Mass splitting of heavy-baryons from QCD sum rules

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Abstract. We extract the heavy-baryons mass-splittings due to $SU(3)$ breaking using double ratios of QCD sum rules. We evaluate the masses of the strange-heavy baryons Ξ_Q , Ξ_Q^* , Ω_Q and Ω_Q^* using as input the masses of the associated non-strange Λ_Q , Σ_Q and Σ_Q^* baryons. We notice that the leading term controlling the mass-splittings is the ratio $\langle \bar{s}s \rangle / \langle \bar{q}q \rangle$ of the quark condensate and not the running mass $\bar{m}_s$. We also predict the hyperfine splittings $\Omega_Q^* - \Omega_Q$ and $\Xi_Q^* - \Xi_Q$.

1 Introduction

The quantum numbers and quark content of the bottom baryons are shown in Table 1.

Table 1. Properties and valence quark content ($q = u$ or d) of the bottom baryons. j is the total spin of the light quark pair

Baryon	quark content	J^P	I	j
Λ_b	qqb	$\frac{1}{2}^+$	0	0
Σ_b	qqb	$\frac{1}{2}^+$	1	1
Σ_b^*	qqb	$\frac{3}{2}^+$	1	1
Ξ_b	qsb	$\frac{1}{2}^+$	$\frac{1}{2}$	0
Ξ_b'	qsb	$\frac{1}{2}^+$	$\frac{1}{2}$	1
Ξ_b^*	qsb	$\frac{3}{2}^+$	$\frac{1}{2}$	1
Ω_b	ssb	$\frac{1}{2}^+$	0	1
Ω_b^*	ssb	$\frac{3}{2}^+$	0	1

The Λ_b was the first observed bottom baryon. It was observed at the CERN ISR [1] and was confirmed by several collaborations. The PDG mass for this state is [2]:

$$M_{\Lambda_b} = (5620.2 \pm 1.6) \text{ MeV.} \quad (1)$$

Only in 2007, more than 15 years after the observation of Λ_b , the Σ_b and Σ_b^* were observed by the CDF Collaboration [3]. They were observed in the decay channel $\Sigma_b^{(*)\pm} \rightarrow \Lambda^0 \pi^\pm$ with the masses given in Table 2.

Following this discovery, the D0 Collaboration reported the observation of the Ξ_b^- baryon in the decay channel $\Xi_b^- \rightarrow$

Table 2. Masses of the $\Sigma_b^{(*)}$ baryons observed by the CDF Collaboration.

Baryon	Mass (MeV)
Σ_b^+	$5807.8^{+2.0}_{-2.2} \pm 1.7$
Σ_b^-	$5815.2 \pm 1.0 \pm 1.7$
Σ_b^{*+}	$5829.0^{+1.6+1.7}_{-1.8-1.8}$
Σ_b^{*-}	$5836.4 \pm 2.0^{+1.8}_{-1.7}$

$J/\psi \Xi^-$ with the mass [4]:

$$M_{\Xi_b}^{(D0)} = (5774 \pm 11 \pm 15) \text{ MeV.} \quad (2)$$

This observation was confirmed by the CDF Collaboration but with a somewhat bigger mass [5]:

$$M_{\Xi_b}^{(CDF)} = (5792.9 \pm 2.5 \pm 1.7) \text{ MeV.} \quad (3)$$

The CDF result is in a excellent agreement with the prediction made by Karliner *et al.* [6], $M_{\Xi_b} = (5795 \pm 5) \text{ MeV}$, and also with the prediction from Jenkins [7], $M_{\Xi_b} = (5805.7 \pm 8.1) \text{ MeV}$.

The Ω_b^- was first observed by the D0 Collaboration in the decay channel $\Omega_b^- \rightarrow J/\psi \Omega^-$ with the mass [8]:

$$M_{\Omega_b}^{(D0)} = (6165 \pm 10 \pm 13) \text{ MeV.} \quad (4)$$

This mass is much higher than expected [9] and is higher than the predictions from different calculations shown in Table 3. However, a new observation of the Ω_b^- , from CDF Collaboration, measured [15]:

$$M_{\Omega_b}^{(CDF)} = (6054.4 \pm 6.8 \pm 0.9) \text{ MeV,} \quad (5)$$

in a much better agreement with the predictions in Table 3. The prediction in ref. [14] was done in a QCD sum rule (QCDSR) calculation [16]. Earlier uses of QCDSR [17–19] for understanding charm and beauty baryons masses

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Table 3. Prediction for the Ω_b baryon mass from different calculations.

ref.	Mass (MeV)
[10]	6052.1 ± 5.6
[11]	6039.1 ± 8.3
[12]	$6006 \pm 10 \pm 19$
[13]	6036 ± 81
[14]	5820 ± 230

have been performed in full QCD [20,21] and in HQET [22], where the results are in quite good agreement with recent experimental findings but with relatively large uncertainties.

Here, we concentrate on the analysis of the heavy baryons mass-splittings due to $SU(3)$ breaking using double ratios (DR) of QCDSR, which are less sensitive to the exact value and definition of the heavy quark mass and to the QCD continuum contributions than the simple ratios used in the literature to determine the absolute value of heavy baryon masses.

2 QCD Sum rules

The QCDSR is constructed from the two-point correlation function

$$\Pi(q) = i \int d^4x e^{iq \cdot x} \langle 0 | T[\eta(x) \bar{\eta}(0)] | 0 \rangle. \quad (6)$$

Lorentz covariance, parity and time reversal imply that the correlator in Eq. (6) has the form

$$\Pi(q) = \not{q} \Pi_1(q^2) + \Pi_2(q^2). \quad (7)$$

For each invariant function Π_1 and Π_2 , a sum rule can be obtained.

The QCD sum rule approach represents an attempt to link the hadron phenomenology with the interactions of quarks and gluons. The method is based on three ingredients: a phenomenological description of the correlator, a theoretical description of the same correlator via an operator product expansion (OPE), and a procedure for matching these two descriptions and extracting the parameters that characterize the hadronic state of interest.

In the phenomenology side, each one of the invariant functions in Eq. (7) can be expressed as a dispersion integral over a physical spectral density ρ_i :

$$\Pi_i^{phen}(q^2) = - \int ds \frac{\rho_i(s)}{q^2 - s + i\epsilon} + \dots, \quad (8)$$

where the dots represent subtraction terms. The spectral density is described, as usual, as a single sharp pole representing the lowest resonance plus a smooth continuum representing higher mass states:

$$\begin{aligned} \rho_1(s) &= \lambda^2 m_B \delta(s - m_B^2) + \rho_1^{cont}(s), \\ \rho_2(s) &= \lambda^2 \delta(s - m_B^2) + \rho_2^{cont}(s), \end{aligned} \quad (9)$$

where λ^2 gives the coupling of the current with the low mass hadron of interest and m_B denotes the heavy baryon mass. It is assumed that the continuum contribution to the spectral density, $\rho_i^{cont}(s)$ in Eq. (9), vanishes below a certain continuum threshold s_0 . Above this threshold, it is assumed to be given by the result obtained with the OPE:

$$\rho_i^{cont}(s) = \frac{1}{\pi} \text{Im}[\Pi_i(s)] \Theta(s - s_0). \quad (10)$$

In the OPE side, we work at leading order in α_s and consider the contributions of condensates up to dimension six. We keep the terms which are linear in the strange-quark mass m_s . After making a Borel transform of both sides, and transferring the continuum contribution to the OPE side, the sum rules can be written as:

$$\lambda^2 e^{-m_B^2/M^2} = \int_{m_Q^2}^{s_0} ds e^{-s/M^2} \frac{1}{\pi} \text{Im}[\Pi_1(s)], \quad (11)$$

$$\lambda^2 m_B e^{-m_B^2/M^2} = \int_{m_Q^2}^{s_0} ds e^{-s/M^2} \frac{1}{\pi} \text{Im}[\Pi_2(s)], \quad (12)$$

where M^2 is the sum rule parameter (Borel mass).

An important point of this method is the choice of appropriate interpolating current. The lowest dimension general currents for the spin 1/2 baryons with one heavy quark Q are:

$$\begin{aligned} \eta_{\Xi_Q} &= \epsilon_{abc} [(q_a^T C \gamma_5 s_b) + b(q_a^T C s_b) \gamma_5] Q_c, \\ \eta_{\Lambda_Q} &= \eta_{\Xi_Q} \quad (s \rightarrow q) \\ \eta_{\Omega_Q} &= \epsilon_{abc} [(s_a^T C \gamma_5 Q_b) + b(s_a^T C Q_b) \gamma_5] s_c, \\ \eta_{\Sigma_Q} &= \eta_{\Omega_Q} \quad (s \rightarrow q), \end{aligned} \quad (13)$$

where b is an arbitrary mixing parameter.

For the spin 3/2 baryons, we follow Ref. [21] and work with the interpolating currents:

$$\begin{aligned} \eta_{\Xi_Q^*}^\mu &= \sqrt{\frac{2}{3}} [(q^T C \gamma_\mu Q) s + (s^T C \gamma_\mu Q) q + (q^T C \gamma_\mu s) Q] \\ \eta_{\Omega_Q^*}^\mu &= \frac{1}{\sqrt{2}} \eta_{\Xi_Q^*}^\mu \quad (q \rightarrow s) \\ \eta_{\Sigma_Q^*}^\mu &= \frac{1}{\sqrt{2}} \eta_{\Xi_Q^*}^\mu \quad (s \rightarrow q), \end{aligned} \quad (14)$$

where an anti-symmetrization over colour indices is understood. In general one can estimate the baryon masses from the following ratios:

$$m_B^2 = \mathcal{R}_i^q = \frac{\int_{m_Q^2}^{s_0} ds s e^{-s/M^2} \text{Im}[\Pi_i(s)]}{\int_{m_Q^2}^{s_0} ds e^{-s/M^2} \text{Im}[\Pi_i(s)]}, \quad i = 1, 2, \quad (15)$$

$$m_B = \mathcal{R}_{21}^q = \frac{\int_{m_Q^2}^{s_0} ds e^{-s/M^2} \text{Im}[\Pi_2(s)]}{\int_{m_Q^2}^{s_0} ds e^{-s/M^2} \text{Im}[\Pi_1(s)]}, \quad (16)$$

Only as an example, we give here the results obtained for the Ω_b . The expressions for the spectral densities are:

- Π_1 :

$$\begin{aligned}
 \text{Im}[\Pi_1]^{\text{pert}}(s) &= \frac{(5+2b+5b^2)m_Q^4}{2^{11}\pi^3} \left(\frac{1}{x^2} - \frac{8}{x} + 8x - x^2 - 12 \ln x \right), \\
 \text{Im}[\Pi_1]^{m_s}(s) &= \frac{3m_s m_Q^3}{2^8 \pi^3} (1-b^2) \left(\frac{2}{x} + 3 - 6x + x^2 + 6 \ln x \right), \\
 \text{Im}[\Pi_1]^{\langle \bar{s}s \rangle}(s) &= -\frac{3m_Q \langle \bar{s}s \rangle}{2^5 \pi} (1-b^2)(1-x)^2, \\
 \text{Im}[\Pi_1]^{m_s \langle \bar{s}s \rangle}(s) &= \frac{3m_s \langle \bar{s}s \rangle}{2^6 \pi} (1+b)^2 (1-x^2), \\
 \text{Im}[\Pi_1]^{\langle G^2 \rangle}(s) &= -\frac{\langle g^2 G^2 \rangle}{2^{12} 3 \pi^3} (1-x) \left[(1+b^2)(11x-5) + 2b(7x-1) \right], \\
 \text{Im}[\Pi_1]^{\langle \bar{s}g\sigma.Gs \rangle}(s) &= \frac{\langle \bar{s}g\sigma.Gs \rangle}{m_Q 2^7 \pi} (1-b^2)(13x^2-6x), \\
 \text{Im}[\Pi_1]^{m_s \langle \bar{s}g\sigma.Gs \rangle}(s) &= -\frac{m_s \langle \bar{s}g\sigma.Gs \rangle}{2^7 3 \pi} \left[(7+22b + 7b^2)\delta(s-m_Q^2) - 6(1+b)^2 \frac{m_Q^2}{s^2} \right], \\
 \Pi_1^{\langle \bar{s}s \rangle^2}(q^2) &= \frac{\langle \bar{s}s \rangle^2}{24} (1-b)^2 \frac{1}{m_Q^2 - q^2}, \\
 \Pi_1^{m_s \langle \bar{s}s \rangle^2}(q^2) &= -\frac{m_Q m_s \langle \bar{s}s \rangle^2 (1-b^2)}{8(m_Q^2 - q^2)^2}. \tag{17}
 \end{aligned}$$

- Π_2 :

$$\begin{aligned}
 \text{Im}[\Pi_2]^{\text{pert}}(s) &= \frac{(1-b)^2 m_Q^5}{2^9 \pi^3} \left[(1-x) \left(\frac{1}{x^2} + \frac{10}{x} + 1 \right) + 6 \left(1 + \frac{1}{x} \right) \ln x \right], \\
 \text{Im}[\Pi_2]^{m_s}(s) &= \frac{3m_s m_Q^4}{2^8 \pi^3} (1-b^2) \left(\frac{1}{x^2} - \frac{6}{x} + 3 + 2x - 6 \ln x \right), \\
 \text{Im}[\Pi_2]^{\langle \bar{s}s \rangle}(s) &= -\frac{3m_Q^2 \langle \bar{s}s \rangle}{2^5 \pi} (1-b^2)x \left(1 - \frac{1}{x} \right)^2, \\
 \text{Im}[\Pi_2]^{m_s \langle \bar{s}s \rangle}(s) &= -\frac{3m_s m_Q \langle \bar{s}s \rangle}{2^5 \pi} (3+2b+3b^2)(1-x), \\
 \text{Im}[\Pi_2]^{\langle G^2 \rangle}(s) &= -\frac{m_Q \langle g^2 G^2 \rangle}{2^{11} 3 \pi^3} (1-b)^2 (1-x) \left(5 - \frac{2}{x} \right), \\
 \text{Im}[\Pi_2]^{\langle \bar{s}g\sigma.Gs \rangle}(s) &= \frac{\langle \bar{s}g\sigma.Gs \rangle}{2^7 \pi} (1-b^2)(6+x), \\
 \text{Im}[\Pi_2]^{m_s \langle \bar{s}g\sigma.Gs \rangle}(s) &= \frac{m_s m_Q \langle \bar{s}g\sigma.Gs \rangle}{2^7 3 \pi} \left[(25+22b + 25b^2)\delta(s-m_Q^2) - 3(5+6b+5b^2)\frac{1}{s} \right],
 \end{aligned}$$

$$\begin{aligned}
 \Pi_2^{\langle \bar{s}s \rangle^2}(q^2) &= \frac{m_Q \langle \bar{s}s \rangle^2}{24} (5+2b+5b^2) \frac{1}{m_Q^2 - q^2}, \\
 \Pi_2^{m_s \langle \bar{s}s \rangle^2}(q^2) &= -\frac{m_s \langle \bar{s}s \rangle^2 (1-b^2)}{8(m_Q^2 - q^2)} \left[1 + \frac{m_Q^2}{m_Q^2 - q^2} \right], \tag{18}
 \end{aligned}$$

where $x = m_Q^2/s$. The expressions for the spectral densities for the other currents are given in ref. [23].

3 Results for Ω_b

In the numerical analysis of the sum rules for the Ω_b we use the same values used in ref. [14]: $m_s = (0.10 \pm 0.03)$ GeV, $m_b(m_b) = (4.24 \pm 0.06)$ GeV, $\langle \bar{q}q \rangle = -(0.23 \pm 0.03)^3$ GeV³, $\langle \bar{s}s \rangle = 0.8 \langle \bar{q}q \rangle$, $\langle \bar{q}g\sigma.Gq \rangle = m_0^2 \langle \bar{q}q \rangle$ with $m_0^2 = 0.8$ GeV², $\langle g^2 G^2 \rangle = 0.88$ GeV⁴.

In Fig. 1 we show the contribution of each term in Eq. (17) to the sum rule in Eq. (11), for $b = 2$ and $\sqrt{s_0} = 6.5$ GeV. We see that we get an excellent OPE convergence. We get a similar OPE convergence for other values of b .

Fig. 1. The OPE convergence for the sum rule Eq. (11) for Ω_b , using $\sqrt{s_0} = 6.5$ GeV and $b = 2$. The dotted, long-dashed, dot-dashed, dotted and solid with squares lines give, respectively, the perturbative, quark condensate, gluon condensate, mixed condensate and four-quark condensate contributions. The solid line gives the total OPE contribution to the sum rule.

However, this is not the case of the sum rule in Eq. (12), since for $b = 1$, for instance, the perturbative the quark condensate and the gluon condensate contributions vanishes. For values of $b > 1$ the perturbative term becomes negative. Therefore, we will use only the sum rule in Eq. (11). To obtain the mass of the baryon we use the ratio defined in Eq. (15) with $i = 1$.

In fig. 2 we show the results obtained for m_{Ω_b} , as a function of the Borel mass, for different values of b . We see from this figure that we get a reasonable Borel stability, and we get

$$m_{\Omega_b} = (5.92 \pm 0.21) \text{ GeV}, \tag{19}$$

in a very good agreement with the experimental value in Eq. (5), but with a relatively large uncertainty.

Fig. 2. Ω_b mass obtained from Eq. (15) with $i = 1$ $\sqrt{s_0} = 6.5$ GeV for $b = 2$ (solid line) and $b = -1$ (dashed line).

4 Double ratio sum rules

The uncertainty in Eq. (19) is only due to the values of b and the Borel parameter. Therefore, taking into account the uncertainties due to the values of s_0 and to the QCD parameters (quark masses and condensates), the uncertainty in Eq. (19) should be even bigger. Besides, there is not a precise criterium to determine the value of the parameter b . To try to circumvent these problems, instead of working with the ratios defined in Eqs. (15) and (16), we are going to use the double ratio sum rules (DR) [24]:

$$r_i^{sd} \equiv \sqrt{\frac{\mathcal{R}_i^s}{\mathcal{R}_i^d}}, \quad r_{21}^{sd} \equiv \frac{\mathcal{R}_{21}^s}{\mathcal{R}_{21}^d}, \quad (20)$$

which take directly into account the $SU(3)$ breaking effects. These quantities are less sensitive to the choice of the heavy quark masses and to the value of the continuum threshold than the simple ratios $\mathcal{R}_i$ and $\mathcal{R}_{21}$.

For the numerical analysis we shall introduce the RGI quantities $\hat{m}$ and $\hat{m}_q$ [25]:

$$\begin{aligned} \bar{m}_q(\tau) &= \frac{\hat{m}_q}{(-\log \sqrt{\tau}\Lambda)^{2/\beta_1}} \\ \langle \bar{q}q \rangle(\tau) &= \hat{\mu}_q^3 (-\log \sqrt{\tau}\Lambda)^{2/\beta_1} \\ \langle \bar{q}g\sigma.Gq \rangle(\tau) &= \hat{\mu}_q^3 (-\log \sqrt{\tau}\Lambda)^{1/-3\beta_1} m_0^2, \end{aligned} \quad (21)$$

where $\beta_1 = -(1/2)(11 - 2n/3)$ is the first coefficient of the β function for n flavours, and $\tau = 1/M^2$. We have used the quark mass and condensate anomalous dimensions reviewed in [17]. We use the same QCD parameters as in ref. [23], given in Table 4.

As an example of the DR sum rule we show here the results obtained analysing the $\Omega_c(css)/\Sigma_c(cqq)$ and $\Omega_b(bss)/\Sigma_b(bqq)$ cases.

4.1 $\Omega_c(css)/\Sigma_c(cdd)$

Analysing the Borel mass behavior and continuum threshold behavior of the DR sum rules in Eq. (20), we found

Table 4. QCD input parameters.

Parameters	Values
Λ	(353 ± 15) MeV
$\hat{m}_d$	(6.1 ± 0.5) MeV
$\hat{m}_s$	(114.5 ± 20.8) MeV
$\hat{\mu}_d$	(263 ± 7) MeV
$\kappa \equiv \langle \bar{s}s \rangle / \langle \bar{d}d \rangle$	(0.7 ± 0.1)
m_0^2	(0.8 ± 0.1) GeV ²
$\langle \alpha_s G^2 \rangle$	$(6.8 \pm 1.3) \times 10^{-2}$ GeV ⁴
m_c	$(1.18 \sim 1.47)$ GeV
m_b	$(4.18 \sim 4.72)$ GeV

that r_{21}^{sd} does not give a result consistent with the one from r_i^{sd} ($i = 1, 2$) and it is also less stable in M^2 than the two others [23]. Therefore, we do not consider r_{21}^{sd} . In Fig. 3 we show the b behavior of r_i^{sd} , $i = 1, 2$ for fixed values of M^2 and s_0 . We see that r_1^{sd} is not stable versus b .

Fig. 3. The b behavior of the DR sum rules r_1^{sd} (dashed-dotted line) and r_2^{sd} (dotted line) for $M^2 = 1.25$ GeV² and $s_0 = 11$ GeV².

The final result from r_2^{sd} is:

$$r_{\Omega_c}^{sd} = 1.141 \pm 0.039, \quad (22)$$

which is determined using $6 \leq s_0 \leq 11$ GeV². The sources of the errors come from M^2 , b , s_0 , m_c , m_s and κ . The other QCD parameters give negligible errors. Using this result together with the experimental averaged value [2]:

$$M_{\Sigma_c}^{exp} = 2453.6 \text{ MeV}, \quad (23)$$

we arrive at:

$$M_{\Omega_c}^{exp} = 2800 \pm 96 \text{ MeV}, \quad (24)$$

which is a little higher, but still in agreement (considering the errors) with the experimental result.

4.2 $\Omega_b(bss)/\Sigma_b(bdd)$

We repeat the previous analysis in the case of the b -quark. The curves present the same qualitative behaviour as in

the case of the charm, where, only r_2^{ds} survives the different tests of stabilities. Here, the s_0 -behaviour is almost flat from $s_0 = 34 \text{ GeV}^2$. The optimal value is taken at $M^2 = 4 \text{ GeV}^2$. We obtain:

$$r_{\Omega_b}^{sd} = 1.051 \pm 0.012, \quad (25)$$

with the same sources of errors as before. Using this value together with the experimental averaged value [2]:

$$M_{\Sigma_b}^{exp} = 5811.2 \text{ MeV}, \quad (26)$$

one obtain

$$M_{\Omega_b} = (6108 \pm 71) \text{ MeV}, \quad (27)$$

which is in a very good agreement with the mean value between the CDF and D0 measurements for M_{Ω_b} .

5 Summary and Conclusions

We have extracted the heavy baryons mass-splittings due to $SU(3)$ breaking using double ratios of the QCD sum rules, which are less sensitive to the heavy quark mass and to the QCD continuum contributions. As a result, we have provided predictions of the $\Xi_Q^{(*)}$ and $\Omega_Q^{(*)}$ masses using the associated non-strange heavy baryons masses from the data. The different results are summarized in Table 5 [23].

Table 5. QSSR predictions of the strange heavy baryon masses in units of MeV from the double ratio sum rules using, as input, the observed masses of the associated non-strange heavy baryons.

Baryons	$r_{B_Q}^{sd}$	Mass	Data
Ξ_c	1.075 ± 0.021	2458 ± 50	2467.9 ± 0.4
Ω_c	1.141 ± 0.039	2800 ± 96	2697.5 ± 2.6
Ξ_b	1.048 ± 0.015	5888 ± 81	5792.4 ± 3.0
Ω_b	1.051 ± 0.012	6108 ± 71	6165.0 ± 13
Ξ_c^*	1.065 ± 0.021	2682 ± 53	2646.1 ± 1.3
Ω_c^*	1.135 ± 0.037	2858 ± 92	2768.3 ± 3.0
Ξ_b^*	1.024 ± 0.008	5973 ± 44	–
Ω_b^*	1.051 ± 0.017	6130 ± 99	–

Combining the predictions for the spin 3/2 baryons with the ones for the spin 1/2 baryons, we give in Table 6 predictions for the hyperfine mass-splittings. These results agree quite well with the data and with some expectations from quark models.

Like in the case of the light baryons [26], it is remarkable to notice that the leading term controlling the mass-splittings is the ratio $\kappa \equiv \langle \bar{s}s \rangle / \langle \bar{d}d \rangle$ of the condensate rather than the running mass $\bar{m}_s$. This ratio gives, after the choice of the continuum threshold, the largest errors in $r_{B_Q}^{sd}$.

Table 6. QSSR predictions of the strange heavy baryon hyperfine splittings in units of MeV from the double ratio sum rules using, as input, the predicted values in Table 5

Hyperfine Splittings	Observed
$M_{\Xi_c^*} - M_{\Xi_c} = 224 \pm 52$	179 ± 1
$M_{\Omega_c^*} - M_{\Omega_c} = 58 \pm 94$	70 ± 3
$M_{\Xi_b^*} - M_{\Xi_b} = 85 \pm 63$	–
$M_{\Omega_b^*} - M_{\Omega_b} = 22 \pm 85$	–

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