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Two-pion exchange electromagnetic current in chiral effective field theory using the method of unitary transformation

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Abstract. Using the method of unitary transformation in combination with chiral effective field theory we derive the two-pion exchange contributions to the two-nucleon electromagnetic current. A formal definition of the current operator in this scheme and the power counting is presented. We discuss the implications of additional unitary transformations that have to be present to ensure the renormalizability of the one-pion exchange current. Further, we give explicit and compact results for the current in coordinate-space.

1 Introduction

Chiral effective field theory (EFT) has been fruitfully applied to the model independent study of hadronic processes. In particular the nuclear forces have been successfully derived [1]. To deepen our understanding of hadronic processes an analysis of further quantities like vector- or axial-currents is necessary. This allows us to to make predictions, for example for electron and photon few-nucleon reactions with momentum transfer of order M_π.

An application to these reactions, where a lot of experimental data are available, see [2] for a recent review article on the theoretical achievements in this field based on conventional framework, is very interesting. Recent progress in the accurate description of the two- [3,4] and more-nucleon systems [5] within the chiral EFT, see also [6] and references therein, gives an additional motivation to apply this framework to the abovementioned processes.

This motivates to calculate the electromagnetic current operator in EFT. Additionally, the current will automatically be consistent with the nuclear force, i.e. fulfill a continuity equation. While the leading two-nucleon contributions to the exchange current arise from one-pion exchange and are well known, the corrections at the one-loop level have not yet been completely worked out.

Recently, Pastore et al. [7,8] have performed an important step towards the description of the exchange current operator. They calculated the electromagnetic current operator at leading one-loop order based on time-ordered perturbation theory.

In this contribution, we discuss the two-pion exchange current operator at leading one-loop order in the framework of unitary transformation. Since there is a freedom of choice of the unitary transformation, we also calculate the leading-loop order one-pion exchange. This enables us to constrain the additional unitary transformations and we briefly described the procedure in this report. Since we use a completely different formalism, our results provide a non-trivial check of the results of Pastore et al. Further, we also present results for the exchange current density. The results are given in extremely compact formulae in configuration space. An expression in momentum space and a more details can be found in [9].

This manuscript is structured as follows. In section 2 we briefly review the method of unitary transformation and discuss how nuclear currents can be calculated. Section 3 contains discussion of additional unitary transformations needed to obtain a renormalizable one-pion exchange current. Results in configuration space are given in section 4. We end with a summary and outlook.

2 Nuclear currents using the method of unitary transformation

We begin with a brief reminder about the method of unitary transformation, applied in the calculation of nuclear potentials, for details see [1].

Following Okubo [10] the unitary transformation U can be parameterized as

$$U = \begin{pmatrix} \eta(1+A^\dagger A)^{-1/2} & -A^\dagger(1+AA^\dagger)^{-1/2} \\ A(1+A^\dagger A)^{-1/2} & \lambda(1+AA^\dagger)^{-1/2} \end{pmatrix}, \qquad (1)$$

in terms of the operator A with the property $A = \lambda A \eta$, where we introduced projection operators η (λ) on the purely nucleonic (the remaining) part of the Fock space satisfying $\eta^2 = \eta$, $\lambda^2 = \lambda$, $\eta\lambda = \lambda\eta = 0$ and $\lambda + \eta = \mathbf{1}$. The operator A has to be chosen in a way that the transformed

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Hamiltonian $\tilde{H} \equiv U^\dagger H U$ is block-diagonal in the $\eta-$ and $\lambda-$ subspaces.

In Ref. [11], a convenient formulation of the power counting has been presented. The low-momentum dimension ν of the effective potential, V, $V \sim \mathcal{O}(Q/\Lambda)^\nu$ with Q and Λ refering to the soft and hard scales of the order of the pion and ρ-meson masses, respectively, is given (modulo the normalization constant -2) by the overall inverse mass dimension of the coupling constants entering the expression for V:

$$\nu = -2 + \sum V_i \kappa_i \,, \qquad \kappa_i = d_i + \frac{3}{2} n_i + p_i - 4 \,. \quad (2)$$

Here, where V_i is the number of vertices of type i while d_i, n_i and p_i refer to the number of derivatives or M_π-insertions, nucleon field and pion field operators, respectively. Further, κ_i is simply the canonical field dimension of a vertex of type i (up to the additional constant -4). Writing the effective chiral Hamiltonian H as

$$H = \sum_{\kappa=1}^{\infty} H^{(\kappa)} \,, \quad (3)$$

the operator A can be calculated recursively from the requirement that the transformed Hamiltonian is block-diagonal,

$$\begin{aligned} A &= \sum_{\alpha=1}^{\infty} A^{(\alpha)} \,, \\ A^{(\alpha)} &= \frac{1}{E_\eta - E_\lambda} \lambda \Bigg[H^{(\alpha)} + \sum_{i=1}^{\alpha-1} H^{(i)} A^{(\alpha-i)} - \sum_{i=1}^{\alpha-1} A^{(\alpha-i)} H^{(i)} \\ &\quad - \sum_{i=1}^{\alpha-2} \sum_{j=1}^{\alpha-j-1} A^{(i)} H^{(j)} A^{(\alpha-i-j)} \Bigg] \eta \,. \end{aligned} \quad (4)$$

Here, E_η (E_λ) refers to the free energy of nucleons (nucleons and pions) in the state η (λ). It is important to emphasize that Eq. (1) does not provide the most general parameterization of the unitary operator. Moreover, as found in Ref. [11], the subleading contributions to the three-nucleon force obtained using the parameterization in Eq. (1) cannot be renormalized. To restore renormalizability at the level of the Hamilton operator additional unitary transformation U' in the η-subspace of the Fock space had to be employed, $\eta U' \eta U'^\dagger \eta = \eta U'^\dagger \eta U' \eta = \eta$, whose explicit form at lowest non-trivial order is given in that work.

It is, in principle, straightforward to extend this formalism to low-energy electromagnetic reactions such as e.g. electron scattering off light nuclei. Here and in what follows, we restrict ourselves to the one-photon-exchange approximation to the scattering amplitude. The effective nuclear current operator $\eta J^\mu_{\rm eff}(x) \eta$ acting in the η-space is then defined according to

$$\begin{aligned} \langle \Psi_f | J^\mu(x) | \Psi_i \rangle &= \langle \phi_f | \eta U'^\dagger \eta U^\dagger J^\mu(x) U \eta U' \eta | \phi_i \rangle \\ &\equiv \langle \phi_f | \eta J^\mu_{\rm eff}(x) \eta | \phi_i \rangle \,, \end{aligned} \quad (5)$$

where $\eta|\phi_{i,f}\rangle = \eta U'^\dagger \eta U^\dagger |\Psi_{i,f}\rangle$ denote the transformed states and we have omitted the components $\lambda|\phi_{i,f}\rangle$ which is justified as long as one stays below the pion production threshold. In the above expression, $J^\mu(x)$ denotes the hadronic current density which enters the effective Lagrangian $\mathcal{L}_{\pi N \gamma}$ describing the interaction of pions and nucleons with an external electromagnetic field $\mathcal{A}^\mu$ and is given by

$$J^\mu(x) = \partial_\nu \frac{\partial \mathcal{L}_{\pi N\gamma}}{\partial(\partial_\nu \mathcal{A}_\mu)} - \frac{\partial \mathcal{L}_{\pi N\gamma}}{\partial \mathcal{A}_\mu} \,. \quad (6)$$

Notice that contrary to the Hamilton operator, the unitarily transformed current does, in general, not have the block-diagonal form, i.e. $\eta U^\dagger J^\mu(x) U \lambda \neq 0$.

We emphasize that the power counting employed in the present work implies the following restrictions on the photon momentum k in the two-nucleon rest frame

$$|\mathbf{k}| \sim \mathcal{O}(M_\pi) \,, \qquad k^0 \sim \mathcal{O}\left(\frac{M_\pi^2}{m}\right) \ll M_\pi \,, \quad (7)$$

where M_π and m refer to the pion and nucleon masses, respectively. For the kinematics with $k^0 \sim \mathcal{O}(M_\pi)$, one will have to systematically keep track of the new soft momentum scale $\sqrt{M_\pi m}$. This goes beyond the scope of the present work.

For the calculation of the leading two-pion exchange two-nuclear current operator in the present work we only need the leading pion and pion-nucleon terms in the effective Lagrangian

$$\begin{aligned} \mathcal{L}^{(2)}_{\pi\pi} &= \frac{F_\pi^2}{4} \operatorname{tr}\left[D_\mu U D^\mu U^\dagger + M_\pi^2 (U + U^\dagger) \right] \,, \\ \mathcal{L}^{(1)}_{\pi N} &= N^\dagger (i v \cdot D + g_A S \cdot u) N \,, \end{aligned} \quad (8)$$

where the superscript i in $\mathcal{L}^{(i)}$ denotes the number of derivatives and/or pion mass insertions. Here, F_π (g_A) is the pion decay constant (the nucleon axial-vector coupling), N represents a nucleon field in the heavy-baryon formulation and $S_\mu = \frac{1}{2}\gamma_5 \sigma_{\mu\nu} v^\nu$ is the Pauli-Lubanski spin vector which reduces to $S^\mu = (0, \frac{1}{2}\boldsymbol{\sigma})$ for $v_\mu = (1,0,0,0)$. At the order we are working and for the contributions to the current operator considered in the present work, all LECs entering Eq. (8) should be taken at their physical values. Further, the SU(2) matrix $U = u^2$ collects the pion fields and various covariant derivatives are defined according to

$$\begin{aligned} D_\mu U &= \partial_\mu U - i r_\mu U + i U l_\mu \,, \\ u_\mu &= i \left[u^\dagger (\partial_\mu - i r_\mu) u - u (\partial_\mu - i l_\mu) u^\dagger \right] \,, \\ D_\mu N &= \left[\partial_\mu + \Gamma_\mu - i v_\mu^{(s)} \right] N \\ \text{with} \quad \Gamma_\mu &= \frac{1}{2} \left[u^\dagger (\partial_\mu - i r_\mu) u + u (\partial_\mu - i l_\mu) u^\dagger \right] \,. \end{aligned} \quad (9)$$

To describe the coupling to an external electromagnetic field, the left- and right-handed currents r_μ and l_μ and the isoscalar current $v_\mu^{(s)}$ have to be chosen as

$$r_\mu = l_\mu = \frac{e}{2} \mathcal{A}_\mu \tau_3 \,, \qquad v_\mu^{(s)} = \frac{e}{2} \mathcal{A}_\mu \,, \quad (10)$$

where e denotes the elementary charge. Expanding the various terms in the effective Lagrangian in powers of the pion field and using the canonical formalism along the lines of Ref. [12], we end up with the following interaction terms in the Hamilton density

$$\begin{aligned}\mathcal{H}_{21}^{(1)} &= \frac{g_A}{2F_\pi} N^\dagger \left(\boldsymbol{\sigma}\boldsymbol{\tau}\cdot\cdot\nabla\boldsymbol{\pi}\right) N\,,\\ \mathcal{H}_{22}^{(2)} &= \frac{1}{4F_\pi^2} N^\dagger \left[\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}}\right]\cdot\boldsymbol{\tau} N\,,\\ \mathcal{H}_{42}^{(4)} &= \frac{1}{32F_\pi^4}\left(N^\dagger\left[\boldsymbol{\tau}\times\boldsymbol{\pi}\right]N\right)\cdot\left(N^\dagger\left[\boldsymbol{\tau}\times\boldsymbol{\pi}\right]N\right)\,, \end{aligned} \tag{11}$$

and the electromagnetic current density is of the form

$$\begin{aligned} J_{20}^{0\;(-1)} &= \frac{e}{2} N^\dagger\left(\mathbb{1}+\tau_3\right)N\,,\\ J_{02}^{0\;(-1)} &= e\left[\boldsymbol{\pi}\times\dot{\boldsymbol{\pi}}\right]_3\,,\\ \mathbf{J}_{02}^{\;(-1)} &= -e\left[\boldsymbol{\pi}\times\nabla\pi\right]_3\,,\\ \mathbf{J}_{21}^{\;(0)} &= e\,\frac{g_A}{2F_\pi} N^\dagger \boldsymbol{\sigma}\left[\boldsymbol{\tau}\times\boldsymbol{\pi}\right]_3 N\,. \end{aligned} \tag{12}$$

In the above expressions we adopt the notation of Ref. [11]. In particular, the subscripts a and b in $\mathcal{H}_{ab}^{(\kappa)}$ and $J_{ab}^{\mu\;(\kappa)}$ refer to the number of the nucleon and pion fields, respectively, while the superscript κ gives the dimension of the operator as defined in Eq. (2). Further, the symbol $\cdot\cdot$ in Eq. (11) denotes a scalar product in the spin and isospin spaces.

3 Additional unitary transformations

The unitary transformations described so far are not of the most general form. Especially, we can use additional transformations $U(\mathcal{A}^\mu)$ depending on the external electromagnetic field. Obviously, the operator $U(\mathcal{A}^\mu)$ has to be chosen in such a way, that the transformed Hamiltonian $U^\dagger(\mathcal{A}^\mu)H_{\pi N\gamma}U(\mathcal{A}^\mu)$ is block-diagonal with respect to η- and λ-spaces. Further, the operator $U(\mathcal{A}^\mu)$ has to have the property

$$U(\mathcal{A}^\mu)\to\mathbb{1}, \quad \text{for} \quad \mathcal{A}^\mu\to 0\,. \tag{13}$$

To determine the additional transformations, we calculate the one-pion exchange contribution to the effective current operator $\eta U(\mathcal{A}^\mu)^\dagger\eta J^\mu_{\rm eff}(x)\eta U(\mathcal{A}^\mu)\eta$ at one-loop order.

In the one-pion exchange sector low-energy constants (LECs) l_i from $\mathcal{L}_\pi$ [13,14] and d_i from $\mathcal{L}_{\pi N}$ [15,16] contribute. Since all β-functions of the LECs are fixed, the ultra violet divergencies stemming from the loop-integrals are severely constrained. It is not obvious, and in fact not true, that the divergencies of the one-pion exchange graphs of the current defined by Eq. (5) can be absorbed by these LECs. Thus, we have to choose the additional transformations in a way that makes the renormalizability of the current explicit.

Let us now show an example of this procedure, a complete treatment can be found in a separate publication [17] devoted to the one-pion exchange current. In Fig. 1 we can see a typical one-loop contribution to the one-pion exchange and the contributions from the LECs. We compute the loop-diagram in dimensional regularization and expand in $1/(d-4)$, where d is the space-time dimension that enters the integral. To see how the renormalization works it is sufficient to focus on the part proportional to $1/(d-4)$ with a spin-isospin structure proportional to $[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^3\ (\mathbf{q}_1+\mathbf{q}_2)\ \mathbf{q}_1\cdot\boldsymbol{\sigma}_1\ \mathbf{q}_2\cdot\boldsymbol{\sigma}_2$. Here $\mathbf{q}_{1/2}$ is the momentum transfer of nucleon one/two. Also, we will neglect the dependence on the pion energies $\sqrt{q_{1/2}^2+M_\pi^2}$. The only LECs that enter are d_{21} and d_{22}. The part proportional to $1/(d-4)$ is then given by $d_{21/22}=\beta_{21/22}/(16\pi^2(d-4))$. The result of the diagrams then reads

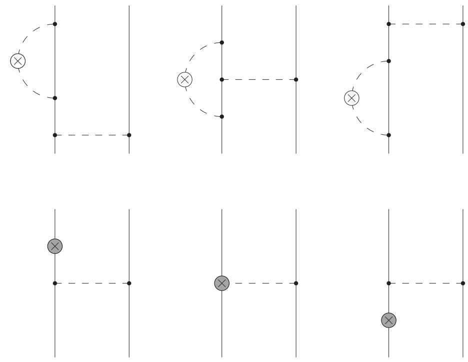

Fig. 1. Example of one-loop and LEC contributions to the one-pion exchange. Solid and dashed lines refer to nucleons and pions, respectively. Solid dots are the lowest-order vertices from the effective Lagrangian while the crosses represent insertions of the electromagnetic vertices as explained in the text. The grey cross denotes insertions of electromagnetic vertices from a higher order Lagrangian.

$$\begin{aligned} &e\,\frac{g_A\,i}{8F_\pi^2}\left(\frac{2\beta_{21}+\beta_{22}}{16\pi^2(d-4)}+\frac{22g_A^3}{96\pi^2(d-4)}\right)\\ &=e\,\frac{g_A\,i}{4F_\pi^2}\left(-\frac{g_A^3}{16\pi^2(d-4)}+\frac{11g_A^3}{96\pi^2(d-4)}\right)\neq 0\,, \end{aligned} \tag{14}$$

where we have used that $\beta_{21}=-g_A^3$ and $\beta_{22}=0$ [15,16]. Clearly, the divergence of the loop-digram is not absorbed by the LECs.

Let us now introduce an additional transformation $U(\mathcal{A}^\mu)$ of the form

$$\begin{aligned} U(\mathcal{A}^\mu) &= e^{S_1+S_2+S_3}\,,\\ S_1 &= -\frac{\alpha}{2}\eta\left[J_{02}^{(-1)}\frac{\lambda^2}{E_\pi^2}\mathcal{H}_{21}^{(1)}\frac{\lambda^1}{E_\pi}\mathcal{H}_{21}^{(1)}-\mathcal{H}_{21}^{(1)}\frac{\lambda^1}{E_\pi}\mathcal{H}_{21}^{(1)}\frac{\lambda^2}{E_\pi^2}J_{02}^{(-1)}\right]\eta\,,\\ S_2 &= -\frac{\alpha}{2}\eta\left[J_{02}^{(-1)}\frac{\lambda^2}{E_\pi}\mathcal{H}_{21}^{(1)}\frac{\lambda^1}{E_\pi^2}\mathcal{H}_{21}^{(1)}-\mathcal{H}_{21}^{(1)}\frac{\lambda^1}{E_\pi^2}\mathcal{H}_{21}^{(1)}\frac{\lambda^2}{E_\pi}J_\pi\right]\eta\,,\\ S_3 &= -\alpha\eta\left[\mathcal{H}_{21}^{(1)}\frac{\lambda^1}{E_\pi}J_{02}^{(-1)}\frac{\lambda^1}{E_\pi^2}\mathcal{H}_{21}^{(1)}\right.\\ &\qquad\left.-\mathcal{H}_{21}^{(1)}\frac{\lambda^1}{E_\pi^2}J_{02}^{(-1)}\frac{\lambda^1}{E_\pi}\mathcal{H}_{21}^{(1)}\right]\eta\,, \end{aligned} \tag{15}$$

where α is an arbitrary parameter and E_π stands for the energy of all pions in the intermediate state. This changes the contribution of the diagrams to

$$e\frac{g_A\, i}{4F_\pi^2}\left(-\frac{g_A^3}{16\pi^2(d-4)}+\frac{g_A^3}{16\pi^2(d-4)}+(1-\alpha)\frac{5g_A^3}{96\pi^2(d-4)}\right). \tag{16}$$

We see that when we set $\alpha = 1$, the divergence is completely absorbed into the LEC, as it should be. By this procedure it is possible to obtain a renormalized one-pion exchange current that is consistent with the known β-functions of the LECs. A more detailed study of the one-pion exchange current will be published in a future work [17]. It is important to notice, that non of the additional unitary transformation affects the two-pion exchange currents.

4 Results in configuration space

For the sake of convenience, we distinguish between seven classes of contributions according to the power of the LEC g_A (i.e. proportional to g_A^0, g_A^2 and g_A^4) and the type of the hadronic current J_{20}^μ, J_{21}^μ or J_{02}^μ as shown in Fig. 2. Notice that there are no contributions proportional to g_A^0 and involving J_{20}^μ and J_{21}^μ. We also emphasize that the second diagram in the class 3 does not generate any contribution. It results from the term in the Hamilton density which is absent in the Lagrangian and arises through the application of the canonical formalism. Finally, it should be understood that the meaning of diagrams in the method of unitary transformation is different from the one arising in the context of covariant and/or time-ordered perturbation theory.

Below, we give explicit results for the current and charge densities, $J^\mu = (\rho, \mathbf{J})$, resulting from the individual classes using the notation

$$\begin{aligned}&\langle \mathbf{p}_1{}'\, \mathbf{p}_2{}'|J^\mu|\mathbf{p}_1\, \mathbf{p}_2\rangle\\ &\qquad \delta(\mathbf{p}_1{}'+\mathbf{p}_2{}'-\mathbf{p}_1-\mathbf{p}_2-\mathbf{k})\left[\sum_{X=\mathrm{c}1}^{\mathrm{c}7} J_X^\mu+(1\leftrightarrow 2)\right],\end{aligned} \tag{17}$$

where $\mathbf{p}_i$ ($\mathbf{p}_i{}'$) denotes the incoming (outgoing) momentum of nucleon i and $\mathbf{k}$ is the photon momentum. In order to avoid confusion with the nucleon labels, in the following we label the Cartesian components of various vectors in isospin space by the superscripts rather than subscripts. Further, $(1 \leftrightarrow 2)$ refers to the contribution resulting from the interchange of the nucleon labels. We find the following compact results for the current density in configuration-space from the individual classes of the diagrams shown in Fig. 2:

$$\begin{aligned}
&\mathbf{J}_{\mathrm{c}1}(\mathbf{r}_{10},\mathbf{r}_{20}) =\\
&\quad e\,\frac{g_A^2\,M_\pi^7}{128\pi^3F_\pi^4}\left[\boldsymbol{\nabla}_{10}\,[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^3+2\,[\boldsymbol{\nabla}_{10}\times\boldsymbol{\sigma}_2]\,\tau_1^3\right]\\
&\quad\times\delta(\mathbf{x}_{20})\,\frac{K_1(2x_{10})}{x_{10}^2}\,,\\
&\mathbf{J}_{\mathrm{c}2}(\mathbf{r}_{10},\mathbf{r}_{20}) =\\
&\quad -e\,\frac{g_A^4\,M_\pi^7}{256\pi^3F_\pi^4}\left(3\nabla_{10}^2-8\right)\Big[\boldsymbol{\nabla}_{10}\,[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^3\\
&\quad +2\,[\boldsymbol{\nabla}_{10}\times\boldsymbol{\sigma}_2]\,\tau_1^3\Big]\,\delta(\mathbf{x}_{20})\,\frac{K_0(2x_{10})}{x_{10}}\\
&\quad +e\,\frac{g_A^4\,M_\pi^7}{32\pi^3F_\pi^4}\,[\boldsymbol{\nabla}_{10}\times\boldsymbol{\sigma}_1]\,\tau_2^3\,\delta(\mathbf{x}_{20})\,\frac{K_1(2x_{10})}{x_{10}^2}\,,\\
&\mathbf{J}_{\mathrm{c}3}(\mathbf{r}_{10},\mathbf{r}_{20}) =\\
&\quad -e\,\frac{M_\pi^7}{512\pi^4F_\pi^4}\,[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^3\,(\boldsymbol{\nabla}_{10}-\boldsymbol{\nabla}_{20})\\
&\quad\times\frac{K_2(x_{10}+x_{20}+x_{12})}{(x_{10}\,x_{20}\,x_{12})(x_{10}+x_{20}+x_{12})}\,,\\
&\mathbf{J}_{\mathrm{c}5}(\mathbf{r}_{10},\mathbf{r}_{20}) =\\
&\quad -e\,\frac{g_A^2\,M_\pi^7}{256\pi^4F_\pi^4}\,(\boldsymbol{\nabla}_{10}-\boldsymbol{\nabla}_{20})\Big[[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^3\,\boldsymbol{\nabla}_{12}\cdot\boldsymbol{\nabla}_{20}\\
&\quad -2\tau_1^3\boldsymbol{\sigma}_2\cdot[\boldsymbol{\nabla}_{12}\times\boldsymbol{\nabla}_{20}]\Big]\frac{K_1(x_{10}+x_{20}+x_{12})}{(x_{10}\,x_{20}\,x_{12})}\,,\\
&\mathbf{J}_{\mathrm{c}7}(\mathbf{r}_{10},\mathbf{r}_{20}) =\\
&\quad e\,\frac{g_A^4\,M_\pi^7}{512\pi^4F_\pi^4}\,(\boldsymbol{\nabla}_{10}-\boldsymbol{\nabla}_{20})\Big[[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^3\,\boldsymbol{\nabla}_{12}\cdot\boldsymbol{\nabla}_{10}\\
&\quad\times\boldsymbol{\nabla}_{12}\cdot\boldsymbol{\nabla}_{20}+4\tau_2^3\boldsymbol{\sigma}_1\cdot[\boldsymbol{\nabla}_{12}\times\boldsymbol{\nabla}_{10}]\;\boldsymbol{\nabla}_{12}\cdot\boldsymbol{\nabla}_{20}\Big]\\
&\quad\times\frac{x_{10}+x_{20}+x_{12}}{x_{10}\,x_{20}\,x_{12}}K_0(x_{10}+x_{20}+x_{12})\,,
\end{aligned} \tag{18}$$

and the charge density

$$\begin{aligned}
&\rho_{\mathrm{c}1}(\mathbf{r}_{10},\mathbf{r}_{20}) = \rho_{\mathrm{c}2}(\mathbf{r}_{10},\mathbf{r}_{20}) = \rho_{\mathrm{c}3}(\mathbf{r}_{10},\mathbf{r}_{20}) = 0\,,\\
&\rho_{\mathrm{c}4}(\mathbf{r}_{10},\mathbf{r}_{20}) = e\,\frac{g_A^2\,M_\pi^7}{256\pi^2F_\pi^4}\,\tau_1^3\,\delta(\mathbf{x}_{20})\left(\nabla_{10}^2-2\right)\frac{e^{-2x_{10}}}{x_{10}^2}\,,\\
&\rho_{\mathrm{c}5}(\mathbf{r}_{10},\mathbf{r}_{20}) = -e\,\frac{g_A^2\,M_\pi^7}{256\pi^2F_\pi^4}\,\tau_2^3\,\delta(\mathbf{x}_{20})\left(\nabla_{10}^2-2\right)\frac{e^{-2x_{10}}}{x_{10}^2}\,,\\
&\rho_{\mathrm{c}6}(\mathbf{r}_{10},\mathbf{r}_{20}) = -e\,\frac{g_A^4\,M_\pi^7}{256\pi^2F_\pi^4}\,\delta(\mathbf{x}_{20})\Big[\tau_1^3\left(2\nabla_{10}^2-4\right)\\
&\qquad\qquad +\tau_2^3\,\boldsymbol{\sigma}_1\cdot\boldsymbol{\nabla}_{10}\,\boldsymbol{\sigma}_2\cdot\boldsymbol{\nabla}_{10}\;-\tau_2^3\boldsymbol{\sigma}_1\cdot\boldsymbol{\sigma}_2\Big]\frac{e^{-2x_{10}}}{x_{10}^2}\\
&\qquad\qquad -e\,\frac{g_A^4\,M_\pi^7}{128\pi^2F_\pi^4}\,\delta(\mathbf{x}_{20})\,\tau_1^3\,\left(3\nabla_{10}^2-11\right)\frac{e^{-2x_{10}}}{x_{10}}\,,\\
&\rho_{\mathrm{c}7}(\mathbf{r}_{10},\mathbf{r}_{20}) = -e\,\frac{g_A^4\,M_\pi^7}{512\pi^3F_\pi^4}\Big[(\tau_1^3+\tau_2^3)\Big(\boldsymbol{\nabla}_{12}\cdot\boldsymbol{\nabla}_{10}\boldsymbol{\nabla}_{12}\cdot\boldsymbol{\nabla}_{20}\\
&\qquad\qquad +\boldsymbol{\nabla}_{12}\cdot[\boldsymbol{\nabla}_{10}\times\boldsymbol{\sigma}_1]\,\boldsymbol{\nabla}_{12}\cdot[\boldsymbol{\nabla}_{20}\times\boldsymbol{\sigma}_2]\Big)\\
&\qquad\qquad +[\boldsymbol{\tau}_1\times\boldsymbol{\tau}_2]^3\,\boldsymbol{\nabla}_{12}\cdot\boldsymbol{\nabla}_{10}\,\boldsymbol{\nabla}_{12}\cdot[\boldsymbol{\nabla}_{20}\times\boldsymbol{\sigma}_2]\Big]\\
&\qquad\qquad\times\frac{e^{-x_{10}}}{x_{10}}\,\frac{e^{-x_{20}}}{x_{20}}\,\frac{e^{-x_{12}}}{x_{12}}\,.
\end{aligned} \tag{19}$$

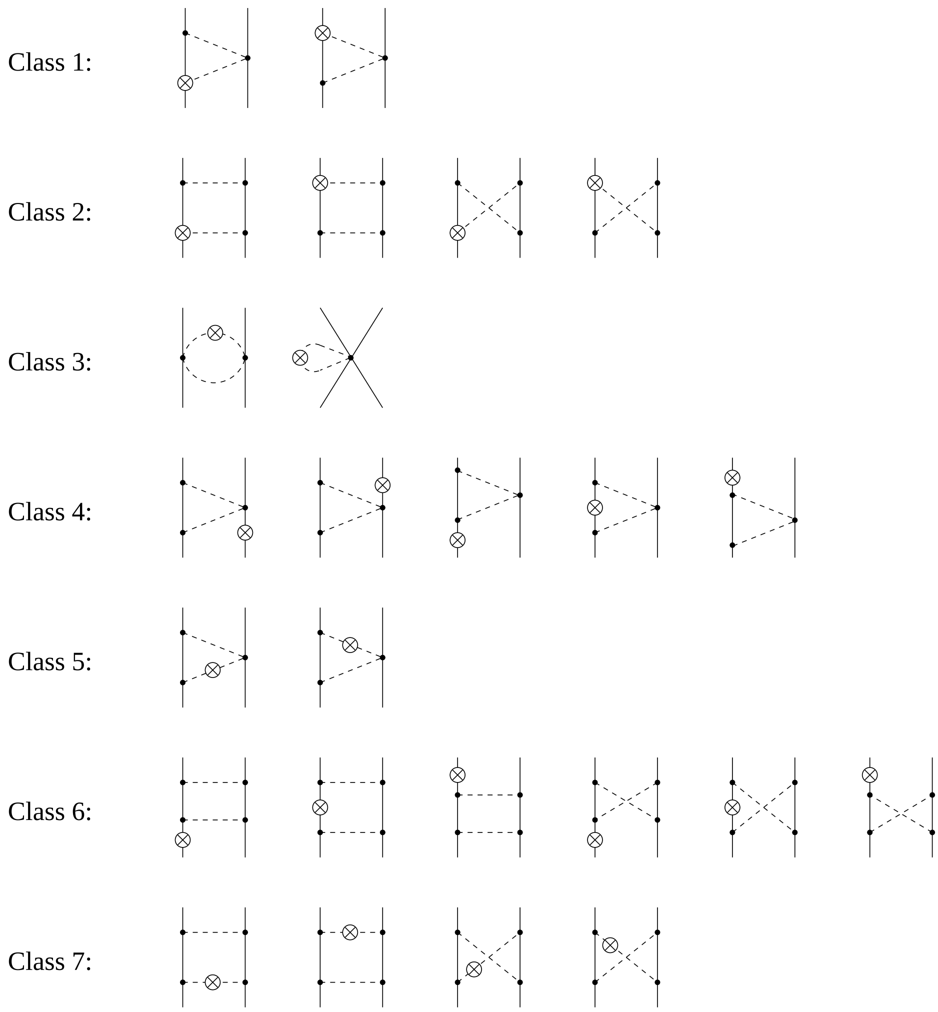

Fig. 2. Diagrams showing contributions to the leading two-pion exchange currents. Solid and dashed lines refer to nucleons and pions, respectively. Solid dots are the lowest-order vertices from the effective Lagrangian while the crosses represent insertions of the electromagnetic vertices as explained in the text. Diagrams resulting from interchanging the nucleon lines are not shown.

In the above expressions, $K_n(x)$ denote the modified Bessel functions of the second kind and we have introduced dimensionless variables $\mathbf{x}_{10} = M_\pi \mathbf{r}_{10}$, $\mathbf{x}_{20} = M_\pi \mathbf{r}_{20}$ and $\mathbf{x}_{12} = M_\pi \mathbf{r}_{12} = M_\pi(\mathbf{r}_1 - \mathbf{r}_2)$. Further, $x_{ij} \equiv |\mathbf{x}_{ij}|$ and all derivatives with respect to $\mathbf{x}_{10}$, $\mathbf{x}_{20}$ and $\mathbf{x}_{12}$ are to be evaluated as if these variables were independent of each other. We also emphasize that the above expressions are valid for $x_{10} + x_{20} > 0$. Finally, it should be understood that the behavior of the current and charge densities at short distances will be affected if one uses a regularization with a finite value of the cutoff.

5 Summary and Outlook

The application of the method of unitary transformation to derive the leading two-pion exchange two-nucleon charge and current densities based on chiral effective field theory was presented. The resulting nuclear current is given configuration space, where we were able to evaluate all loop integrals analytically leading to very compact expressions in terms of the modified Bessel functions of the second kind. We have also explicitly verified that the derived exchange currents fulfill the continuity equation, see [9] for details.

In addition to the two-pion exchange contributions, there are also one-pion exchange and short-range terms at order $\mathcal{O}(eQ)$, see Refs. [7,8] for a recent work based on time-ordered perturbation theory. As we have demonstrated in this contribution, renormalization of the one-pion exchange contributions at the one-loop level strongly restricts the ambiguity in the definition of the current and provides a highly non-trivial consistency check of the calculation. In particular, one needs to ensure that *all* appearing ultraviolet divergences are absorbed into redefinition of the LECs d_i and l_i from $\mathcal{L}_{\pi N}^{(3)}$ and $\mathcal{L}_{\pi}^{(4)}$, respectively, with already known β-functions, see e.g. [13–16]. This work is in progress [17].

Finally, in the future, one also needs to test the convergence of the chiral expansion for the one- and two-pion exchange currents by calculating the corrections at order $\mathcal{O}(eQ^2)$. Given the large numerical values of the LECs $c_{3,4}$

from $\mathcal{L}_{\pi N}^{(2)}$, one might expect sizeable corrections which, indeed, is well known to be the case for the two-pion exchange potential [18]. In this context, it might be advantageous to include the $\Delta(1232)$ isobar as an explicit degree of freedom in effective field theory utilizing the small scale expansion [19].

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