

## $\Sigma$ -admixture in neutron-rich $\Lambda$ hypernuclei in a microscopic shell-model calculation

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**Abstract.** We systematically investigate the structures of the  $\Lambda$  hypernuclei  ${}_{\Lambda}\text{Li}$  with the mass number  $A = 7-10$  in shell-model calculations considering the  $\Lambda N$ - $\Sigma N$  coupling in the first-order perturbation method. We find that the calculated  $\Sigma$ -mixing probabilities and energy shifts due to the  $\Lambda N$ - $\Sigma N$  coupling increase with the neutron number. The Fermi-type and Gamow-Teller-type couplings, which are related to the  $\beta$ -transition properties of the nuclear core state, coherently contribute to the energy shift in neutron-rich hypernuclei.

### 1 Introduction

One of the most important subjects in strangeness nuclear physics is a study of neutron-rich  $\Lambda$  hypernuclei [1]. It is expected that a  $\Lambda$  hyperon plays a glue-like role in neutron-rich nuclei, together with a strong  $\Lambda N$ - $\Sigma N$  coupling [2,3], which might induce a  $\Sigma$ -mixing in nuclei. The knowledge of the behavior of hyperons in a neutron-excess environment will significantly affect our understanding of neutron stars, because adding hyperons softens the Equation of State [4]. The purpose of our study is to theoretically clarify the structure of neutron-rich  $\Lambda$  hypernuclei and contribution of the  $\Lambda N$ - $\Sigma N$  coupling by a shell model, which has successfully been applied for description of the neutron-excess nuclei [5-7].

The overbinding problem in  ${}^5_{\Lambda}\text{He}$ , which is regarded as the underbinding problem in  ${}^4_{\Lambda}\text{H}$  and  ${}^4_{\Lambda}\text{He}$ , was known in the calculation of energy levels of  $s$ -shell hypernuclei [8]. The spin-spin component of the effective  $\Lambda N$  interaction contributes to the splitting of the  $0^+$  and  $1^+$  doublet states of  ${}^4_{\Lambda}\text{H}$  and  ${}^4_{\Lambda}\text{He}$  but cannot fully explain this splitting. Recently, Akaishi and his collaborators [2,3] suggested that a coherent  $\Lambda N$ - $\Sigma N$  coupling, which is not included in effective  $\Lambda N$  interactions, can resolve the underbinding problem. Because the coherent  $\Lambda N$ - $\Sigma N$  coupling is not so large in  $N \approx Z$   $p$ -shell hypernuclei [9], the  $\Sigma$ -mixing probabilities and the energy shifts is not significant. However, the authors [10] found first an enhancement of the  $\Sigma$ -mixing probabilities and the energy shifts in the neutron-rich hypernucleus  ${}^{10}_{\Lambda}\text{Li}$  in the shell-model calculation, where  $\Sigma$ -nuclear states are included in the model space by the first-order perturbation. Thus it is expected that an investigation of the  $\Lambda N$ - $\Sigma N$  coupling in neutron-rich hypernuclei gives a new knowledge of behavior of hyperons.

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Recently, Saha and his collaborators have performed the first successful measurement of a neutron-rich  $\Lambda$  hypernucleus  ${}^{10}_{\Lambda}\text{Li}$  by the double-charge exchange reaction ( $\pi^-, K^+$ ) on a  ${}^{10}\text{B}$  target at  $p_{\pi} = 1.2$  GeV/ $c$  [11]. The theoretical analysis of this reaction [12] suggests that a one-step process,  $\pi^- p \rightarrow K^+ \Sigma^-$  via  $\Sigma^-$  doorways due to the  $\Sigma^- p \leftrightarrow \Lambda n$  coupling is dominant rather than two-step processes. This means the importance of the  $\Sigma$ -mixing in the  $\Lambda$  hypernuclei. More experiments for productions of neutron-rich hypernuclei by the ( $\pi^-, K^+$ ) reactions are planned at J-PARC [13] and analyses of these reactions provide to examine precisely a wave function involving the  $\Sigma$ -mixing in neutron-rich  $\Lambda$  hypernuclei as well as mechanisms of these reactions.

In this report, we investigate the structure of  ${}_{\Lambda}\text{Li}$  hypernuclei with the mass number  $A = 7-10$  by focusing on the  $\Sigma$ -mixing probabilities and the energy shifts in shell-model calculations including  $\Lambda N$ - $\Sigma N$  coupling. Also, we discuss the  $\Lambda N$ - $\Sigma N$  coupling strengths in neutron-rich  $\Lambda$  hypernuclei in terms of the  $\beta$ -transition properties of the nuclear core state.

### 2 Formalism

#### 2.1 Shell model for $\Lambda$ hypernuclei

A shell-model Hamiltonian is given as

$$H = H_{\Lambda} + H_{\Sigma} + V_{\Lambda\Sigma} + V_{\Sigma\Lambda}, \quad (1)$$

where  $H_{\Lambda}$  and  $H_{\Sigma}$  are the Hamiltonians in the  $\Lambda$  and  $\Sigma$  configuration spaces, respectively.  $V_{\Lambda\Sigma}$  and its Hermitian conjugate  $V_{\Sigma\Lambda}$  denote the two-body  $\Lambda N$ - $\Sigma N$  coupling interaction,  $\Lambda N \leftrightarrow \Sigma N$ . We treat the  $\Lambda N$ - $\Sigma N$  coupling interactions,  $V_{\Lambda\Sigma}$  and  $V_{\Sigma\Lambda}$ , as perturbation because a  $\Sigma$  hyperon has a larger mass than a  $\Lambda$  hyperon by 80 MeV. When taking into account up to the first-order terms, we obtain the

$\nu$ -th eigenstate with the isospin  $T$  and the angular momentum  $J$  of the Hamiltonian  $H$  as

$$|({}^{\Lambda}Z)\nu TJ\rangle = \sum_{\mu} C_{\nu,\mu} |\psi_{\mu}^{\Lambda}; TJ\rangle + \sum_{\mu'} D_{\nu,\mu'} |\psi_{\mu'}^{\Sigma}; TJ\rangle, \quad (2)$$

where the  $\Lambda$ -nuclear and  $\Sigma$ -nuclear eigenstates,  $|\psi_{\mu}^{\Lambda}; TJ\rangle$  and  $|\psi_{\mu'}^{\Sigma}; TJ\rangle$ , are obtained by solving the eigenvalue equations,

$$H_{\Lambda} |\psi_{\mu}^{\Lambda}; TJ\rangle = E_{\mu}^{\Lambda} |\psi_{\mu}^{\Lambda}; TJ\rangle, \quad (3)$$

$$H_{\Sigma} |\psi_{\mu'}^{\Sigma}; TJ\rangle = E_{\mu'}^{\Sigma} |\psi_{\mu'}^{\Sigma}; TJ\rangle, \quad (4)$$

respectively, and the coefficients are given by

$$C_{\nu,\mu} = \delta_{\nu\mu}, \quad (5)$$

$$D_{\nu,\mu'} = -\frac{\langle \psi_{\mu'}^{\Sigma} | V_{\Sigma\Lambda} | \psi_{\nu}^{\Lambda} \rangle_{TJ}}{E_{\mu'}^{\Sigma} - E_{\nu}^{\Lambda}}. \quad (6)$$

The  $\Sigma$ -mixing probability  $P_{\Sigma;\nu}$  and the eigenenergy  $E_{\nu}$  for the  $|({}^{\Lambda}Z)\nu TJ\rangle$  eigenstate are given by

$$P_{\Sigma;\nu} = \sum_{\mu'} |D_{\nu,\mu'}|^2, \quad (7)$$

$$E_{\nu} = E_{\nu}^{\Lambda} - \Delta\epsilon_{\nu}, \quad (8)$$

in the first-order perturbation, where  $E_{\nu}^{\Lambda} = E_{\mu=\nu}^{\Lambda}$  is given in Eq. (4) and

$$\Delta\epsilon_{\nu} = \sum_{\mu'} (E_{\mu'}^{\Sigma} - E_{\nu}^{\Lambda}) |D_{\nu,\mu'}|^2 \quad (9)$$

is the energy shift by the  $\Lambda N$ - $\Sigma N$  coupling.

## 2.2 Shell-model setup

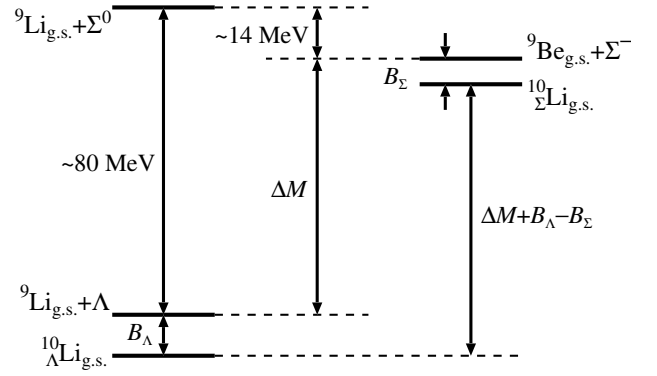
In the present shell-model calculations, we construct wave functions in the following model space: Basis states are described on the isospin base. The nucleon part of the wave function consists of the inert  ${}^4\text{He}$  core and valence nucleons in the  $p$ -shell ( $0p_{3/2}$  and  $0p_{1/2}$ ) orbits. The hyperon part consists of a hyperon ( $\Lambda$  or  $\Sigma$ ) in the  $0s_{1/2}$  orbit.

For the effective  $NN$  interaction, we adopt the Cohen-Kurath (8–16) 2BME [14] that is obtained by a fit to 35 energy data of  $A = 8$ –16 nuclei by using two single-particle energies and fifteen two-body matrix elements as parameters. The effective  $YN$  interaction is written as

$$V_Y = \bar{V} + \Delta s_N \cdot \mathbf{s}_Y + S_+ \ell_N \cdot (\mathbf{s}_N + \mathbf{s}_Y) + S_- \ell_N \cdot (-\mathbf{s}_N + \mathbf{s}_Y) + T S_{12}, \quad (10)$$

where  $\bar{V}$ ,  $\Delta$ ,  $S_+$ ,  $S_-$  and  $T$  are radial integrals [15–17].  $\mathbf{s}_N$  and  $\mathbf{s}_Y$  are spin operators for nucleons and the hyperon, respectively.  $\ell_N$  is the angular momentum operator for nucleons and is proportional to the relative  $\ell$  for states with the  $0s_{1/2}$  hyperon. The tensor operator  $S_{12}$  is defined by

$$S_{12} = 3(\hat{\mathbf{r}} \cdot \boldsymbol{\sigma}_N)(\hat{\mathbf{r}} \cdot \boldsymbol{\sigma}_Y) - (\boldsymbol{\sigma}_N \cdot \boldsymbol{\sigma}_Y) \quad (11)$$



**Fig. 1.** Schematic energy levels for  $\Lambda$ -nuclear and  $\Sigma$ -nuclear ground states of  ${}^{10}\text{Li}$ .

with  $\boldsymbol{\sigma} = 2\mathbf{s}$  and  $\hat{\mathbf{r}} = (\mathbf{r}_N - \mathbf{r}_Y)/|\mathbf{r}_N - \mathbf{r}_Y|$ . We use the effective  $\Lambda N$  interaction given in Ref. [9] and the effective  $\Lambda N$ - $\Sigma N$  and  $\Sigma N$  interactions [9, 18] based on the NSC97e,f potentials [19]. The values of radial integrals for these effective  $YN$  interactions are listed in Ref. [10].

## 3 Numerical results and discussion

### 3.1 $\Sigma$ -mixing probabilities and energy shifts

We perform numerical calculations for the  ${}^{\Lambda}\text{Li}$  hypernuclei and evaluate the  $\Sigma$ -mixing probabilities and the energy shifts which are obtained by Eqs. (7)–(9). In Fig. 1, we show the schematic energy levels for the  $\Lambda$ -nuclear and  $\Sigma$ -nuclear ground states of  ${}^{10}\text{Li}$ . Here we assume that the difference between  $\Lambda$  and  $\Sigma$  threshold energies is

$$E({}^0\text{Li}_{\text{g.s.}+\Sigma}) - E({}^0\text{Li}_{\text{g.s.}+\Lambda}) = 80 \text{ MeV}, \quad (12)$$

and then the energy of the  $\Sigma$ -nuclear ground state  $|\psi_{\text{g.s.}}^{\Sigma}\rangle$  is calculated to be

$$E_{\text{g.s.}}^{\Sigma} - E_{\text{g.s.}}^{\Lambda} = \Delta M + B_{\Lambda} - B_{\Sigma} = 69.3 \text{ MeV}, \quad (13)$$

measured from that of the  $\Lambda$ -nuclear ground state  $|\psi_{\text{g.s.}}^{\Lambda}\rangle$ .

In Table 1, we show the calculated  $\Sigma$ -mixing probabilities and energy shifts and find that these values are the order of 0.1 % and 0.1 MeV, respectively, and increase with the neutron number (or isospin). The  $\Sigma$ -mixing probability is about 0.34 % and the energy shift is about 0.28 MeV

**Table 1.** The calculated  $\Sigma$ -mixing probabilities  $P_{\Sigma}$  and energy shifts  $\Delta\epsilon$  for ground states of  ${}^{\Lambda}\text{Li}$  isotopes.

	$J^{\pi}$	$T$	$P_{\Sigma}$ (%)	$\Delta\epsilon$ (MeV)	$\Delta\epsilon^{(a)}$ (MeV)
${}^7_{\Lambda}\text{Li}$	$\frac{1}{2}^+$	0	0.098	0.085	0.078
${}^8_{\Lambda}\text{Li}$	$1^-$	$\frac{1}{2}$	0.172	0.139	
${}^9_{\Lambda}\text{Li}$	$\frac{3}{2}^+$	1	0.211	0.172	
${}^{10}_{\Lambda}\text{Li}$	$1^-$	$\frac{3}{2}$	0.345	0.280	
${}^{11}_{\Lambda}\text{B}$	$\frac{5}{2}^+$	0	0.076	0.073	0.066

(a) Ref. [9]

for the neutron-rich  ${}^{10}_\Lambda\text{Li}$  ground state, which are about 3 times larger than those for  ${}^7_\Lambda\text{Li}$ . In order to check our shell-model calculation, we also compared our numerical results for the  $Z = N$   $\Lambda$  hypernuclei to Millener's work [9]. We obtain the energy shifts, e.g.,  $\Delta\epsilon = 0.085$  and  $0.073$  MeV for the ground states of  ${}^7_\Lambda\text{Li}$  and  ${}^{11}_\Lambda\text{B}$ , which are comparable to the Millener's results of  $0.078$  and  $0.066$  MeV, respectively. Therefore, we conclude that our calculations agree with Millener's results.

### 3.2 $\Lambda N$ - $\Sigma N$ coupling strengths

We examine an enhancement of the  $\Sigma$ -mixing probability and the energy shift in  ${}^{10}_\Lambda\text{Li}$  isotope by the configuration mixing in the  $\Sigma$ -nuclear states, which couple to the  $\Lambda$ -nuclear states by the  $\Lambda N$ - $\Sigma N$  coupling. The calculated  $\Lambda N$ - $\Sigma N$  coupling strengths  $|D_{\mu'}|^2$  between the  $\Sigma$ -nuclear eigenstates  $|\psi_{\mu'}^\Sigma\rangle$  and the  $\Lambda$ -nuclear ground state  $|\psi_{\text{g.s.}}^\Lambda\rangle$  are shown in Fig. 2. It should be noticed that a contribution of the  $\Sigma$ -nuclear ground state  $|\psi_{\text{g.s.}}^\Sigma\rangle$  to the  $\Sigma$ -mixing of the  ${}^{10}_\Lambda\text{Li}$  ground state is reduced to  $|D_{\text{g.s.}}^\Sigma|^2 = 0.002\%$ , whereas the several  $\Sigma$ -nuclear excited states with  $E_{\mu'}^\Sigma - E_{\text{g.s.}}^\Lambda \approx 80$  MeV considerably contribute to the  $\Sigma$ -mixing. These contributions are coherently enhanced by the configuration mixing which is caused by the  $\Sigma N$  interaction. It is shown that the nature of the  $\Sigma$ -nuclear states plays an important role in the  $\Lambda N$ - $\Sigma N$  coupling.

It is worth discussing microscopically one mechanism of  $\Lambda N$ - $\Sigma N$  coupling in  $\Lambda$  hypernuclei. When a  $\Lambda$ -nuclear state in  ${}_\Lambda\text{Li}$  converts to a  $\Sigma^-$ -nuclear state through the  $\Lambda N$ - $\Sigma N$  coupling interaction, the Li core state changes into the Be core state. In other words, the  $\beta^-$ -transition,  $\text{Li} \rightarrow \text{Be}$ , occurs in the core-nuclear state. We stress that the  $\Lambda N$ - $\Sigma N$  coupling strengths  $|D_{\mu'}|^2$  are extremely sensitive to the strengths of  $\beta$ -transitions between the core-nuclear states. The two-body  $\Lambda N$ - $\Sigma N$  coupling interaction  $V_{\Sigma\Lambda}$  can be approximately rewritten as

$$V_{\Sigma\Lambda} \approx V_{\Sigma\Lambda}^{\text{F}} + V_{\Sigma\Lambda}^{\text{GT}}, \quad (14)$$

where

$$V_{\Sigma\Lambda}^{\text{F}} = \bar{V}_{\Sigma\Lambda}, \quad (15)$$

$$V_{\Sigma\Lambda}^{\text{GT}} = \Delta_{\Sigma\Lambda} \mathbf{s}_N \cdot \mathbf{s}_{\Sigma\Lambda} \quad (16)$$

in the particle representation:  $\bar{V}_{\Sigma\Lambda}$  and  $\Delta_{\Sigma\Lambda}$  are radial integrals of central potentials. In the isospin representation, these terms can be rewritten as

$$V_{\Sigma\Lambda}^{\text{F}} \propto \mathbf{t}_N \cdot \boldsymbol{\phi}_{\Sigma\Lambda}, \quad (17)$$

$$V_{\Sigma\Lambda}^{\text{GT}} \propto (\boldsymbol{\sigma}_N \cdot \boldsymbol{\sigma}_{\Sigma\Lambda}) \mathbf{t}_N \cdot \boldsymbol{\phi}_{\Sigma\Lambda}, \quad (18)$$

where  $\mathbf{t}_N$  is the isospin operator for a nucleon and  $\boldsymbol{\phi}_{\Sigma\Lambda}$  is the operator that changes a  $\Lambda$  hyperon into a  $\Sigma$  hyperon,

$$|j_\Sigma\rangle = \boldsymbol{\phi}_{\Sigma\Lambda} |j_\Lambda\rangle. \quad (19)$$

Because  $\mathbf{t}_N$  and  $\boldsymbol{\sigma}_N \mathbf{t}_N$  denote the Fermi and Gamow-Teller  $\beta$ -transition operators for a nucleon, respectively,  $V_{\Sigma\Lambda}^{\text{F}}$  and

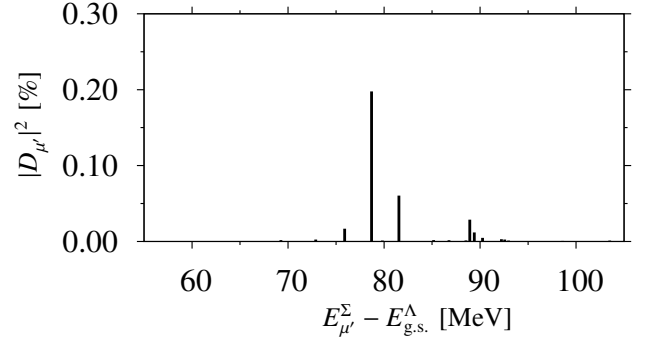


Fig. 2.  $\Lambda N$ - $\Sigma N$  coupling strengths  $|D_{\mu'}|^2$  of the  $\Sigma$ -nuclear eigenstates in the ground state of  ${}^{10}_\Lambda\text{Li}$ .

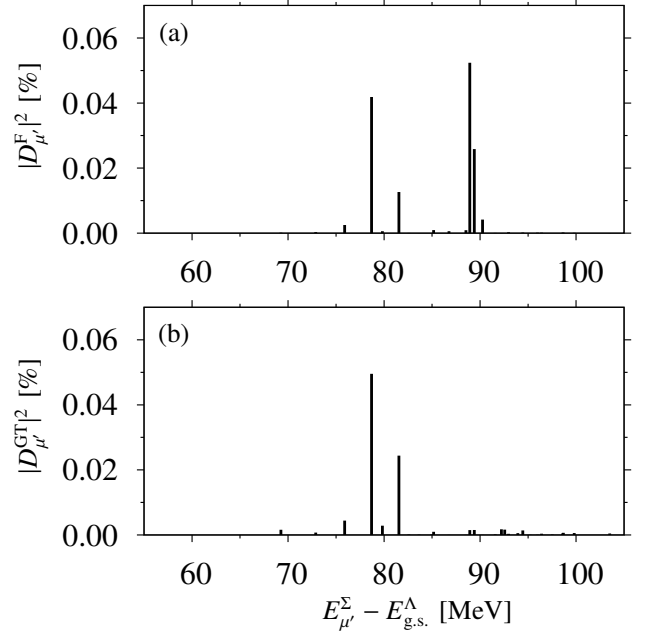


Fig. 3. (a) Fermi-type coupling strengths  $|D_{\mu'}^{\text{F}}|^2$  and (b) Gamow-Teller-type coupling strengths  $|D_{\mu'}^{\text{GT}}|^2$  of the  $\Sigma$ -nuclear eigenstates in the ground state of  ${}^{10}_\Lambda\text{Li}$ .

$V_{\Sigma\Lambda}^{\text{GT}}$  are regarded as the Fermi-type and Gamow-Teller-type coupling interactions. The calculated strength distributions of these couplings,

$$|D_{\mu'}^{\text{F}}|^2 = \left| \frac{\langle \psi_{\text{g.s.}}^\Lambda; T J | V_{\Sigma\Lambda}^{\text{F}} | \psi_{\mu'}^\Sigma; T J \rangle}{E_{\mu'}^\Sigma - E_{\text{g.s.}}^\Lambda} \right|^2, \quad (20)$$

$$|D_{\mu'}^{\text{GT}}|^2 = \left| \frac{\langle \psi_{\text{g.s.}}^\Lambda; T J | V_{\Sigma\Lambda}^{\text{GT}} | \psi_{\mu'}^\Sigma; T J \rangle}{E_{\mu'}^\Sigma - E_{\text{g.s.}}^\Lambda} \right|^2, \quad (21)$$

are shown in Figs. 3 (a) and (b), respectively. Comparing them with Fig. 2, we see the Fermi and Gamow-Teller components coherently contribute to the  $\Lambda N$ - $\Sigma N$  coupling strengths in the energy region of  $E_{\mu'}^\Sigma - E_{\text{g.s.}}^\Lambda \approx 80$  MeV. The calculated  $\Sigma$ -mixing probability involving both couplings is

$$\sum_{\mu'} \left| \frac{\langle \psi_{\text{g.s.}}^\Lambda; T J | (V_{\Sigma\Lambda}^{\text{F}} + V_{\Sigma\Lambda}^{\text{GT}}) | \psi_{\mu'}^\Sigma; T J \rangle}{E_{\mu'}^\Sigma - E_{\text{g.s.}}^\Lambda} \right|^2 = 0.350\%, \quad (22)$$

which is close to the full calculated probability  $P_{\Sigma} = 0.345$  %, while  $\sum_{\mu'} |D_{\mu'}^F|^2 = 0.144$  % is obtained for the Fermi type and  $\sum_{\mu'} |D_{\mu'}^{GT}|^2 = 0.098$  % for the Gamow-Teller type.

In order to see the potentiality of the neutron-rich nuclei clearly, we consider the reason why the energy shift in  ${}_{\Lambda}^{10}\text{Li}$  is about 3 times larger than that in  ${}_{\Lambda}^7\text{Li}$ . The enhancement of the  $\Sigma$ -mixing probabilities in neutron-rich  $\Lambda$  hypernuclei is mainly due to the Fermi-type coupling interaction  $V_{\Sigma\Lambda}^F$ , which might correspond to the coherent  $\Lambda N$ - $\Sigma N$  coupling suggested in Refs. [2,3]. When we assume the weak-coupling limit for the  $\Lambda N$  interaction and the approximation in which all the energy differences  $E_{\mu'}^{\Sigma} - E_{\nu}^{\Lambda}$  between  $\Lambda$ -nuclear and  $\Sigma$ -nuclear states are replaced by a constant value  $\Delta\mathcal{E}$ , we can write a sum over the Fermi-type coupling strengths as

$$\sum_{\mu'} |D_{\mu'}^F|^2 \sim \frac{1}{\Delta\mathcal{E}} \frac{4}{3} \bar{V}^2 T (T + 1). \quad (23)$$

Thus the Fermi-type coupling strengths play an important role in the  $\Sigma$ -mixing probability of neutron-rich  ${}_{\Lambda}^{10}\text{Li}$  eigenstates because these states have  $T = \frac{3}{2}$ . On the other hand, in the case of  ${}_{\Lambda}^7\text{Li}$  with  $T = 0$ , the Fermi-type coupling strengths vanish and then the  $\Sigma$ -mixing probability is generated by the Gamow-Teller-type coupling strengths.

In the Gamow-Teller transitions for ordinary nuclei, the model independent formula [20]

$$\sum B(\text{GT}-) - \sum B(\text{GT}+) = 3(N - Z) \quad (24)$$

is well known, where  $B(\text{GT}\mp)$  is a strength of the Gamow-Teller  $\beta^{\mp}$ -transition,  $|^AZ\rangle \rightarrow |^AZ\pm 1\rangle$ . In general,  $\sum B(\text{GT}+)$  becomes smaller as neutron-excess grows larger, leading to  $\sum B(\text{GT}-) \approx 3(N - Z)$ . Therefore, the Gamow-Teller-type coupling is very important in  $\Lambda$  hypernuclei with large neutron excess.

## 4 Summary

We have investigated the structure of the  ${}_{\Lambda}\text{Li}$  hypernuclei in shell-model calculations including  $\Lambda N$ - $\Sigma N$  coupling in perturbation theory. We found that the  $\Sigma$ -mixing probabilities and the energy shifts are the order of 0.1 % and 0.1 MeV, respectively, and they increase with the neutron number (or isospin). The reasons why the  $\Sigma$ -mixing probabilities are enhanced are summarized as follows: (i) The  $\Sigma$ -nuclear excited states can be strongly coupled with the  $\Lambda$ -nuclear ground state with the help of the  $\Sigma N$  interaction. (ii) The strong  $\Lambda N$ - $\Sigma N$  coupling is coherently enhanced by the Fermi-type and Gamow-Teller-type coupling components. (iii) The Fermi-type coupling becomes more effective in a neutron-rich environment, increasing as  $T(T + 1)$ .

## References

1. L. Majling, Nucl. Phys. **A585**, (1995) 211c.

2. Khin Swe Myint, T. Harada, S. Shinmura, and Y. Akaishi, Few-Body Syst. Suppl. **12**, (2000) 383.
3. Y. Akaishi, T. Harada, S. Shinmura, and Khin Swe Myint, Phys. Rev. Lett. **84**, (2000) 3539.
4. M. Baldo, G. F. Burgio, and H.-J. Schulze, Phys. Rev. C **61**, (2000) 055801.
5. O. Sorlin and M.-G. Porquet, Prog. Part. Nucl. Phys. **61**, (2008) 602, and references therein.
6. A. Umeya and K. Muto, Phys. Rev. C **69**, (2004) 024306; **74**, (2006) 034330.
7. A. Umeya, G. Kaneko, T. Haneda, and K. Muto, Phys. Rev. C **77**, (2008) 044301.
8. R. H. Dalitz, R. C. Herndon, and Y. C. Tang, Nucl. Phys. **B47**, (1972) 109.
9. D. J. Millener, Lect. Notes. Phys. **724**, (2007) 31; Nucl. Phys. **A804**, (2008) 84.
10. A. Umeya and T. Harada, Phys. Rev. C **79**, (2009) 024315.
11. P. K. Saha, T. Fukuda, W. Imoto, J. K. Ahn, S. Ajimura, K. Aoki, H. C. Bhang, H. Fujioka, H. Hotchi, J. I. Hwang, T. Itabashi, B. H. Kang, H. D. Kim, M. J. Kim, T. Kishimoto, A. Krutenkova, T. Maruta, Y. Miura, K. Miwa, T. Nagae, H. Noumi, H. Outa, T. Ohtaki, A. Sakaguchi, Y. Sato, M. Sekimoto, Y. Shimizu, H. Tamura, K. Tanida, A. Toyoda, M. Ukai, and H. J. Yim, Phys. Rev. Lett. **94**, (2005) 052502.
12. T. Harada, A. Umeya, and Y. Hirabayashi, Phys. Rev. C **79**, (2009) 014603.
13. A. Sakaguchi, S. Ajimura, H. Bhang, L. Busso, M. Endo, D. Faso, T. Fukuda, T. Kishimoto, K. Matsuda, K. Matsuoka, Y. Mizoi, O. Morra, H. Noumi, P. K. Saha, C. Samanta, Y. Shimizu, T. Takahashi, T. N. Takahashi, and K. Yoshida, arXiv:0904.0298 [nucl-ex], 2009.
14. S. Cohen and D. Kurath, Nucl. Phys. **73**, (1965) 1.
15. A. Gal, J. M. Soper, and R. H. Dalitz, Ann. Phys. (NY) **63**, (1971) 53; **72**, (1972) 445; **113**, (1978) 79.
16. R. H. Dalitz and A. Gal, Ann. Phys. (NY) **116**, (1978) 167.
17. D. J. Millener, A. Gal, C. B. Dover, and R. H. Dalitz, Phys. Rev. C **31**, (1985) 499.
18. D. J. Millener (private communication).
19. Th. A. Rijken, V. G. J. Stoks, and Y. Yamamoto, Phys. Rev. C **59**, (1999) 21.
20. K. Ikeda, S. Fujii, and J. I. Fujita, Phys. Lett. **3**, (1963) 271.