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Baryon resonances in the baryon meson scattering coupled to the q^3-state

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Abstract. We investigate $\Lambda(1405)$, $\Delta(1232)$, and N*(1440) as resonances in the baryon-meson scattering with bound states embedded in the continuum (BSEC). This BSEC is introduced by hand, as a state not originated from a simple baryon-meson system. We assume it comes from the three-quark state. The resonances are successfully reproduced by this scheme. As for the $\Lambda(1405)$ resonance, it is found that our calculation can give an appropriate energy and width for the peak and reproduces the N$\overline{\mathrm{K}}$ scattering length when the BSEC contribution to the resonance is roughly half of that of the N$\overline{\mathrm{K}}$ channel. The energy of the three-quark pole for $\Delta(1232)$ is taken to be 1327 MeV, which comes down to 1232 MeV after the mixing to the Nπ and $\Delta\pi$ scattering states is included. The nucleon also gets the self energy of about 130 MeV, so the NΔ mass difference, which is one of the key values in the quark model, does not change much. It also found that the positive parity excited nucleon mass can be reduced by more than 400 MeV and become around 1440 MeV after the mixing to the continuum is taken into account.

1 Introduction

It is known that the features of the ground state baryons can be well reproduced by the quark models. They, however, are less successful in describing the excited light baryons [3–5]. For example, the $\Lambda(1405)$ resonance is known to have much lighter mass than the one predicted by the quark model. It is the lowest negative-parity baryon in spite of its non-zero strangeness. The mass of positive parity baryon, N*(1440), is lower than the mass of the negative parity N* although the N*(1440) is supposed to be by 2 $\hbar\omega$ above the nucleon while the lowest negative parity baryons belong to the 1 $\hbar\omega$ excitation.

As one can see from the large width in the mass spectrum, the coupling of such states to the baryon-meson scattering states is important to understand the behavior of the baryon resonances. Many works have been done employing the baryon meson interaction based on the chiral perturbation approach [6–12]. In this work, we investigate both of the above $\Lambda(1405)$ and N*(1440) resonances in our simple baryon-meson model with a 'bound states embedded in the continuum' (BSEC).

We also investigate $\Delta(1232)$ in this work. This resonance is a ground state from a quark model point of view. There, the mass difference between the nucleon and $\Delta(1232)$ is assumed to come from the hyperfine interaction of the gluon exchange between quarks. Or, inversely, the empirical strength of the qq interaction is usually taken so as to give this mass difference [3,4,13]. Therefore, suppose the mass difference between the nucleon pole and the delta pole in our calculation is found to be very different from the observed 293 MeV, then the quark model may require a serious modification. As we will discuss later, both of the nucleon and Δ poles get a large self energy from the coupling to the baryon meson continuum. Their sizes, however, are similar to each other, and the energy difference between the N and Δ poles does not change much. So the quark model scheme holds after the meson effects are introduced.

2 Λ(1405)

2.1 Model

The flavor-octet baryon and meson system can be classified into six irreducible representations of the flavor SU(3):

$$\mathbf{8}_B \times \mathbf{8}_M = \mathbf{1}_{BM} + \mathbf{8}^A_{BM} + \mathbf{8}^S_{BM} + \mathbf{10}_{BM} + \overline{\mathbf{10}}_{BM} + \mathbf{27}_{BM}. \quad (1)$$

The strangeness=-1 and isospin $T = 0$ state appears in the $\mathbf{1}_{BM}$, $\mathbf{8}^A_{BM}$, $\mathbf{8}^S_{BM}$, and $\mathbf{27}_{BM}$ states. These four states are given by a linear combination of the four baryon-meson systems, $\Sigma\pi$, N$\overline{\mathrm{K}}$, $\Lambda\eta$, and ΞK. For the $\Lambda(1405)$ resonance, the scattering channels are taken to be the S-wave $\Sigma\pi$-N$\overline{\mathrm{K}}$-$\Lambda\eta$ because the ΞK threshold is much higher than these states.

We also take into account a BSEC, which is considered to come from the three-quark state: the flavor-singlet negative parity Λ^*. We solve the Lippmann-Schwinger equation with the semirelativistic kinematics in the momentum space [1,2].

The hamiltonian as well as the wave function is divided into the baryon-meson space (P-space) and the q^3

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Table 1. The factor c_{ij} and c_{iQ} for the FF or CMI-type

	FF-type				CMI-type			
c_{ij}	$\Sigma\pi$	$N\overline{K}$	$\Lambda\eta$	ΞK	$\Sigma\pi$	$N\overline{K}$	$\Lambda\eta$	ΞK
$\Sigma\pi$	-8	$\sqrt{6}$	0	$-\sqrt{6}$	$-\frac{16}{3}$	$\frac{116\sqrt{7}}{21}$	$-\frac{16\sqrt{105}}{105}$	0
$N\overline{K}$		-6	$3\sqrt{2}$	0		0	$\frac{28\sqrt{15}}{15}$	0
$\Lambda\eta$			0	$-3\sqrt{2}$			$\frac{112}{15}$	$-\frac{40\sqrt{70}}{21}$
ΞK				-6				$-\frac{160}{21}$
c_{iQ}	$\sqrt{\frac{3}{8}}$	$-\frac{1}{2}$	$\sqrt{\frac{1}{8}}$	$\frac{1}{2}$	0.82	-0.50	0.31	-0.31

pole space (Q-space):

$$H = \begin{pmatrix} H_P & V_{PQ} \\ V_{QP} & E_Q \end{pmatrix} \quad \text{and} \quad \psi = \begin{pmatrix} \psi_P \\ \psi_Q \end{pmatrix}. \tag{2}$$

The potential between baryon and meson, V_P, is assumed to be a central separable potential. The one between the i-th and j-th baryon-meson channels is defined by

$$\begin{aligned} V_{ij}(\boldsymbol{p}, \boldsymbol{p}') &= c_{ij} \frac{\pi V_0}{2} u \exp[-\frac{1}{4}a^2(p^2 + p'^2)] Y_{00}(\Omega_p) Y_{00}(\Omega_{p'}) \\ &= c_{ij} \frac{V_0}{8} u \exp[-\frac{1}{4}a^2(p^2 + p'^2)]. \end{aligned} \tag{3}$$

Here,the strength V_0 is taken so as to give the strength of the chiral unitary model for FF type, or that of the quark model for CMI-type. The range a corresponds to the typical baryon size in the quark model, and the factor u is multiplied to express the energy dependence if necessary [2,8].

Note that the potential acts only on the S-wave baryon-meson states. For the FF type, the factor c_{ij} is taken to be

$$c_{ij} = (\boldsymbol{F}_B \cdot \boldsymbol{F}_M)_{ij}, \tag{4}$$

where $\boldsymbol{F}_B$ and $\boldsymbol{F}_M$ are the flavor SU(3) generators for baryons and mesons. In the CMI-type, c_{ij}'s are given by the color-magnetic interaction with the the quark exchanges. The matrix elements of c_{ij} for both cases with strangeness$=-1$ and isospin $T=0$ are shown in Table 2.1.

The coupling of the baryon-meson state and the Q state, $V_{PQ} = V_{QP}$, is given by

$$V_{PQ}(p) = V_0^{PQ} c_{iQ} \{c_1 + c_2(a_Q p)^2\} \exp[-\frac{1}{4}a_Q^2 p^2] \sqrt{4\pi} Y_{00}(\Omega_p), \tag{5}$$

where V_0^{PQ} is the strength and a_Q describes the form factor of the coupling potential. The channel dependence c_{iQ} as well as V_0^{PQ} used for the CMI-type is obtained from the quark pair annihilation by the one-gluon exchange. As for the FF type, we fix the values by assuming the state to be a flavor singlet state. The overall strength is taken so as to give the same size as the CMI-type does for the $N\overline{K}$ channel. The c_{iQ}'s in both of the cases are shown also in Table 2.1. We use the factor $\{c_1 + c_2(a_Q p)^2\}$ to express the possible p^2-dependence of the transition potential. We call the (c_1, c_2)=(1,0) case as the 1-type, and the (0,1) case as the p^2-type as shown in Table 2. The 1-type coupling appears when one considers the internal structure of the baryons.

2.2 Lippmann-Schwinger equation with BSEC

The Lippmann-Schwinger equation for $H = H_0 + V$ is written as

$$T = V + VG^{(0)}T, \quad G^{(0)} = \frac{1}{E - H_0 + \mathrm{i}\varepsilon}. \tag{6}$$

We assume that the Q space has only one state, then we can set $QHQ \equiv E_Q$ or $QVQ \equiv V_{QQ} = 0$. Using $P + Q = 1$, we obtain the T matrix of the P space, T_{PP}, as the following form [14].

$$T_{PP} = T^{(P)} + (1 + V_{PP}G_P)V_{PQ}G_Q V_{QP}(1 + G_P V_{PP}) \; . \tag{7}$$

Here $T^{(P)}$ is the T-matrix solved within the P space, G_P is a propagator in P space, and G_Q is a propagator for the Q space, which contains the coupling with the P space. The term $(1 + V_{PP}G_P)$ in the above equation describes a distortion in the P space due to the potential V_{PP}.

Because we consider light meson systems, we take the semirelativistic kinematics for the baryon and meson propagators. The free propagator for the initial energy E_{tot} in the P space becomes

$$\begin{aligned} G_P^{(0)} &= \mathrm{i} \int \frac{d^4 q}{(2\pi)^4} \frac{M}{\Omega} \frac{1}{E_{tot} - q^0 - \Omega + \mathrm{i}\varepsilon} \frac{2m}{q_0^2 - \boldsymbol{q}^2 - m^2 + \mathrm{i}\varepsilon} \\ &= \int \frac{d^3 \boldsymbol{q}}{(2\pi)^3} \frac{M}{\Omega} \frac{1}{E_{tot} - \omega - \Omega + \mathrm{i}\varepsilon} \frac{m}{\omega}, \end{aligned} \tag{8}$$

where

$$\begin{aligned} \Omega &= \sqrt{M^2 + \boldsymbol{q}^2}, \quad \omega = \sqrt{m^2 + \boldsymbol{q}^2}, \\ E_{tot} &= \sqrt{M^2 + \boldsymbol{k}^2} + \sqrt{m^2 + \boldsymbol{k}^2}. \end{aligned} \tag{9}$$

Here the $\boldsymbol{k}$ is the relative momentum of the baryon-meson system and M and m are the baryon and meson masses, respectively. The factor $\frac{mM}{\omega\Omega}$ plays a very important role to reduce the strength of the potential in a high momentum region. The effect is strong when the mass is small. Therefore this factor strongly cuts off the $\Sigma\pi$ potential. This cancels the $\Sigma\pi$ attraction at least partially and enables the system to have a peak just below the $N\overline{K}$ threshold.

2.3 Wave functions

The wave function ψ with the initial wave function ϕ_{ini} can be obtained by using the T-matrix. The wave function of the P-space is

$$|\psi_P\rangle = |\phi_{ini}\rangle + G_P^{(0)} T_{PP} |\phi_{ini}\rangle, \tag{10}$$

whereas the wave function for the Q-space becomes

$$|\psi_Q\rangle = G_Q^{(0)} T_{QP} |\phi_{ini}\rangle. \tag{11}$$

The wave function of the coordinate space for the P-space can be obtained by the Fourier transformation.

Table 2. The $N\overline{K}$ scattering length and the probability of the q^3 pole.

Potentials	BSEC	kinematics	E_{res}	width	$R_{q^3/N\overline{K}}$	$N\overline{K}$ scatt. length	self energy
strong FF		E-dep	1407	50	-	$-2.1 + 0.59i$	
strong FF			1408	24	-	$-1.9 + 0.25i$	
weak FF	p^2-type		1404	18	0.7	$-1.1 + 0.18i$	$-119 - 17i$
weak FF	1-type		1404	41	0.4	$-1.7 + 0.43i$	$-104 - 49i$
CMI	1-type	nonrela	1406	44	2.8	$-0.6 + 0.25i$	$-83 - 29i$
CMI	1-type		1406	56	2.7	$-0.7 + 0.34i$	$-78 - 40i$
CMI	p^2-type		1403	10	13.3	$-0.0 + 0.03i$	$-88 - 4i$
Exp. [15, 16]			1406	50		$-1.7 + 0.68i$	

Relative importance between the closed P-space ϕ_c and the Q-space, or between the $N\overline{K}$ channel to the q^3 state in this case, $R_{q^3/N\overline{K}}$, can be found by taking a ratio of the following two probabilities.

$$|\langle\phi_c|\psi_P\rangle|^2 = |\langle\phi_c|G_P^{(0)}T_{PP}|\phi_{ini}\rangle|^2 , \qquad (12)$$

$$|\langle Q|\psi_Q\rangle|^2 = |\langle Q|G_Q^{(0)}T_{QP}|\phi_{ini}\rangle|^2 . \qquad (13)$$

2.4 Results

It is found that this approach can give an appropriate peak for the $\Lambda(1405)$ [2]. The one whose baryon-meson interaction is the SU(3) FF-type is shown in Fig. 1(a), whereas the one obtained from the color-magnetic interaction between quarks is shown in Fig. 1(b). The shape of the peak differs slightly from each other. The values of the parameters used for each potential are given in Ref. [2].

Since the interaction from the quark model is not attractive in the $N\overline{K}$ channel, the calculated peak consists mostly of the q^3 state there. So, the $N\overline{K}$ scattering length is negative but rather small: about half of the observed value. On the other hand, the peak obtained from the one with the FF-type interaction consists mostly of the $N\overline{K}$ bound state. Therefore the $N\overline{K}$ scattering length is negative and large. As is shown in Table 2 and in Fig. 2, our calculation where the probability of the q^3 state is about half of that of the $N\overline{K}$ channel at the resonance seems to reproduce the observed $N\overline{K}$ scattering length.

3 N(939) and N*(1440)

It is hard to reproduce the N*(1440) resonance if one assumes a simple q^3 configuration. The masses of the other positive parity baryons are around 1680~1720 MeV. The interaction between quarks may reduce the mass, but it is not sufficient [4, 5].

We employ the baryon meson interaction (Fig. 3(b)), which arises from the cross term (Fig. 3(a)). The q^3 states which correspond to the nucleon and N*(1440) are placed as BSECs in the P-wave $N\pi$-$\Delta\pi$ scattering states (Fig. 3(c)) [17].

The baryon meson interaction we employs is again separable as in eq. (3), and the strength between baryons and pion are taken from the quark model. The spin and the isospin factors of the strength are shown in Table 3. The quark model values for the coupling constants are $f_{\pi N\Delta}$ ~

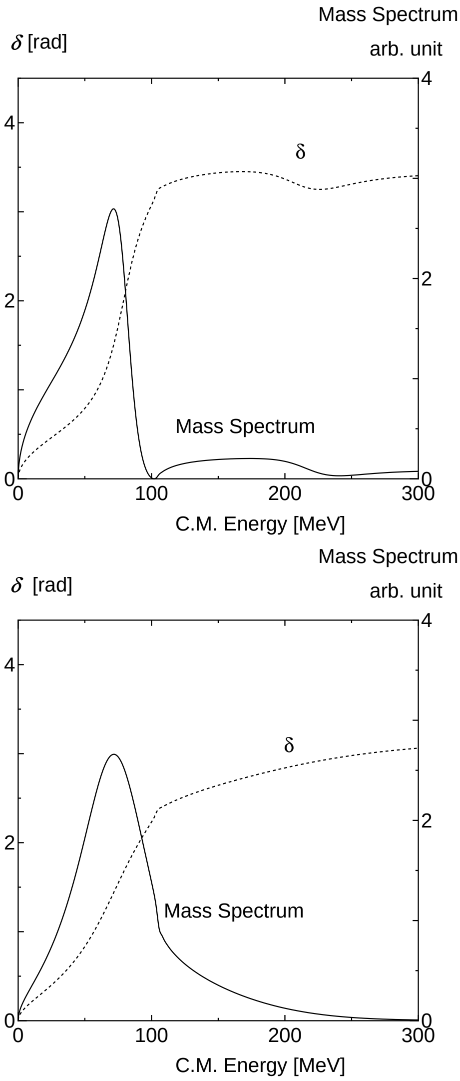

Fig. 1. Phase shifts and mass spectra of the $\Lambda(1405)$. The interaction is (a) the SU(3) FF-type and (b) the one obtained from the color-magnetic interaction between quarks.

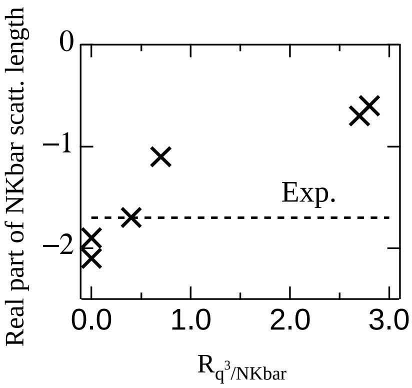

Fig. 2. Real part of the $N\overline{K}$ scattering length and the ratio of the probability of the q^3 pole over $N\overline{K}$ at the resonance energy.

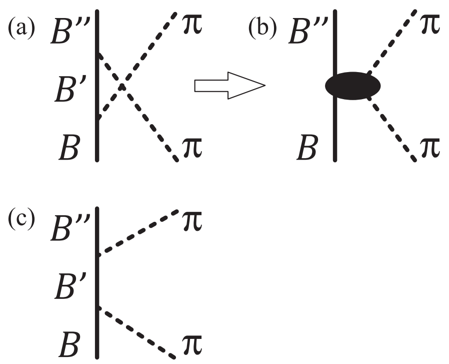

Fig. 3. Baryon meson interaction and a baryon pole.

Table 3. Coupling strength of the baryon meson interaction.

B	B''	(b) B'-exchange N-exchange	(b) B'-exchange Δ-exchange	(c) B'-pole
$S = T = 1/2$				B' = N or N*
N	N	$f_{\pi NN}^2$	$16/9\ f_{\pi N\Delta}^2$	$9\ f_{\pi NB'}^2$
N	Δ	$8/3\ f_{\pi NN} f_{\pi N\Delta}$	$50/3\ f_{\pi N\Delta} f_{\pi\Delta\Delta}$	$6\ f_{\pi NB'} f_{\pi B'\Delta}$
Δ	Δ	$1/9\ f_{\pi N\Delta}^2$	$100\ f_{\pi\Delta\Delta}^2$	$4 f_{\pi B'\Delta}^2$
$S = T = 3/2$				$B' = \Delta$
N	N	$4 f_{\pi NN}^2$	$1/9\ f_{\pi N\Delta}^2$	$f_{\pi N\Delta}^2$
N	Δ	$5/3\ f_{\pi NN} f_{\pi N\Delta}$	$20/3\ f_{\pi N\Delta} f_{\pi\Delta\Delta}$	$15\ f_{\pi N\Delta} f_{\pi\Delta\Delta}$
Δ	Δ	$4/9\ f_{\pi N\Delta}^2$	$121\ f_{\pi\Delta\Delta}^2$	$225 f_{\pi\Delta\Delta}^2$

$1.7 f_{\pi NN}$ and $f_{\pi\Delta\Delta} \sim 0.2 f_{\pi NN}$. So, the effects from the B'-poles become most important in the $S = T = 1/2$ case.

To fit the N*(1440) peak, we have to assume $f_{\pi NN^*} = 2 f_{\pi NN}$ and $f_{\pi\Delta N^*} = 2 f_{\pi N\Delta}$. The origin of this factor 2 is not specified here; it may come from the difference in the orbital part or other modes such as $N\pi\pi$. Then, with the N*-pole at 1822 MeV and the N-pole at 1067 MeV, the resonance shape is well reproduced as seen in Fig. 4. The energy of the N*-pole becomes reasonably close to the other positive parity baryons.

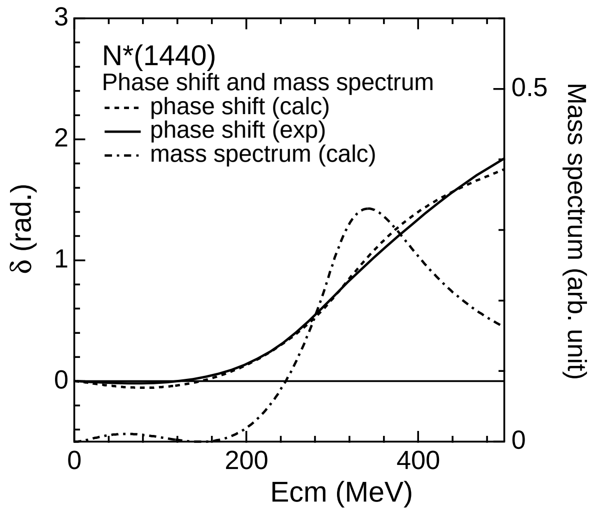

Fig. 4. Phase shifts and mass spectrum of the N*(1440).

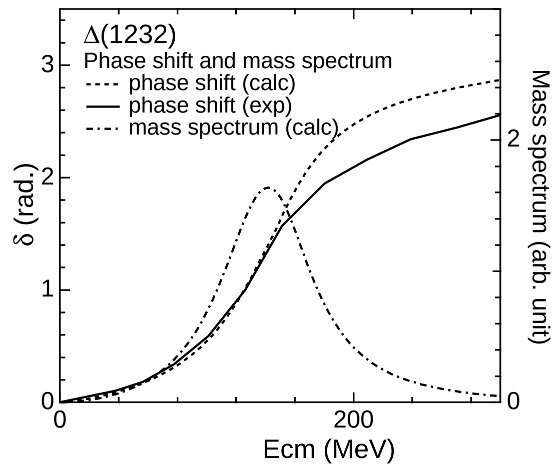

Fig. 5. Phase shifts and mass spectrum of the Δ(1232).

4 Δ(1232)

The Δ resonance appears in the $S = T = 3/2$ $N\pi$ and $\Delta\pi$ scattering. We also apply the same scheme to this resonance and obtain a reasonable peak as shown in Fig. 5. As seen in Table 3, $\Delta\pi$ scattering with the Δ pole diagram is most important to form the peak. The energy of the q^3 Δ-pole is found to be 1327 MeV. The difference between the N and Δ pole energies then becomes 280 MeV. In the quark model viewpoint, this difference is considered to come from hyperfine interaction in the gluon exchange between quarks [4, 13]. In the usual quark model, this difference is taken from the observed $N\Delta$ mass difference, 293MeV. Reflecting the fact that pion cloud produces similar self energy to N and Δ-pole, the difference or the empirical strength of the quark-quark interaction does not change much when one considers the meson effects in the quark model.

5 Summary

We have investigated $\Lambda(1405)$, $N^*(1440)$ and $\Delta(1232)$ as resonances in the baryon-meson scattering. The S-wave interaction between the baryon and the meson is taken to be CMI-type or FF-type for the $\Lambda(1405)$. The P-wave interaction for $N^*(1440)$ and $\Delta(1232)$ is taken to be the one arising from the pion-quark interaction. The q^3 state is introduced in the systems as a BSEC. The peaks of these resonances are reproduced successfully by our baryon meson model though it is necessary to modify the coupling constant $f_{\pi BN^*}$ to have a proper width for $N^*(1440)$.

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