosley, and R.D. Hoffman, Astrophys. J. **399**, (1992) 656
2. B.S. Meyer, Annu. Rev. Astron. Astrophys. **32**, (1994) 153
3. J. Görres, H. Herndl, I.J. Thompson, and M. Wiescher, Phys. Rev. C **52**, (1995) 2231
4. A. Bartlett, J. Görres, G.J. Mathews, K. Otsuki, M. Wiescher, D. Frekers, A. Mengoni, and J. Tostevin, Phys. Rev. C **74**, (2006) 015802
5. V.D. Efros, W. Balogh, H. Herndl, R. Hofinger, and H. Oberhummer, Z. Phys. A **355**, (1996) 101
6. M. Arnould and K. Takahashi, Rep. Prog. Phys. **62**, (1999) 395
7. C. Romero-Redondo, E. Garrido, D.V. Fedorov, and A.S. Jensen, Phys. Rev. C **77**, (2008) 054313
8. W.A. Fowler, G.R. Caughlan, and B.A. Zimmerman, Annu. Rev. Astron. Astrophys. **13**, (1975) 69
9. K. Sumiyoshi, H. Utsunomiya, S. Goko, and T. Kajino, Nucl. Phys. A **709**, (2002) 467