

Σ -admixture in neutron-rich Λ hypernuclei in a microscopic shell-model calculation

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Abstract. We systematically investigate the structures of the Λ hypernuclei ${}_{\Lambda}^A\text{Li}$ with the mass number $A = 7-10$ in shell-model calculations considering the ΛN - ΣN coupling in the first-order perturbation method. We find that the calculated Σ -mixing probabilities and energy shifts due to the ΛN - ΣN coupling increase with the neutron number. The Fermi-type and Gamow-Teller-type couplings, which are related to the β -transition properties of the nuclear core state, coherently contribute to the energy shift in neutron-rich hypernuclei.

1 Introduction

One of the most important subjects in strangeness nuclear physics is a study of neutron-rich Λ hypernuclei [1]. It is expected that a Λ hyperon plays a glue-like role in neutron-rich nuclei, together with a strong ΛN - ΣN coupling [2,3], which might induce a Σ -mixing in nuclei. The knowledge of the behavior of hyperons in a neutron-excess environment will significantly affect our understanding of neutron stars, because adding hyperons softens the Equation of State [4]. The purpose of our study is to theoretically clarify the structure of neutron-rich Λ hypernuclei and contribution of the ΛN - ΣN coupling by a shell model, which has successfully been applied for description of the neutron-excess nuclei [5-7].

The overbinding problem in ${}_{\Lambda}^5\text{He}$, which is regarded as the underbinding problem in ${}_{\Lambda}^4\text{H}$ and ${}_{\Lambda}^4\text{He}$, was known in the calculation of energy levels of s -shell hypernuclei [8]. The spin-spin component of the effective ΛN interaction contributes to the splitting of the 0^+ and 1^+ doublet states of ${}_{\Lambda}^4\text{H}$ and ${}_{\Lambda}^4\text{He}$ but cannot fully explain this splitting. Recently, Akaishi and his collaborators [2,3] suggested that a coherent ΛN - ΣN coupling, which is not included in effective ΛN interactions, can resolve the underbinding problem. Because the coherent ΛN - ΣN coupling is not so large in $N \approx Z$ p -shell hypernuclei [9], the Σ -mixing probabilities and the energy shifts is not significant. However, the authors [10] found first an enhancement of the Σ -mixing probabilities and the energy shifts in the neutron-rich hypernucleus ${}_{\Lambda}^{10}\text{Li}$ in the shell-model calculation, where Σ -nuclear states are included in the model space by the first-order perturbation. Thus it is expected that an investigation of the ΛN - ΣN coupling in neutron-rich hypernuclei gives a new knowledge of behavior of hyperons.

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Recently, Saha and his collaborators have performed the first successful measurement of a neutron-rich Λ hypernucleus ${}_{\Lambda}^{10}\text{Li}$ by the double-charge exchange reaction (π^-, K^+) on a ${}^{10}\text{B}$ target at $p_{\pi} = 1.2$ GeV/ c [11]. The theoretical analysis of this reaction [12] suggests that a one-step process, $\pi^- p \rightarrow K^+ \Sigma^-$ via Σ^- doorways due to the $\Sigma^- p \leftrightarrow \Lambda n$ coupling is dominant rather than two-step processes. This means the importance of the Σ -mixing in the Λ hypernuclei. More experiments for productions of neutron-rich hypernuclei by the (π^-, K^+) reactions are planned at J-PARC [13] and analyses of these reactions provide to examine precisely a wave function involving the Σ -mixing in neutron-rich Λ hypernuclei as well as mechanisms of these reactions.

In this report, we investigate the structure of ${}_{\Lambda}^A\text{Li}$ hypernuclei with the mass number $A = 7-10$ by focusing on the Σ -mixing probabilities and the energy shifts in shell-model calculations including ΛN - ΣN coupling. Also, we discuss the ΛN - ΣN coupling strengths in neutron-rich Λ hypernuclei in terms of the β -transition properties of the nuclear core state.

2 Formalism

2.1 Shell model for Λ hypernuclei

A shell-model Hamiltonian is given as

$$H = H_{\Lambda} + H_{\Sigma} + V_{\Lambda\Sigma} + V_{\Sigma\Lambda}, \quad (1)$$

where H_{Λ} and H_{Σ} are the Hamiltonians in the Λ and Σ configuration spaces, respectively. $V_{\Lambda\Sigma}$ and its Hermitian conjugate $V_{\Sigma\Lambda}$ denote the two-body ΛN - ΣN coupling interaction, $\Lambda N \leftrightarrow \Sigma N$. We treat the ΛN - ΣN coupling interactions, $V_{\Lambda\Sigma}$ and $V_{\Sigma\Lambda}$, as perturbation because a Σ hyperon has a larger mass than a Λ hyperon by 80 MeV. When taking into account up to the first-order terms, we obtain the

ν -th eigenstate with the isospin T and the angular momentum J of the Hamiltonian H as

$$|({}^{\Lambda}Z)\nu TJ\rangle = \sum_{\mu} C_{\nu,\mu} |\psi_{\mu}^{\Lambda}; TJ\rangle + \sum_{\mu'} D_{\nu,\mu'} |\psi_{\mu'}^{\Sigma}; TJ\rangle, \quad (2)$$

where the Λ -nuclear and Σ -nuclear eigenstates, $|\psi_{\mu}^{\Lambda}; TJ\rangle$ and $|\psi_{\mu'}^{\Sigma}; TJ\rangle$, are obtained by solving the eigenvalue equations,

$$H_{\Lambda} |\psi_{\mu}^{\Lambda}; TJ\rangle = E_{\mu}^{\Lambda} |\psi_{\mu}^{\Lambda}; TJ\rangle, \quad (3)$$

$$H_{\Sigma} |\psi_{\mu'}^{\Sigma}; TJ\rangle = E_{\mu'}^{\Sigma} |\psi_{\mu'}^{\Sigma}; TJ\rangle, \quad (4)$$

respectively, and the coefficients are given by

$$C_{\nu,\mu} = \delta_{\nu\mu}, \quad (5)$$

$$D_{\nu,\mu'} = -\frac{\langle \psi_{\mu'}^{\Sigma} | V_{\Sigma\Lambda} | \psi_{\nu}^{\Lambda} \rangle_{TJ}}{E_{\mu'}^{\Sigma} - E_{\nu}^{\Lambda}}. \quad (6)$$

The Σ -mixing probability $P_{\Sigma;\nu}$ and the eigenenergy E_{ν} for the $|({}^{\Lambda}Z)\nu TJ\rangle$ eigenstate are given by

$$P_{\Sigma;\nu} = \sum_{\mu'} |D_{\nu,\mu'}|^2, \quad (7)$$

$$E_{\nu} = E_{\nu}^{\Lambda} - \Delta\epsilon_{\nu}, \quad (8)$$

in the first-order perturbation, where $E_{\nu}^{\Lambda} = E_{\mu=\nu}^{\Lambda}$ is given in Eq. (4) and

$$\Delta\epsilon_{\nu} = \sum_{\mu'} (E_{\mu'}^{\Sigma} - E_{\nu}^{\Lambda}) |D_{\nu,\mu'}|^2 \quad (9)$$

is the energy shift by the ΛN - ΣN coupling.

2.2 Shell-model setup

In the present shell-model calculations, we construct wave functions in the following model space: Basis states are described on the isospin base. The nucleon part of the wave function consists of the inert ${}^4\text{He}$ core and valence nucleons in the p -shell ($0p_{3/2}$ and $0p_{1/2}$) orbits. The hyperon part consists of a hyperon (Λ or Σ) in the $0s_{1/2}$ orbit.

For the effective NN interaction, we adopt the Cohen-Kurath (8–16) 2BME [14] that is obtained by a fit to 35 energy data of $A = 8$ –16 nuclei by using two single-particle energies and fifteen two-body matrix elements as parameters. The effective YN interaction is written as

$$V_Y = \bar{V} + \Delta s_N \cdot \mathbf{s}_Y + S_+ \boldsymbol{\ell}_N \cdot (\mathbf{s}_N + \mathbf{s}_Y) + S_- \boldsymbol{\ell}_N \cdot (-\mathbf{s}_N + \mathbf{s}_Y) + T S_{12}, \quad (10)$$

where \bar{V} , Δ , S_+ , S_- and T are radial integrals [15–17]. \mathbf{s}_N and \mathbf{s}_Y are spin operators for nucleons and the hyperon, respectively. $\boldsymbol{\ell}_N$ is the angular momentum operator for nucleons and is proportional to the relative $\boldsymbol{\ell}$ for states with the $0s_{1/2}$ hyperon. The tensor operator S_{12} is defined by

$$S_{12} = 3(\hat{\mathbf{r}} \cdot \boldsymbol{\sigma}_N)(\hat{\mathbf{r}} \cdot \boldsymbol{\sigma}_Y) - (\boldsymbol{\sigma}_N \cdot \boldsymbol{\sigma}_Y) \quad (11)$$

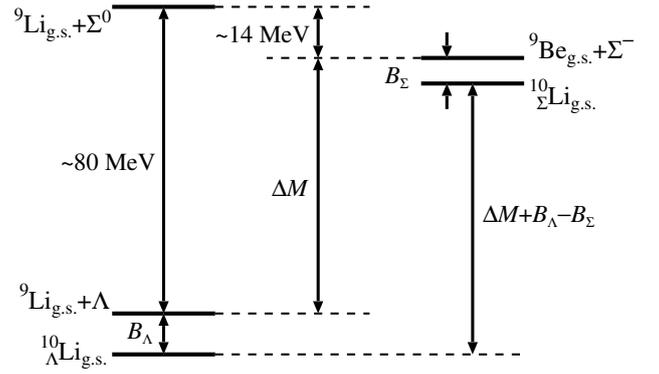


Fig. 1. Schematic energy levels for Λ -nuclear and Σ -nuclear ground states of ${}^{10}\text{Li}$.

with $\boldsymbol{\sigma} = 2\mathbf{s}$ and $\hat{\mathbf{r}} = (\mathbf{r}_N - \mathbf{r}_Y)/|\mathbf{r}_N - \mathbf{r}_Y|$. We use the effective ΛN interaction given in Ref. [9] and the effective ΛN - ΣN and ΣN interactions [9, 18] based on the NSC97e,f potentials [19]. The values of radial integrals for these effective YN interactions are listed in Ref. [10].

3 Numerical results and discussion

3.1 Σ -mixing probabilities and energy shifts

We perform numerical calculations for the ${}^{\Lambda}\text{Li}$ hypernuclei and evaluate the Σ -mixing probabilities and the energy shifts which are obtained by Eqs. (7)–(9). In Fig. 1, we show the schematic energy levels for the Λ -nuclear and Σ -nuclear ground states of ${}^{10}\text{Li}$. Here we assume that the difference between Λ and Σ threshold energies is

$$E({}^0\text{Li}_{\text{g.s.}}+\Sigma) - E({}^0\text{Li}_{\text{g.s.}}+\Lambda) = 80 \text{ MeV}, \quad (12)$$

and then the energy of the Σ -nuclear ground state $|\psi_{\text{g.s.}}^{\Sigma}\rangle$ is calculated to be

$$E_{\text{g.s.}}^{\Sigma} - E_{\text{g.s.}}^{\Lambda} = \Delta M + B_{\Lambda} - B_{\Sigma} = 69.3 \text{ MeV}, \quad (13)$$

measured from that of the Λ -nuclear ground state $|\psi_{\text{g.s.}}^{\Lambda}\rangle$.

In Table 1, we show the calculated Σ -mixing probabilities and energy shifts and find that these values are the order of 0.1 % and 0.1 MeV, respectively, and increase with the neutron number (or isospin). The Σ -mixing probability is about 0.34 % and the energy shift is about 0.28 MeV

Table 1. The calculated Σ -mixing probabilities P_{Σ} and energy shifts $\Delta\epsilon$ for ground states of ${}^{\Lambda}\text{Li}$ isotopes.

	J^{π}	T	P_{Σ} (%)	$\Delta\epsilon$ (MeV)	$\Delta\epsilon^{(a)}$ (MeV)
${}^7_{\Lambda}\text{Li}$	$\frac{1}{2}^+$	0	0.098	0.085	0.078
${}^8_{\Lambda}\text{Li}$	1^-	$\frac{1}{2}$	0.172	0.139	
${}^9_{\Lambda}\text{Li}$	$\frac{3}{2}^+$	1	0.211	0.172	
${}^{10}_{\Lambda}\text{Li}$	1^-	$\frac{3}{2}$	0.345	0.280	
${}^{11}_{\Lambda}\text{B}$	$\frac{5}{2}^+$	0	0.076	0.073	0.066

(a) Ref. [9]

for the neutron-rich $^{10}_\Lambda\text{Li}$ ground state, which are about 3 times larger than those for $^7_\Lambda\text{Li}$. In order to check our shell-model calculation, we also compared our numerical results for the $Z = N$ Λ hypernuclei to Millener's work [9]. We obtain the energy shifts, e.g., $\Delta\epsilon = 0.085$ and 0.073 MeV for the ground states of $^7_\Lambda\text{Li}$ and $^{11}_\Lambda\text{B}$, which are comparable to the Millener's results of 0.078 and 0.066 MeV, respectively. Therefore, we conclude that our calculations agree with Millener's results.

3.2 ΛN - ΣN coupling strengths

We examine an enhancement of the Σ -mixing probability and the energy shift in $^{10}_\Lambda\text{Li}$ isotope by the configuration mixing in the Σ -nuclear states, which couple to the Λ -nuclear states by the ΛN - ΣN coupling. The calculated ΛN - ΣN coupling strengths $|D_{\mu'}|^2$ between the Σ -nuclear eigenstates $|\psi_{\mu'}^\Sigma\rangle$ and the Λ -nuclear ground state $|\psi_{\text{g.s.}}^\Lambda\rangle$ are shown in Fig. 2. It should be noticed that a contribution of the Σ -nuclear ground state $|\psi_{\text{g.s.}}^\Sigma\rangle$ to the Σ -mixing of the $^{10}_\Lambda\text{Li}$ ground state is reduced to $|D_{\text{g.s.}}^\Sigma|^2 = 0.002\%$, whereas the several Σ -nuclear excited states with $E_{\mu'}^\Sigma - E_{\text{g.s.}}^\Lambda \approx 80$ MeV considerably contribute to the Σ -mixing. These contributions are coherently enhanced by the configuration mixing which is caused by the ΣN interaction. It is shown that the nature of the Σ -nuclear states plays an important role in the ΛN - ΣN coupling.

It is worth discussing microscopically one mechanism of ΛN - ΣN coupling in Λ hypernuclei. When a Λ -nuclear state in $^{10}_\Lambda\text{Li}$ converts to a Σ^- -nuclear state through the ΛN - ΣN coupling interaction, the Li core state changes into the Be core state. In other words, the β^- -transition, $\text{Li} \rightarrow \text{Be}$, occurs in the core-nuclear state. We stress that the ΛN - ΣN coupling strengths $|D_{\mu'}|^2$ are extremely sensitive to the strengths of β -transitions between the core-nuclear states. The two-body ΛN - ΣN coupling interaction $V_{\Sigma\Lambda}$ can be approximately rewritten as

$$V_{\Sigma\Lambda} \approx V_{\Sigma\Lambda}^{\text{F}} + V_{\Sigma\Lambda}^{\text{GT}}, \quad (14)$$

where

$$V_{\Sigma\Lambda}^{\text{F}} = \bar{V}_{\Sigma\Lambda}, \quad (15)$$

$$V_{\Sigma\Lambda}^{\text{GT}} = \Delta_{\Sigma\Lambda} \mathbf{s}_N \cdot \mathbf{s}_{\Sigma\Lambda} \quad (16)$$

in the particle representation: $\bar{V}_{\Sigma\Lambda}$ and $\Delta_{\Sigma\Lambda}$ are radial integrals of central potentials. In the isospin representation, these terms can be rewritten as

$$V_{\Sigma\Lambda}^{\text{F}} \propto \mathbf{t}_N \cdot \boldsymbol{\phi}_{\Sigma\Lambda}, \quad (17)$$

$$V_{\Sigma\Lambda}^{\text{GT}} \propto (\boldsymbol{\sigma}_N \cdot \boldsymbol{\sigma}_{\Sigma\Lambda}) \mathbf{t}_N \cdot \boldsymbol{\phi}_{\Sigma\Lambda}, \quad (18)$$

where \mathbf{t}_N is the isospin operator for a nucleon and $\boldsymbol{\phi}_{\Sigma\Lambda}$ is the operator that changes a Λ hyperon into a Σ hyperon,

$$|j_\Sigma\rangle = \boldsymbol{\phi}_{\Sigma\Lambda} |j_\Lambda\rangle. \quad (19)$$

Because \mathbf{t}_N and $\boldsymbol{\sigma}_N \mathbf{t}_N$ denote the Fermi and Gamow-Teller β -transition operators for a nucleon, respectively, $V_{\Sigma\Lambda}^{\text{F}}$ and

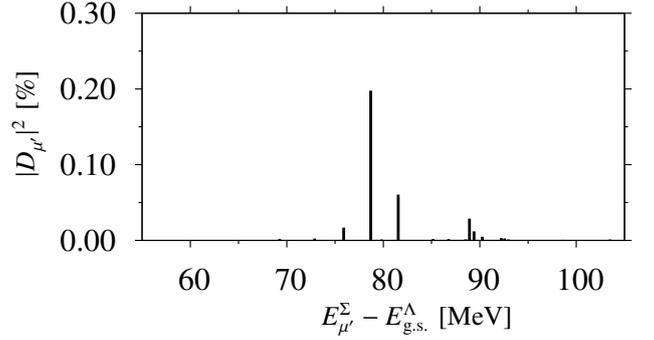


Fig. 2. ΛN - ΣN coupling strengths $|D_{\mu'}|^2$ of the Σ -nuclear eigenstates in the ground state of $^{10}_\Lambda\text{Li}$.

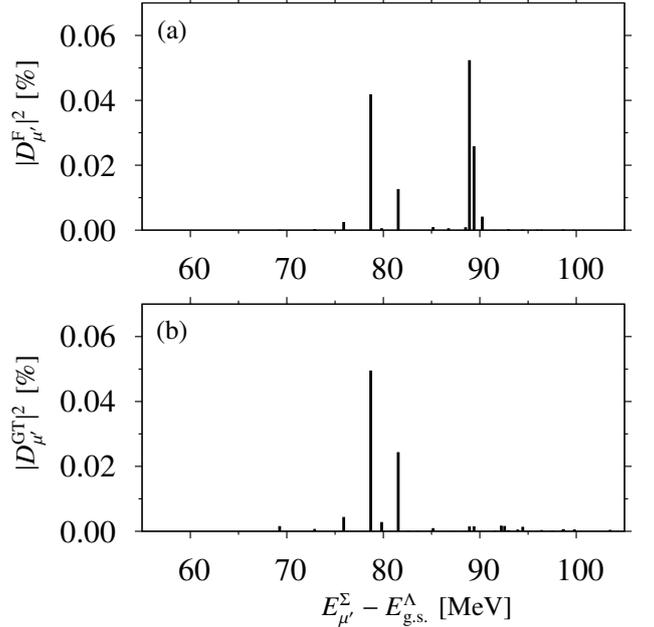


Fig. 3. (a) Fermi-type coupling strengths $|D_{\mu'}^{\text{F}}|^2$ and (b) Gamow-Teller-type coupling strengths $|D_{\mu'}^{\text{GT}}|^2$ of the Σ -nuclear eigenstates in the ground state of $^{10}_\Lambda\text{Li}$.

$V_{\Sigma\Lambda}^{\text{GT}}$ are regarded as the Fermi-type and Gamow-Teller-type coupling interactions. The calculated strength distributions of these couplings,

$$|D_{\mu'}^{\text{F}}|^2 = \left| \frac{\langle \psi_{\text{g.s.}}^\Lambda; T J | V_{\Sigma\Lambda}^{\text{F}} | \psi_{\mu'}^\Sigma; T J \rangle}{E_{\mu'}^\Sigma - E_{\text{g.s.}}^\Lambda} \right|^2, \quad (20)$$

$$|D_{\mu'}^{\text{GT}}|^2 = \left| \frac{\langle \psi_{\text{g.s.}}^\Lambda; T J | V_{\Sigma\Lambda}^{\text{GT}} | \psi_{\mu'}^\Sigma; T J \rangle}{E_{\mu'}^\Sigma - E_{\text{g.s.}}^\Lambda} \right|^2, \quad (21)$$

are shown in Figs. 3 (a) and (b), respectively. Comparing them with Fig. 2, we see the Fermi and Gamow-Teller components coherently contribute to the ΛN - ΣN coupling strengths in the energy region of $E_{\mu'}^\Sigma - E_{\text{g.s.}}^\Lambda \approx 80$ MeV. The calculated Σ -mixing probability involving both couplings is

$$\sum_{\mu'} \left| \frac{\langle \psi_{\text{g.s.}}^\Lambda; T J | (V_{\Sigma\Lambda}^{\text{F}} + V_{\Sigma\Lambda}^{\text{GT}}) | \psi_{\mu'}^\Sigma; T J \rangle}{E_{\mu'}^\Sigma - E_{\text{g.s.}}^\Lambda} \right|^2 = 0.350\%, \quad (22)$$

which is close to the full calculated probability $P_{\Sigma} = 0.345$ %, while $\sum_{\mu'} |D_{\mu'}^F|^2 = 0.144$ % is obtained for the Fermi type and $\sum_{\mu'} |D_{\mu'}^{GT}|^2 = 0.098$ % for the Gamow-Teller type.

In order to see the potentiality of the neutron-rich nuclei clearly, we consider the reason why the energy shift in ${}_{\Lambda}^{10}\text{Li}$ is about 3 times larger than that in ${}_{\Lambda}^7\text{Li}$. The enhancement of the Σ -mixing probabilities in neutron-rich Λ hypernuclei is mainly due to the Fermi-type coupling interaction $V_{\Sigma\Lambda}^F$, which might correspond to the coherent ΛN - ΣN coupling suggested in Refs. [2,3]. When we assume the weak-coupling limit for the ΛN interaction and the approximation in which all the energy differences $E_{\mu'}^{\Sigma} - E_{\nu}^{\Lambda}$ between Λ -nuclear and Σ -nuclear states are replaced by a constant value $\Delta\mathcal{E}$, we can write a sum over the Fermi-type coupling strengths as

$$\sum_{\mu'} |D_{\mu'}^F|^2 \sim \frac{1}{\Delta\mathcal{E}} \frac{4}{3} \bar{V}^2 T (T + 1). \quad (23)$$

Thus the Fermi-type coupling strengths play an important role in the Σ -mixing probability of neutron-rich ${}_{\Lambda}^{10}\text{Li}$ eigenstates because these states have $T = \frac{3}{2}$. On the other hand, in the case of ${}_{\Lambda}^7\text{Li}$ with $T = 0$, the Fermi-type coupling strengths vanish and then the Σ -mixing probability is generated by the Gamow-Teller-type coupling strengths.

In the Gamow-Teller transitions for ordinary nuclei, the model independent formula [20]

$$\sum B(\text{GT}-) - \sum B(\text{GT}+) = 3(N - Z) \quad (24)$$

is well known, where $B(\text{GT}\mp)$ is a strength of the Gamow-Teller β^{\mp} -transition, $|^A Z\rangle \rightarrow |^A Z\pm 1\rangle$. In general, $\sum B(\text{GT}+)$ becomes smaller as neutron-excess grows larger, leading to $\sum B(\text{GT}-) \approx 3(N - Z)$. Therefore, the Gamow-Teller-type coupling is very important in Λ hypernuclei with large neutron excess.

4 Summary

We have investigated the structure of the ${}_{\Lambda}\text{Li}$ hypernuclei in shell-model calculations including ΛN - ΣN coupling in perturbation theory. We found that the Σ -mixing probabilities and the energy shifts are the order of 0.1 % and 0.1 MeV, respectively, and they increase with the neutron number (or isospin). The reasons why the Σ -mixing probabilities are enhanced are summarized as follows: (i) The Σ -nuclear excited states can be strongly coupled with the Λ -nuclear ground state with the help of the ΣN interaction. (ii) The strong ΛN - ΣN coupling is coherently enhanced by the Fermi-type and Gamow-Teller-type coupling components. (iii) The Fermi-type coupling becomes more effective in a neutron-rich environment, increasing as $T(T + 1)$.

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