Three-Body pd Scattering with a Possible Long-Range Force

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Abstract. The nucleon-nucleon (NN) phase shifts are represented by using a hybrid potential which is composed a short range potential of mesonic origin and a possible long range potential where the short range σ -meson potential is replaced by a Van der Waals (VW)-type potential. Therefore, “the on-shell equivalent but off-shell different” hybrid NN potentials are characterized by only two or four parameters instead of the original σ -meson potential. These potentials are used to calculate pd elastic scattering and to compare with that of the Paris (PEST) potential. Since the hybrid potential needs a longer numerical calculation time due to slower Padé convergence, we utilized only ten single channel hybrid potentials together with four coupled-channel short-range PEST potentials for the pd calculation. The pd results seem to be unreasonable in fitting the short range case, although it is not a complete calculation. In spite of using such hybrid off-shell different potentials only for 1S_0 and 3P_0 -states plus the short range NN for the other states, the pd differential cross section could well represent the short range potential results except in the minimum cross section region.

1 Introduction

Three-nucleon scattering problems have been intensely investigated during almost a half century in an effort to constrain models of the nuclear force. Three-body forces, Δ -isobar effects, and the Coulomb force have been taken into account. However, we still see discrepancies between theoretical predictions based on certain nuclear forces and the experimental data. In addition, some attempts to investigate long-range nuclear interactions such as the Van der Waals (VW) force, which can originate from Coulomb-like potentials, have been made; however, the potentials have a very small coupling constant and produce only a minor effect.

Almost two decades ago, one of the authors (T.S.) , in search of a possible long range force between hadrons[1], analysed S-wave phase shift data for proton-proton scattering. In the approach used the “once subtracted” (Kantor) scattering amplitude was employed. The Kantor amplitude is defined in terms of a dispersion treatment for on-the-energy shell analytic structure of the scattering amplitude in the complex energy plane, where the left-hand cut’s contribution to the integral is given by the difference between the real part of the amplitude and the right-hand cut contribution to the Cauchy integral. Because that difference is replaced by the experimental phase shifts, the Cauchy integral for the left-hand cut is given by those differences, or the potential. The integral range runs from $-\infty$ to the inverse potential range squared $-\mu^2$, where the Coulomb range is $\mu^2 = 0$ and just encloses the origin of the energy plane. The long-range force following from the Coulomb

potential could be the VW force. In other words, the Kantor amplitude’s behavior at the threshold causes a cusp at the origin. The cusp was fitted by a long range potential with the asymptotic form of $V(r) \sim -C/r^\alpha$. After a χ^2 search, we obtained α to be $6.1 \sim 7$ and C to be positive with the strength of the strong interaction. It is consistent with a potential corresponding to a strong VW force.

Here, we try to reproduce modern nuclear phase shifts by replacing the σ -meson term of the Paris potential with a VW potential $C'/r(r+a)^5$ having two parameters, the range a and the depth C' . We obtained a reasonable fit to the phase shifts $^1S_0, ^1P_1, ^1D_2, ^1F_3, ^1G_4, ^3P_0, ^3P_1, ^3D_2, ^3F_3, ^3G_4, ^3S_1 - ^3D_1, ^3D_3 - ^3G_3, ^3P_2 - ^3F_2$, and $^3F_4 - ^3H_4$ by using the GSE method[2].

Preliminary calculations for the three-body pd elastic scattering, which employ the long-range modified single channel potentials but the old short range coupled-channel potentials of the Paris (or PEST) potentials[3] [4], were performed to obtain sample physical observables[5]. The preliminary calculation employing such off-shell different hybrid potentials found that the p - d differential cross section could well agree with the short range potential results except in the minimum cross section region from 90° to 140° which is the area sensitive to the three-body force [6] [7]. In the section 2, we will show how to introduce the long-range hybrid two-body potentials. Section 3 presents the two-body calculated results. Section 4 illustrates the three-body calculation results together with the simple Coulomb treatment, and finally we will discuss our new approach in section 5.

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2 The long-range modification of the NN-potential

The short range σ -meson origin potential is defined by

$$V^\sigma(r) = \frac{\mu_s g_s^2}{4\pi} \left[- \left(1 - \frac{\mu_s^2}{8M^2} \right) Y(\mu_s r) - \frac{\mu_s^2}{2M^2} \left(1 - \frac{\mu_s^2}{8M^2} \right) (\mathbf{L} \cdot \mathbf{S}) Z(\mu_s r) \right] \quad (1)$$

$$Y(x) = \frac{e^{-x}}{x}$$

$$Z(x) = \left(\frac{1}{x} + \frac{1}{x^2} \right) Y(x)$$

where μ_s is the σ -meson inverse range with $\mu_s/\mu_\pi = 4$, μ_π the pion mass, g_s the coupling constant with $g_s^2/4\pi = 5.2$, and M the nucleon mass, respectively[8]. We define the modified potential by using the original NN-potential V^{NN} and $V^\sigma(r)$ and the VW potential $V^W(r)$,

$$V^{(NN)}(r) = V^{NN}(r) - V^\sigma(r) + V^W(r) = V^{NN}(r) + V^\phi(r). \quad (2)$$

We adopt the VW-type potential which is equivalent for the long range limit to the London type.

$$V^W(r) = \frac{-c}{r(r+a)^5} \rightarrow \frac{-c}{r^6} (r \rightarrow \infty). \quad (3)$$

2.1 Integral representation of the VW potential

The integral representation of the VW potential is given by,

$$V(r) = -c' \int_0^\infty t^\gamma e^{-a\sqrt{t}} \frac{e^{-\gamma\sqrt{t}}}{r} dt \quad (4)$$

putting $t = z^2$, and $\alpha = r + a$, and $\alpha z = s$, we obtain

$$V(r) = -c' \int_0^\infty z^{2\gamma} \frac{e^{-\alpha z}}{r} dz^2 = -2c' \int_0^\infty z^{(2\gamma+1)} \frac{e^{-\alpha z}}{r} dz \quad (5)$$

$$= -\frac{2c'}{\alpha r} \int_0^\infty \left(\frac{s}{\alpha} \right)^{(2\gamma+1)} e^{-s} ds = -\frac{2c' \Gamma(2\gamma+2)}{\alpha^{(2\gamma+2)} r}. \quad (6)$$

If we take $\gamma = 3/2$, and $c = 2c'4!$, we obtain the London-type Van der Waals potential,

$$V(r) = -\frac{c}{r(a+r)^5} \quad (7)$$

while adopting $\gamma = 4/2$, and $d_c = 2c'5!$ we obtain the Casimira-type VW form,

$$V(r) = -\frac{d_c}{r(a_c+r)^6}. \quad (8)$$

In the present work, the Casimira-type components was negligible.

2.2 The momentum representation of the VW potential

The momentum representation of $V^W(r)$ is given by using (5),

$$V^W(\mathbf{p}, \mathbf{p}') = \int V^W(r) e^{i(\mathbf{p}-\mathbf{p}')\cdot\mathbf{r}} d\mathbf{r} = -8\pi c' \int_0^\infty z^{2\gamma+1} e^{-az} \frac{1}{(\mathbf{p}-\mathbf{p}')^2 + z^2} dz. \quad (9)$$

The partial wave expansion of (9) is obtained by usual way,

$$V_l^W(p, p') = \frac{1}{2} \int_0^\infty P_l(\chi) V^W(\mathbf{p}, \mathbf{p}') d\chi = \frac{-8\pi c'}{2pp'} \int_0^\infty z^{2\gamma+1} e^{-az} Q_l\left(\frac{p^2 + p'^2 + z^2}{2pp'}\right) dz \quad (10)$$

where the singularity of long range limit at $z \rightarrow 0$ could be removed in eq.(10) for $\gamma = 3/2$.

2.3 The yukawa fitting for the VW-potential

We apply the yukawa fitting to VW-potential for an actual calculation.

$$V^W(r) = \frac{-c}{r(r+a)^5} = -c \int_0^\infty \frac{\mu^4 e^{-a\mu}}{24} \frac{e^{-\mu r}}{r} d\mu = -c \sum_i \frac{\mu_i^4 e^{-a\mu_i}}{24} \frac{e^{-\mu_i r}}{r} w_i. \quad (11)$$

The strong point of Yukawa fomulations is that we easily transform momemntum representation.

$$V^W(r) = -c \sum_i \frac{\mu_i^4 e^{-a\mu_i}}{24} \frac{e^{-\mu_i r}}{r} w_i \rightarrow -c \sum_i C_i \frac{2\pi}{pp'} Q_l(x) \quad (12)$$

$$x = \frac{p^2 + p'^2 + \mu_i^2}{2pp'}. \quad (13)$$

2.4 Auxiliary potential

In (2), we define the difference between the σ origin potential and the VW-type potential as an auxiliary potentials V^ϕ .

$$V^\phi = V^W - V^\sigma. \quad (14)$$

The Lippmann-Schwinger (LS) equation for V^ϕ is given by

$$T^\phi = V^\phi + V^\phi G_0 T^\phi. \quad (15)$$

In order to obtain overall fitting, we demanded that $T^\phi(k, k)$ should be vanished in the low energy limit. This means that phase shifts ϕ by $T^\phi(k, k)$ is 0. In the case of the isospin= 1 and spin single states, the relation is satisfied not only for the low energy but also higher energy region, 1G_4 and 1F_3 for example. The results are shown by the dashed lines in Figs.1, 2, 4, and 6. On the contrary, we could require $T^\phi(k, k) = 0$ at the higher energy region which will be discussed in another occasion.

3 VW parameters and Two-Body Results

3.1 VW parameters

We determine the VW potential parameters in (7) by fitting nuclear phase shifts. We show the VW-type potential parameters See Table1.

Table 1. VW-type potential parameters for the central force in (6).

two-body state	c	a	γ
1S_0	214	0.25	3/2
1P_1	226	0.27	3/2
1D_2	190	0.21	3/2
1F_3	178	0.21	3/2
1G_4	138	0.18	3/2
3P_0	1226	0.348	3/2
3P_1	190	0.18	3/2
3D_2	208	0.27	3/2
3F_3	190	0.27	3/2
3G_4	171	0.27	3/2
3S_1	276	0.25	3/2
3D_1	367	0.25	3/2
3D_3	205	0.25	3/2
3G_3	205	0.28	3/2
3P_2	460.9	0.36	3/2
3F_2	528	0.16	3/2
3F_4	265	0.62	3/2
3H_4	528	0.16	3/2

It is noticed that only $^3P_0, ^3P_2 - ^3F_2, ^3F_4 - ^3H_4$ states need LS forces in the VW-type potential,

$$V^{WLS}(r) = -\frac{d}{r(r+b)^5} V_{so} r_{so}^2 (\mathbf{LS}). \quad (16)$$

See Table2.

3.2 Results of Two-Body Phase Shifts

Firstly, we show for single channel phase shifts the two-body results in Fig.1 to Fig.11. Next, we show the coupled state phase shifts in Fig.12 to Fig.15.

Table 2. VW-type potential parameters for LS force in (16)

two-body state	d	b	V_{so}	r_{so}^2
3P_0	100.6	0.17	0.99	1
3P_2	12	0.56	1	1
3F_2	101	0.20	1	1
3F_4	22	0.20	1	1
3H_4	82	0.20	1	1

Fig. 1. 1S_0 phase shift. The solid line Paris potential, circle long range modified potential and the dashed line auxiliary potential. The horizontal axis is log scale energy.

Fig. 2. 1S_0 phase shift.

Fig. 3. 1P_1 phase shift. The solid line represents the long range modified potential and the squares represent the Paris potential.

Fig. 4. 1D_2 phase shift. Fig. 7. 3P_0 phase shift. The square represent the PEST and the solid line represents the long range modified potential. Fig. 5. 1F_3 phase shift. Fig. 8. 3P_1 phase shift. Fig. 6. 1G_4 phase shift. Fig. 9. 3D_2 phase shift.

Fig. 10. 3F_3 phase shift.

Fig. 13. ${}^3D_3 - {}^3G_3$ phase shift.

Fig. 11. 3G_4 phase shift.

Fig. 14. ${}^3P_2 - {}^3F_2$ phase shift. The solid and dashed line represent the PEST and the squares and triangles represent the long range modified potential.

Fig. 12. ${}^3S_1 - {}^3D_1$ phase shift. The solid and dashed line represent the Paris potential and the squares and triangles represent the long range modified potential.

Fig. 15. ${}^3F_4 - {}^3H_4$ phase shift.

4 Three-Body Calculations with the Long Range Potentials

4.1 Formalism

The AGS equation for the composite particles scattering amplitude with the short range plus long range potential $V^{(L)} = V^s + V^L$ was proposed by one of us (S.O) [9] where C is replaced by L , is given in eqs. (133) and (126) of the reference: [9],

$$X_{\alpha\beta n}^{(L)}(q_\alpha q'_\beta) = Z_{\alpha\beta n}^{(L)}(q_\alpha q'_\beta) + \sum_\gamma \sum_{ij} \int Z_{\alpha\gamma i}^{(L)}(q_\alpha q''_\gamma) \tau_{\gamma,ij}^{sL}(z''_\gamma) X_{\gamma\beta n}^{(L)}(q''_\gamma q'_\beta) \frac{q''_\gamma{}^2 dq''_\gamma}{2\pi^2}. \quad (17)$$

Here, $\tau_{\gamma,ij}^{sL}(z''_\gamma)$ is the two-body propagator, which is defined for the two-body LS amplitude by

$$T_\gamma^{(L)}(z_\gamma) = V_\gamma^s + V_\gamma^s G_\gamma^L(z_\gamma) T_\gamma^{(L)}(z_\gamma) \quad (18)$$

$$\equiv G_0^{-1} |\psi_\gamma^s\rangle \langle \psi_\gamma^{sL}(z_\gamma) | \psi_\gamma^s | G_0^{-1} \quad (19)$$

$$\begin{aligned} \tau_\gamma^{sL}(z_\gamma) &= \frac{S_\gamma^{sL}(z_\gamma)}{E - z_\gamma(k_\gamma^0) - q_\gamma^2/2\mu_\gamma} \\ &= \frac{S_\gamma^{sL}(z_\gamma)}{E(q_\gamma^0) - H_0(q_\gamma)} \end{aligned} \quad (20)$$

where only the rank one case is shown for simplicity, but our calculations correspond to ranks of the PEST-potentials. Here, E the three-body energy, z_γ the two-body energy, μ_γ the three-body reduced mass between nucleon and the composite particle, respectively. q_γ is the spectator momentum of the γ -channel, therefore q_γ^0 is the pole of the two-body amplitude in the complex three-body momentum plane, by neglecting the three-body force,

$$\begin{aligned} Z_{\alpha\beta n}^{(L)}(q_\alpha q'_\beta) &\equiv \langle \psi_{\alpha\beta}^s | G_0^{-1} \bar{\delta}_{\alpha\beta} + T_{\alpha\beta}^L - T_\alpha^L \delta_{\alpha\beta} | \psi_{\beta n}^s \rangle \\ &\equiv \mathcal{B}_{\alpha\beta n}^L + \mathcal{V}_{\alpha\beta n}^L \delta_{\alpha\beta}, \end{aligned} \quad (21)$$

where $\bar{\delta}_{\alpha\beta} = 1 - \delta_{\alpha\beta}$, $\mathcal{V}_{\alpha\beta n}^L \delta_{\alpha\beta}$ is the long range potential between the charged composite particle deuteron and the proton. The long range potential consists of the VW-potential and the Coulomb potential. However, we learned that the long range part of the VW-potential is rather mild and the long range behavior could decrease faster than the Coulomb potential. Therefore, the VW part could be treated as a part of the short range nuclear force which was parameterized in the previous section. Therefore, we put, by omitting the physical state quantum numbers m, n ,

$$\mathcal{V}_{\alpha\beta}^L \delta_{\alpha\beta} \approx \mathcal{V}_\alpha^C \delta_{\alpha\beta} \equiv V_\alpha^{C(pd)}, \quad (22)$$

where the last term means the Coulomb potential between the proton and deuteron. Hereafter we omit m, n for simplicity except for the necessity.

On the other hand, the remaining part $\mathcal{B}_{\alpha\beta}^L$ will be shown by using the short range form factor: $g^s(p)$, the long range

modified form factor: $g^{(L)}(p) = g^s G_0 T^L(p)$, and the three-body free Green's function,

$$\mathcal{B}_{\alpha\beta}^L = \langle \psi_\alpha^s | G_0^{-1} \bar{\delta}_{\alpha\beta} + T_{\alpha\beta}^L - T_\alpha^L \delta_{\alpha\beta} | \psi_\beta^s \rangle - \mathcal{V}_\alpha^{C(pd)} \delta_{\alpha\beta}. \quad (23)$$

Since the long range modified form factor is smaller than the short range form factor in the 100 MeV region, then we can neglect such a term, and obtain

$$\mathcal{B}_{\alpha\beta}^L \approx \langle \psi_\alpha^s | G_0^{-1} \bar{\delta}_{\alpha\beta} | \psi_\beta^s \rangle \equiv Z_{\alpha\beta n}^s(q_\alpha q'_\beta). \quad (24)$$

In summary, we obtain

$$\begin{aligned} X_{\alpha\beta n}^{(L)}(q_\alpha q'_\beta) &= \{Z_{\alpha\beta n}^s + \mathcal{V}_{\alpha\beta n}^{C(pd)} \delta_{\alpha\beta}\} (q_\alpha q'_\beta) \\ &+ \sum_\gamma \sum_{ij} \int \{Z_{\alpha\gamma i}^s + \mathcal{V}_{\alpha\gamma i}^{C(pd)} \delta_{\alpha\gamma}\} (q_\alpha q''_\gamma) \\ &\tau_{\gamma,ij}^{sL}(z''_\gamma) X_{\gamma\beta n}^{(L)}(q''_\gamma q'_\beta) \frac{q''_\gamma{}^2 dq''_\gamma}{2\pi^2}, \end{aligned} \quad (25)$$

where $\mathcal{V}_{\alpha\gamma i}^{C(pd)} \delta_{\alpha\gamma}$ exists only for the p - d Coulomb potential.

Therefore the three-body elastic amplitude is given by using two-potential theory for the three-body equation (25), as for the equation (142) of reference [9],

$$X_{\alpha\beta}^{(L)}(q_\alpha q'_\beta) = \bar{\mathcal{Q}}_\alpha^C \chi_{\alpha\beta}^{sL} \mathcal{Q}_\beta^C + X_{\alpha\alpha}^{C(pd)} \delta_{\alpha\beta}. \quad (26)$$

The Møller operator for the p - d Coulomb scattering is defined by

$$X_{\alpha\alpha}^{C(pd)} = \mathcal{V}_\alpha^{C(pd)} + \mathcal{V}_\alpha^{C(pd)} \tau_\alpha^{sC} X_{\alpha\alpha}^{C(pd)} \quad (27)$$

$$\equiv \mathcal{V}_\alpha^{C(pd)} \mathcal{Q}_\alpha^C \equiv \bar{\mathcal{Q}}_\alpha^C \mathcal{V}_\alpha^{C(pd)}. \quad (28)$$

In order to obtain the leading term of the three-body elastic scattering,

$$\chi_{\alpha\beta}^{sL} = \mathcal{B}_{\alpha\beta}^L + \sum_\gamma \mathcal{B}_{\alpha\gamma}^L \mathcal{T}_\gamma^L \chi_{\gamma\beta}^{sL} \quad (29)$$

$$\mathcal{T}_\alpha^L = \tau_\alpha^{sL} + \tau_\alpha^{sL} X_{\alpha\alpha}^{C(pd)} \tau_\alpha^{sL} \quad (30)$$

$$= \frac{S_\gamma^{sL}(z_\gamma)}{E(q_\gamma^0) - H_0(q_\gamma) - \mathcal{V}_\gamma^{C(pd)}}. \quad (31)$$

In our calculation, we adopted the $E = 135$ MeV elastic scattering case, then $\mathcal{V}_\gamma^{C(pd)}$ effect is very small except for forward scattering, then we approximate it by taking

$$\mathcal{T}_\alpha^L \approx \tau_\alpha^{sL}. \quad (32)$$

Furthermore, in order to solve the leading equation (29), the long range VW-type potential is more important than the Coulomb term by the order of the coupling constant. Therefore, we can replace τ_α^{sL} by the VW-type potential,

$$\tau_\alpha^{sL} \approx \tau_\alpha^{sVW}. \quad (33)$$

In conclusion, we solved the following equations,

$$\chi_{\alpha\beta n}^{sL} = Z_{\alpha\beta n}^s + \sum_\gamma \sum_{ij} Z_{\alpha\gamma i}^s \tau_{\gamma ij}^{sVW} \chi_{\gamma\beta n}^{sL}. \quad (34)$$

Table 3. Explanation about Figs.16, and 17.

○ includes the van der Waals type hybrid potential and × is only the original nuclear PEST-potential.

two-body state	CAL A	CAL B
1S_0	○	○
1P_1	○	×
1D_2	○	×
1F_3	○	×
1G_4	○	×
3P_0	○	○
3P_1	○	×
3D_2	○	×
3F_3	○	×
3G_4	○	×
3S_1	×	×
3D_1	×	×
3D_3	×	×
3G_3	×	×
3P_2	×	×
3F_2	×	×
3F_4	×	×
3H_4	×	×

Finally, the total amplitude can be calculated by using the exact p - d elastic scattering amplitude in (26)

$$X_{\alpha\beta}^{(L)}(q_\alpha q'_\beta) = \bar{\Omega}_\alpha^{C(pd)} \chi_{\alpha\beta}^{sVW} \Omega_\beta^{C(pd)} + X_{\alpha\alpha}^{C(pd)} \delta_{\alpha\beta}. \quad (35)$$

The Rutherford scattering amplitude $X_{\alpha\beta}^{C(pd)}$ is given,

$$X_{\alpha\alpha}^{C(pd)} = \frac{\epsilon}{2q \sin^2(\theta/2)} \exp \left[-i\epsilon \ln \left(\sin^2 \frac{\theta}{2} \right) + i\pi + 2i\eta_0 \right],$$

$$\epsilon = M_{pd} \frac{ZZ' e^2}{\hbar^2 q}, \quad \text{and} \quad \eta_l = \arg \Gamma(l+1+i\epsilon), \quad (36)$$

with the reduced mass $M_{pd} = \mu_\alpha$ of proton and deuteron, and $q_\beta \equiv q$ is the final momentum, respectively.

4.2 The Three-Body Calculated Results

The three-body calculated results for the differential cross section in elastic p - d scattering are illustrated in Fig.16 and the analysing power in Fig.17. In spite of good phase shift fitting by the VW-type potentials, ten long range modified single channel states plus PEST for the rest of the states, “CAL A” doesn’t give sufficient results in Fig.16. However, “CAL B” with two modified 1S_0 and 3P_0 states plus all other PEST potentials leads to successful results (the dashed-dotted line).

5 Conclusion and Discussion

Our motivation in the present work came from the serious discrepancy around the low energy S-wave Kantor amplitudes between the meson based short range potential

and the experimental values. It was suggested that a Van der Waals type potential was preferable in the very low energy region. Therefore, we tried to represent the two-body phase shifts with a proper Van der Waals type potential instead of the σ -meson based short range potential. Fortunately, we could obtain a good fit for almost all states using only two parameters for the width and the range. Adding the shape of the potential did not give a better fit. We tried other long-range possible potential frameworks to fit the two-body phase shifts, however, almost all attempts failed. We did not have a reasonable understanding of this fact. Nevertheless, the three-body calculation raises a problem in which the three-body Padé convergence is slower than the short range potential calculation except for the two-body $J = 0$ cases which seems to be a kind of litmus test. (See Fig.18.)

Here we show a three-body calculation with the modified 1P_1 plus the original 3S_1 – 3D_1 states obtained using the Padé approximation[10]. In this process, we found that the Padé convergence for such a long range modified potential is very slow in comparison with the short-range potential only.

Our results suggest that the present VW-type hybrid potential works only for $J = 0$ states. It means that $J = 0$ states have the potential of (7) but for the $J = 1$ – 4 states one should take another type or use short range only. Anyhow, we should investigate the reason why such a long range phenomenon is caused in the lower energy NN interaction.

Fig. 18. The Padé[10] convergence of the three-body calculation. Two-body states are 3S_1 – 3D_1 and 1P_1 . Three-body state is $3/2^+$. The horizontal axis is numbers of iterations based on only nuclear forces which has only 1. The vertical axis is error of Padé and the Padé convergence condition is $\left| \frac{X_{\alpha\beta}^{ON} - X_{\alpha\beta}^{ON, before}}{X_{\alpha\beta}^{ON, before}} \right| < 10^{-8}$ with $X_{\alpha\beta}^{ON}$ on-shell scattering amplitude, $X_{\alpha\beta}^{ON, before}$ previous degree of Padé.

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Fig. 16. Differential Cross Section for elastic proton-deuteron scattering at 135MeV (in Lab energy). Exp.1 is Ref [6] and Exp.2 is Ref [7] . “CAL A”(the dashed line) adopts all the single channel states: 1S_0 , 1P_1 , 1D_2 , 1F_3 , 1G_4 , 3P_0 , 3P_1 , 3D_2 , 3F_3 , 3G_4 with the long range VW-type modified (or hybrid) potential plus the short range potentials (PEST) for four coupled states $^3S_1 - ^3D_1$, $^3P_2 - ^3F_2$, $^3D_3 - ^3G_3$, and $^3F_4 - ^3H_4$. While the “CAL B” (dashed dotted line) is given only by the long range modified 1S_0 , and 3P_0 states and others the original PEST potential are adopted. The “org” denotes the PEST potential results.

Fig. 17. Ay of elastic proton-deuteron scattering at 135MeV in Lab system.

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