EPJ Web of Conferences **7**, 01001 (2010)
DOI:10.1051/epjconf/20100701001

The thermal-statistical model for particle production

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Abstract. We introduce the thermal statistical model for particle production and its applications to heavy-ion collision experiments. We focus on the aspects related to chemical equilibrium, in particular on particle yields.

Acknowledgment

The author acknowledges the support of the JINR - South Africa cooperation agreement.

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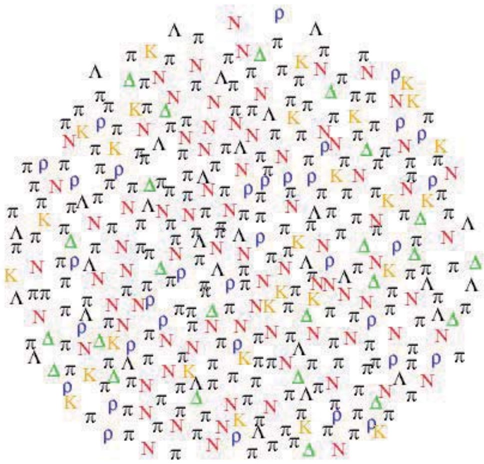
The Thermal-Statistical Model for Particle Production I.

J. Cleymans

23 - 25 July 2008 / JINR, Dubna

Statistical Model | Strangeness | E_T/N_{ch} vs. E/N | The Horn in the K^+/π^+ Ratio

Outline



J.C. and H. Satz, Zeitschrift fuer Physik C57, 135 (1993)

Thermal Equilibrium

In thermal equilibrium

$$Z = \mathrm{Tr}\, e^{-\frac{H}{T} + \frac{\mu N}{T}}$$

$$\langle N \rangle = \frac{\mathrm{Tr}\, N e^{-\frac{H}{T} + \frac{\mu N}{T}}}{\mathrm{Tr}\, e^{-\frac{H}{T} + \frac{\mu N}{T}}}$$

$$\langle E \rangle = \frac{\mathrm{Tr}\, E e^{\frac{-H}{T} + \frac{\mu N}{T}}}{\mathrm{Tr}\, e^{\frac{-H}{T} + \frac{\mu N}{T}}}$$

Full Hydrodynamic Flow

Bjorken scaling + Transverse expansion

After integration over m_T

$$\frac{dN_i/dy}{dN_j/dy} = \frac{N_i^0}{N_j^0}$$

where N_i^0 is the particle yield
as calculated in a fireball **AT REST!**

Effects of hydrodynamic flow cancel out in ratios.

Thermal Equilibrium

Particle Number

$$\begin{aligned}
\langle N \rangle &= \frac{\mathrm{Tr}\, N e^{-\frac{H}{T} + \frac{\mu N}{T}}}{\mathrm{Tr}\, e^{-\frac{H}{T} + \frac{\mu N}{T}}} \\
&= \frac{T}{Z}\frac{\partial}{\partial \mu}\mathrm{Tr}\, e^{\frac{-H}{T} + \frac{\mu N}{T}} \\
&= T\frac{1}{Z}\frac{\partial Z}{\partial \mu} \\
&= T\frac{\partial}{\partial \mu}\ln Z
\end{aligned}$$

Thermal Equilibrium

Average Energy

$$
\begin{aligned}
\langle E \rangle &= \frac{\mathrm{Tr}\, H\, e^{\frac{-H}{T} + \frac{\mu N}{T}}}{\mathrm{Tr}\, e^{\frac{-H}{T} + \frac{\mu N}{T}}} \\
&= -\frac{1}{Z}\frac{\partial Z}{\partial \beta} + \mu \langle N \rangle \\
&= T^2 \frac{\partial}{\partial T} \ln Z + \mu \langle N \rangle
\end{aligned}
$$

Statistical Model Strangeness E_T/N_{ch} vs. E/N The Horn in the K^+/π^+ Ratio

Thermal Equilibrium

$$
\begin{aligned}
N_i &= g_i\, V \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E}{T}\right) e^{\frac{\mu_i}{T}} \\
&= g_i\, V \frac{4\pi}{(2\pi)^3} \int p^2\, dp \exp\left(-\frac{\sqrt{p^2 + m_i^2}}{T}\right) e^{\frac{\mu_i}{T}} \\
&= g_i\, V \frac{4\pi}{(2\pi)^3} T^3 \int x^2\, dx \exp\left(-\sqrt{x^2 + m_i^2/T^2}\right) e^{\frac{\mu_i}{T}} \\
&= g_i\, V \frac{1}{2\pi^2} T m_i^2 K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}}
\end{aligned}
$$

Thermal Equilibrium

$$
\begin{aligned}
n_i &= g_i \frac{1}{2\pi^2} T m_i^2 K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}} \\
\epsilon_i &= g_i \frac{1}{2\pi^2} T m_i^3 \left[K_1\left(\frac{m_i}{T}\right) + 3\frac{T}{m} K_2\left(\frac{m_i}{T}\right)\right] e^{\frac{\mu_i}{T}} \\
s_i &= g_i \frac{1}{2\pi^2} m_i^3 \left[K_1\left(\frac{m_i}{T}\right) + \frac{4T}{m} K_2\left(\frac{m_i}{T}\right) - \frac{\mu_i}{m} K_2\left(\frac{m_i}{T}\right)\right] e^{\frac{\mu_i}{T}} \\
P_i &= g_i \frac{1}{2\pi^2} T^2 m_i^2 K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}}
\end{aligned}
$$

Statistical Model Strangeness E_T/N_{ch} vs. E/N The Horn in the K^+/π^+ Ratio

Chemical Equilibrium

In equilibrium

$$E_1 + E_2 + \cdots = E_3 + E_4 + E_5 + \cdots \quad (1)$$

for the chemical potentials

$$\mu_1 + \mu_2 + \cdots = \mu_3 + \mu_4 + \mu_5 + \cdots \quad (2)$$

As an example

$$\pi^0 + p \leftrightarrow \pi_0 + p + \pi^0 \quad (3)$$

leads to

$$\mu_{\pi^0} + \mu_p = \mu_{\pi^0} + \mu_p + \mu_{\pi^0} \quad (4)$$

which leads to

$$\mu_{\pi^0} = 0 \quad (5)$$

Chemical Equilibrium

In equilibrium

$$B + B \leftrightarrow B + B + B + \bar{B} \tag{6}$$

$$dE = -pdV + TdS + \mu_B dN_B + \mu_{\bar{B}} dN_{\bar{B}}$$

Due to baryon number conservation one has

$$N_B - N_{\bar{B}} = \text{constant}$$

and

$$dN_B = dN_{\bar{B}}$$

The energy is a minimum for

$$dE = (\mu_B + \mu_{\bar{B}})dN_B = 0 \tag{7}$$

$$\mu_B = -\mu_{\bar{B}} \tag{8}$$

Statistical Model Strangeness E_T/N_{ch} vs. E/N The Horn in the K^+/π^+ Ratio

Chemical Equilibrium

In equilibrium

$$N_B = g\ V \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E}{T} + \frac{\mu_B}{T}\right)$$
$$N_{\bar{B}} = g\ V \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E}{T} - \frac{\mu_B}{T}\right)$$

$$\begin{aligned} N_B &= N_{\bar{B}} \rightarrow \mu_B = 0 \\ N_B &\geq N_{\bar{B}} \rightarrow \mu_B \geq 0 \\ N_B &\leq N_{\bar{B}} \rightarrow \mu_B \leq 0 \end{aligned}$$

	Chemical Equilibrium	No Chem. Equil.
π	$\exp\left[-\frac{E_\pi}{T}\right]$	$\exp\left[-\frac{E_\pi}{T}+\frac{\mu_\pi}{T}\right]$
N	$\exp\left[-\frac{E_N}{T}+\frac{\mu_B}{T}\right]$	$\exp\left[-\frac{E_N}{T}+\frac{\mu_N}{T}\right]$
$\overline{N}$	$\exp\left[-\frac{E_N}{T}-\frac{\mu_B}{T}\right]$	$\exp\left[-\frac{E_N}{T}+\frac{\mu_{\overline{N}}}{T}\right]$
Λ	$\exp\left[-\frac{E_\Lambda}{T}+\frac{\mu_B}{T}-\frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_\Lambda}{T}+\frac{\mu_\Lambda}{T}\right]$
$\overline{\Lambda}$	$\exp\left[-\frac{E_\Lambda}{T}-\frac{\mu_B}{T}+\frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_\Lambda}{T}+\frac{\mu_{\overline{\Lambda}}}{T}\right]$
K	$\exp\left[-\frac{E_K}{T}+\frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_K}{T}+\frac{\mu_K}{T}\right]$
$\overline{K}$	$\exp\left[-\frac{E_K}{T}-\frac{\mu_S}{T}\right]$	$\exp\left[-\frac{E_K}{T}+\frac{\mu_{\overline{K}}}{T}\right]$

The number of particles of type i is determined by:

$$N_i = V\, g_i \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E_i}{T}+\frac{\mu_i}{T}\right)$$

For bosons:

$$N_i = V\, g_i \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp\left(\frac{E_i}{T}-\frac{\mu_i}{T}\right)-1}$$

For fermions:

$$N_i = V\, g_i \int \frac{d^3p}{(2\pi)^3} \frac{1}{\exp\left(\frac{E_i}{T}-\frac{\mu_i}{T}\right)+1}$$

Only conserved quantum numbers matter for chemical equilibrium: In equilibrium

$$\mu_i = B_i\mu_B + Q_i\mu_Q + S_i\mu_S + C_i\mu_C + .. \tag{9}$$

g_i	m_i	stat	S_i	B_i	Q_i	Particle i
1	0.140	-1	0	0	1.	π^+
1	0.135	-1	0	0	0.	π^0
1	0.140	-1	0	0	-1.	π^-
1	0.547	-1	0	0	0.	η
3	0.770	-1	0	0	1.	ρ^+
3	0.770	-1	0	0	0.	ρ^0
3	0.770	-1	0	0	-1.	ρ^-
3	0.782	-1	0	0	0.	ω
1	0.958	-1	0	0	0.	η'
1	0.980	-1	0	0	0.	f_0
1	0.982	-1	0	0	1.	a_0^+
1	0.982	-1	0	0	0.	a_0^0
1	0.982	-1	0	0	-1.	a_0^-
3	1.019	-1	0	0	0.	ϕ
3	1.170	-1	0	0	0.	
3	1.230	-1	0	0	1.	
3	1.230	-1	0	0	0.	
3	1.230	-1	0	0	-1.	
3	1.229	-1	0	0	1.	
3	1.229	-1	0	0	0.	
3	1.229	-1	0	0	-1.	
5	1.275	-1	0	0	0.	
3	1.282	-1	0	0	0.	
1	1.297	-1	0	0	0.	
1	1.300	-1	0	0	1.	
1	1.300	-1	0	0	0.	

Statistical Model   Strangeness E_T/N_{ch} vs. E/N The Horn in the K^+/π^+ Ratio

The Role of Resonances

Example: ρ's

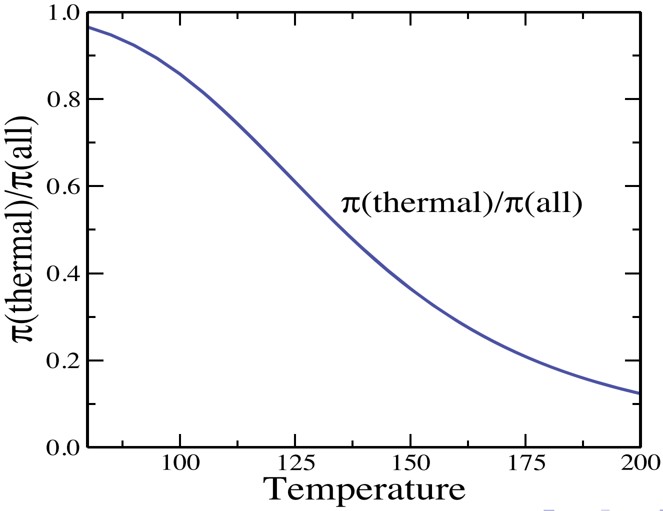
$$\rho \longrightarrow \pi^+\pi^-$$

Final, observed, number of π^+ is given by

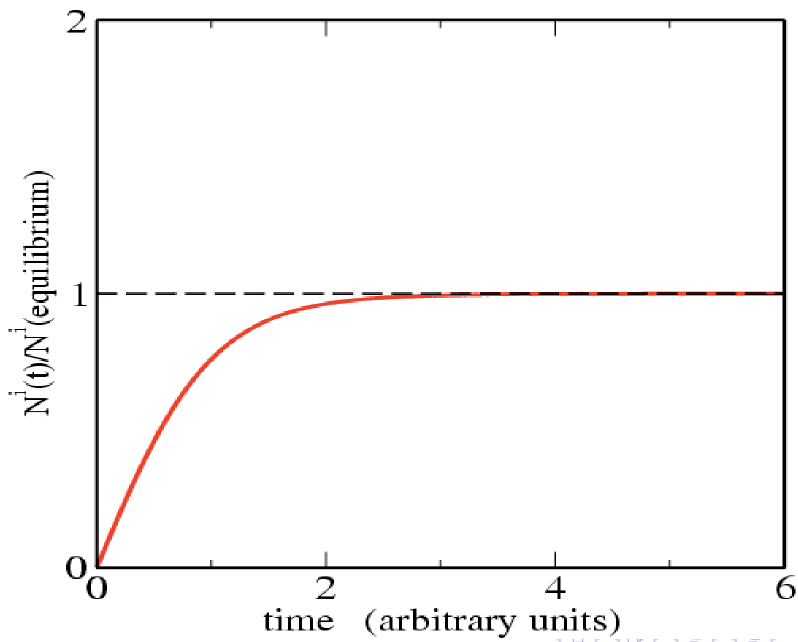
$$N_{\pi^+} = N_{\pi^+}(\text{thermal}) + N_{\pi^+}(\text{resonance decays})$$

depending on the temperature, over 80% of observed pions are due to resonance decays

Importance of Resonances.



Strangeness saturation?



Strangeness saturation?

$$N_i = \boxed{\gamma_s^{|S|}} V\, g_i \int \frac{d^3p}{(2\pi)^3} \exp\left(-\frac{E_i}{T} + \frac{\mu_i}{T}\right)$$

with

$\gamma_s < 1$ strangeness under-saturation
$\gamma_s = 1$ strangeness in chemical equilibrium
$\gamma_s > 1$ strangeness over-saturation

Statistical Model | Strangeness | E_T/N_{ch} vs. E/N | The Horn in the K^+/π^+ Ratio

SPS data.

	Measurement
Pb–Pb 158*A* GeV	
$(\pi^+ + \pi^-)/2.$	600±30
K^+	95 ±10
K^-	50 ±5
K^0_S	60 ±12
p	140±12
$\bar{p}$	10 ±1.7
ϕ	7.6±1.1
Ξ^-	4.42±0.31
$\overline{\Xi^-}$	0.74±0.04
$\overline{\Lambda}/\Lambda$	0.2±0.04

SPS data.

SPS: Chemical Freeze-Out Parameters:

$$T = 156.0 \pm 2.4\mathrm{MeV}$$
$$\mu_B = 239 \pm 12\mathrm{MeV}$$
$$\gamma_s = 0.862 \pm 0.036$$

F. Becattini, J.C., A. Keränen, E. Suhonen and K. Redlich
Physical Review C64 (2001) 024901.

AGS data.

	Measurement
Au–Au 11.6*A* GeV	
Participants	363±10
K^+	23.7±2.9
K^-	3.76±0.47
π^+	133.7±9.9
Λ	20.34±2.74
p/π^+	1.234±0.126
$\bar{p}$	>0.0185±0.0018

AGS data.

AGS: Chemical Freeze-Out Parameters:

$$T = 130.6 \pm 5.5\text{MeV}$$
$$\mu_B = 594 \pm 26\text{MeV}$$
$$\gamma_s = 0.883 \pm 0.124$$

F. Becattini, J.C., A. Keränen, E. Suhonen and K. Redlich
Physical Review C64 (2001) 024901.

SIS data.

	Measurement
Au–Au 1.7*A* GeV	
π^+/p	0.052±0.013
K$^+/\pi^+$	0.003±0.00075
π^-/π^+	2.05±0.51
η/π^0	0.018±0.007

SIS: Chemical Freeze-Out Parameters:

$$T = 49.7 \pm 1.1\text{MeV}$$
$$\mu_B = 818 \pm 15\text{MeV}$$
$$\gamma_s = 1 \text{ (fixed)}$$

J. C., H. Oeschler and K. Redlich)
Physical Review C59, (1999) 1663.

RHIC data.

J. C., B. Kämpfer, M. Kaneta, S. Wheaton, N. Xu, Phys. Rev. C71, 0409071 (2005)

Ratio	Experiment	Central	Mid-Central	Peripheral
$\pi^-_{(2)}/\pi^+_{(2)}$	BRAHMS	0.990±0.100		
	PHENIX	0.960±0.177	0.920±0.170	0.933±0.172
	PHOBOS	1.000±0.022		
	STAR	1.000±0.073	1.000±0.073	1.000 ± 0.073
$K^+_{(2)}/K^-_{(2)}$	PHENIX	1.152±0.240	1.292±0.268	1.322±0.284
	PHOBOS	1.099±0.111		
	STAR	1.109±0.022	1.105±0.036	1.120±0.040
$\bar{p}_{(1)}/p_{(1)}$	PHENIX	0.680±0.149	0.671±0.142	0.717±0.157
$\bar{p}_{(2)}/p_{(2)}$	BRAHMS	0.650±0.092		
	PHOBOS	0.600±0.072		
	STAR	0.714±0.050	0.724±0.050	0.764±0.053
$\bar{\Lambda}_{(1)}/\Lambda_{(1)}$	PHENIX	0.750±0.180	0.798±0.197	0.795±0.197
$\bar{\Lambda}_{(2)}/\Lambda_{(2)}$	STAR	0.719±0.090	0.739±0.092	0.744±0.100
$\bar{\Xi}^+_{(2)}/\Xi^-_{(2)}$	STAR	0.840±0.053	0.822±0.114	0.815±0.096
$\bar{\Omega}^+/\Omega^-$	STAR	1.062±0.410		
$K^-_{(2)}/\pi^-_{(2)}$	PHENIX	0.151±0.030	0.134±0.027	0.116±0.023
	STAR	0.151±0.022	0.147±0.022	0.130±0.019
$K^0_S/\pi^-_{(2)}$	STAR	0.134±0.022	0.131±0.022	0.108±0.018
$\bar{p}_{(1)}/\pi^-_{(2)}$	PHENIX	0.049±0.010	0.047±0.010	0.045±0.009
$\bar{p}_{(2)}/\pi^-_{(2)}$	STAR	0.069±0.019	0.067±0.019	0.067±0.019
$\Lambda_{(1)}/\pi^-_{(2)}$	STAR	0.043±0.008	0.043±0.008	0.039±0.007
$\Lambda_{(2)}/\pi^-_{(2)}$	PHENIX	0.072±0.017	0.068±0.016	0.074±0.017
$< K^{*0} > /\pi^-_{(2)}$	STAR	0.039±0.011		
$\phi/\pi^-_{(2)}$	STAR	0.022±0.003	0.021±0.004	0.022±0.004
$\Xi^-_{(2)}/\pi^-_{(2)}$	STAR	0.0093±0.0012	0.0072±0.0011	0.0060±0.0008

Statistical Model Strangeness E_T/N_{ch} vs. E/N The Horn in the K^+/π^+ Ratio

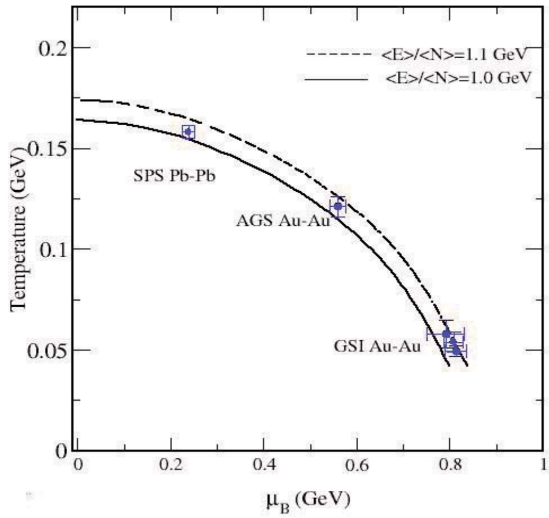
RHIC data.

RHIC: Chemical Freeze-Out Parameters:

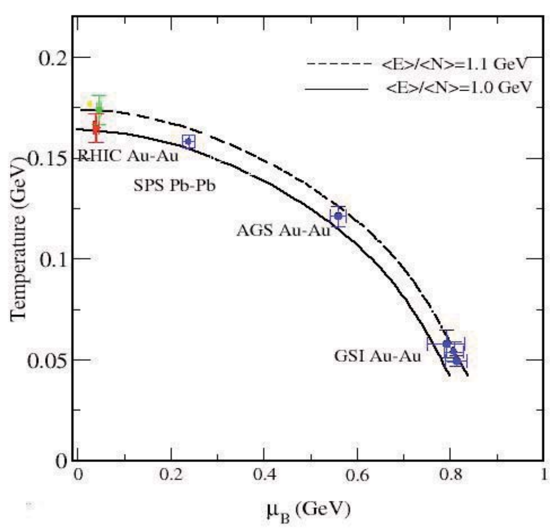
$$T = 169 \pm 4.2\,\mathrm{MeV}$$
$$\mu_B = 39.6 \pm 6\,\mathrm{MeV}$$
$$\gamma_s = 0.9 \pm 0.1$$

J. C., B. Kämpfer, M. Kaneta, S. Wheaton, N. Xu
Phys. Rev. C71, 0409071 (2005)

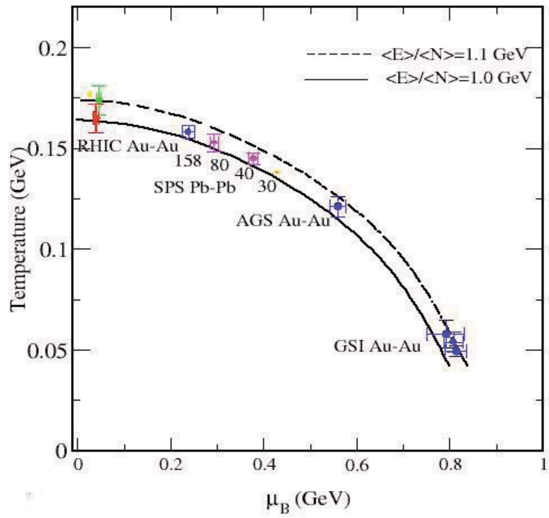
E/N in 1999



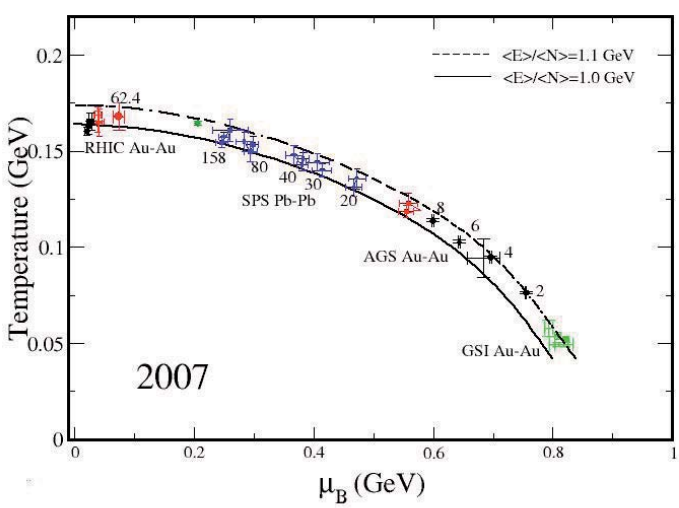
E/N in 2000



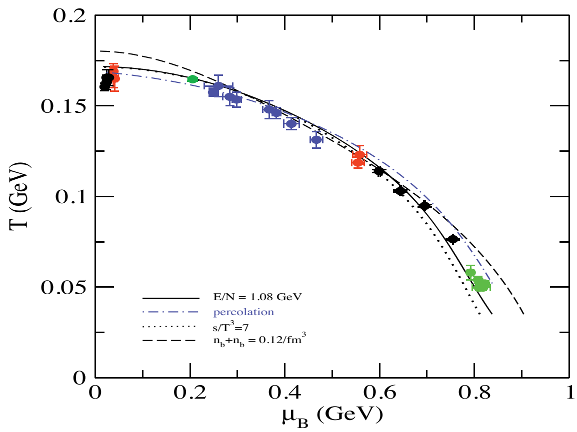
E/N in 2005



Statistical Model Strangeness E_T/N_{ch} vs. E/N The Horn in the K^+/π^+ Ratio



Chemical Freeze-Out: Status in 2005

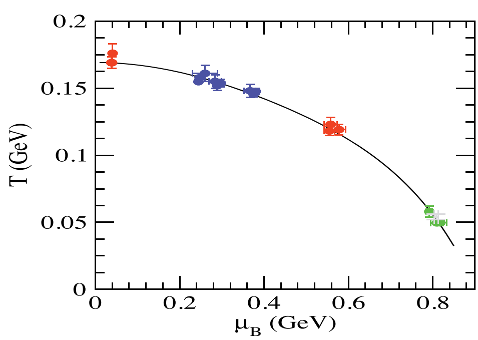


V. Magas and H. Satz, Eur. Phys. J. **C32** 115 (2003).

P. Braun-Munzinger and J. Stachel, J. Phys. G:Nucl. Part. Phys. **28** 1971 (2002).

Statistical Model | Strangeness | E_T/N_{ch} vs. E/N | The Horn in the K^+/π^+ Ratio

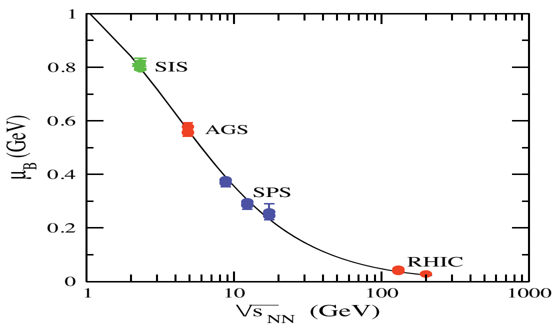
Chemical Freeze-Out: Status in 2005



$$T(\mu_B) = 0.169 - 0.189\mu_B^2 + 0.165\mu_B^4 - 0.229\mu_B^6.$$

J. C., H. Oeschler, K. Redlich, S. Wheaton

μ_B as a function of $\sqrt{s_{NN}}$



$$\mu_B(\sqrt{s}) = \frac{1.273 \text{ GeV}}{1 + 0.258 \text{ GeV}^{-1}\sqrt{s}}.$$

This predicts at LHC $\mu_B \approx 1$ MeV.

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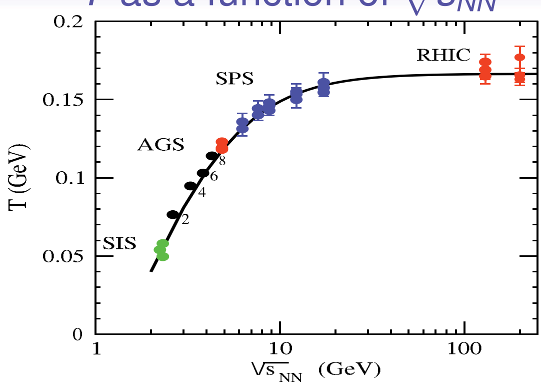
T as a function of $\sqrt{s_{NN}}$



This predicts at LHC $T \approx 170$ MeV.

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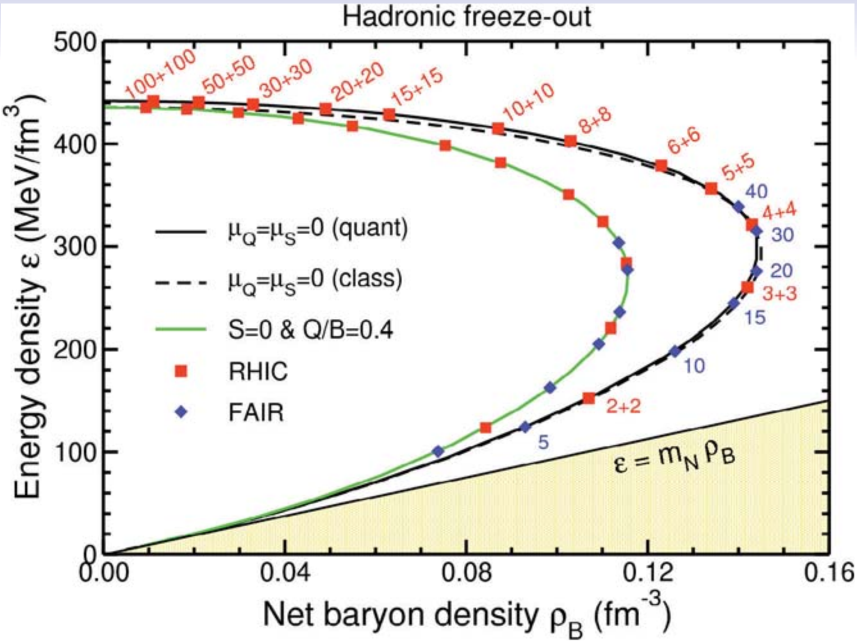


J. Randrup and J.C., Phys. Rev. C74 (2006) 047901

Will it be possible to determine directly E/N ?

E: energy of primordial hadrons
N: number of primordial hadrons

$$\langle E_T \rangle = \langle E \sin\theta \rangle = \frac{\pi}{4} \langle E \rangle$$

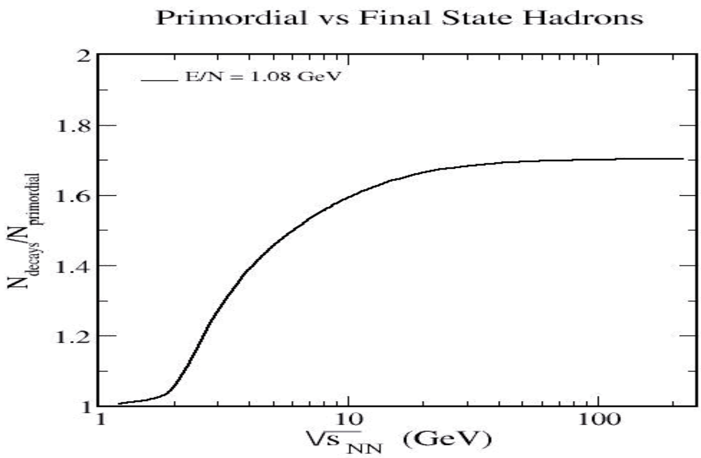
Low energy limit

$$\lim \frac{E_T}{N_{ch}} = \frac{\frac{\pi}{4} m_N}{0.4} \approx 1.8\ \mathrm{GeV}$$

High energy limit

$$\lim \frac{E_T}{N_{ch}} = \frac{\frac{\pi}{4} \langle M \rangle}{2/3} \approx 0.9\ \mathrm{GeV}$$

However E_T : subtract m_N for baryons add m_N for antibaryons.



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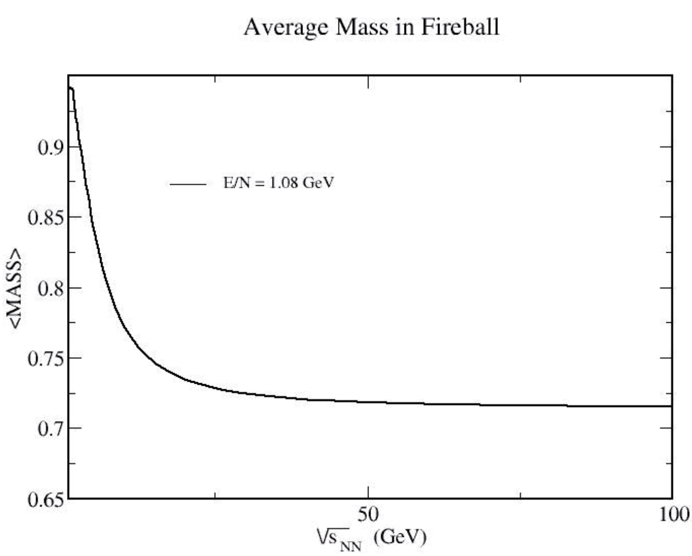

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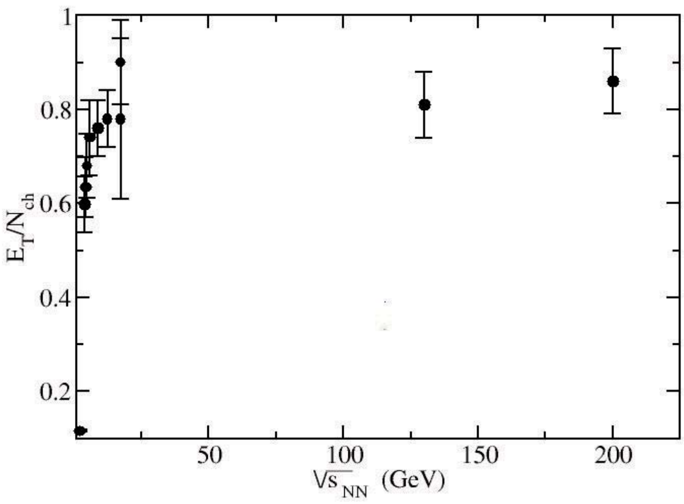
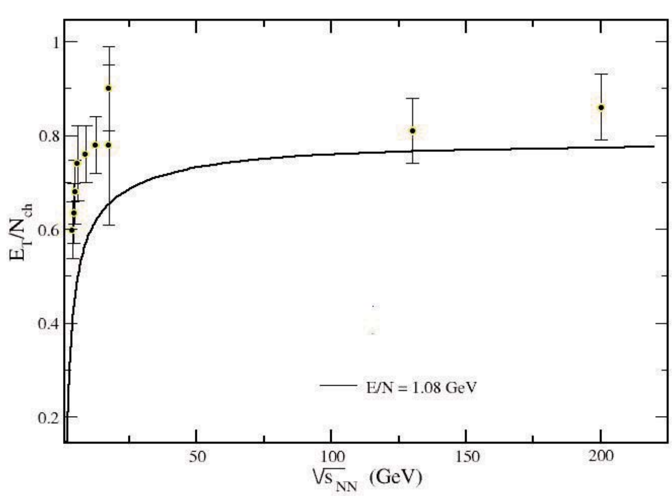
Transverse Energy per Charge

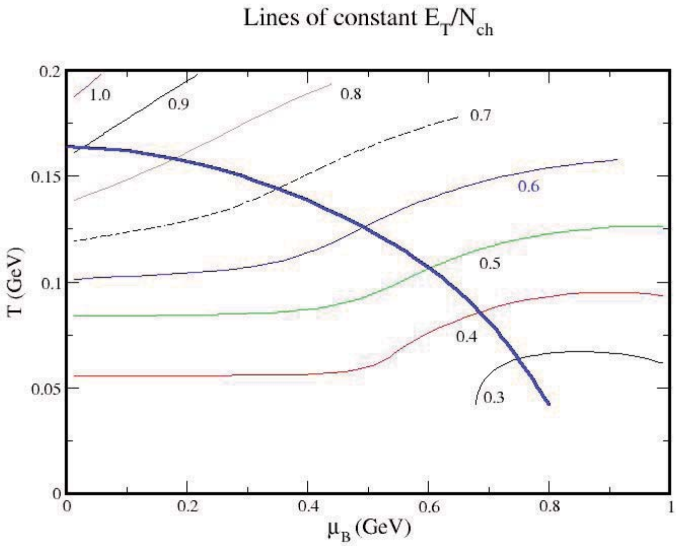


Statistical Model Strangeness E_T/N_{ch} vs. E/N The Horn in the K^+/π^+ Ratio

Transverse Energy per Charged Hadron



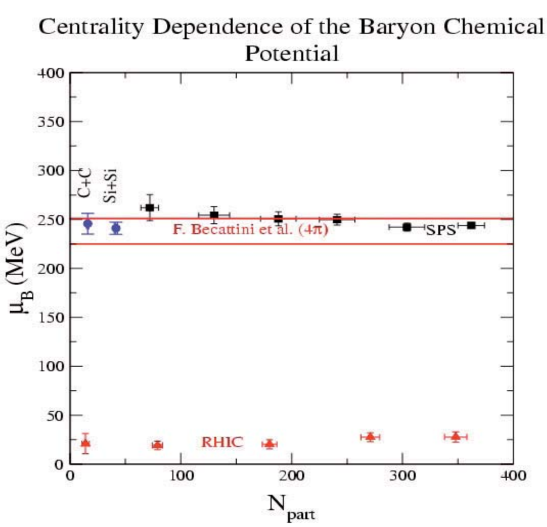
J.C., R. Sahoo, D.K. Srivastava, S. Wheaton



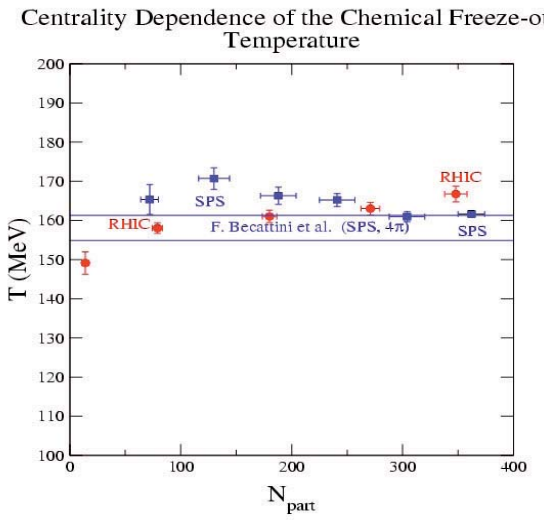
E_T/N_{c}

mainly follows T and is determined by E/N,

J.C., R. Sahoo, D.K. Srivastava, S. Wheaton



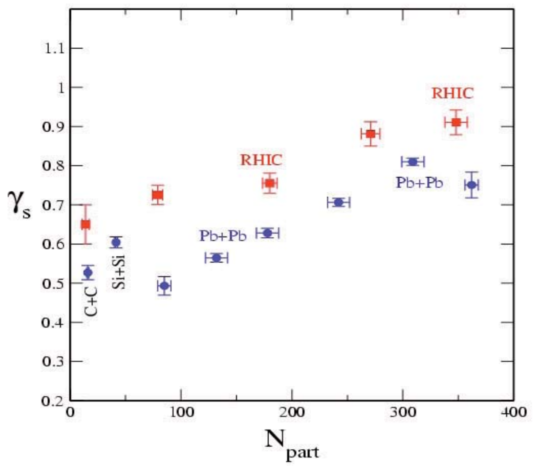
J. C., B. Kämpfer, P. Steinberg and S. Wheaton, Journal of Physics G30 S595-S598 (2004).



J. C., B. Kämpfer, P. Steinberg and S. Wheaton, Journal of Physics G30 S595-S598 (2004).

Statistical Model

K^+/π^+ Ratio

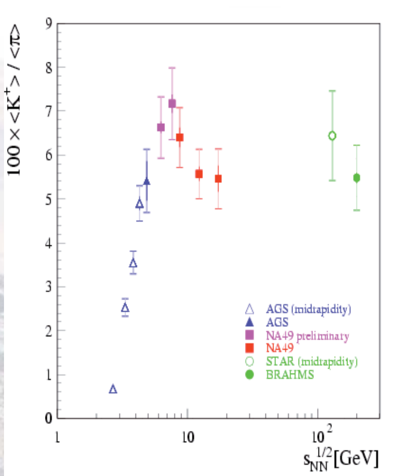


J. C., B. Kämpfer, P. Steinberg and S. Wheaton, Journal of Physics G30 S595-S598 (2004).

The NA49 Collaboration has recently performed a series of measurements of Pb-Pb collisions at 20, 30, 40, 80 and 158 AGeV beam energies . When these results are combined with measurements at lower beam energies from the AGS they reveal an unusually sharp variation with beam energy in the $\Lambda/\langle\pi\rangle$, with $\langle\pi\rangle \equiv 3/2(\pi^+ + \pi^-)$, and K^+/π^+ ratios. Such a strong variation with energy does not occur in pp collisions and therefore indicates a major difference in heavy-ion collisions. This transition has been referred as the “horn”.

The Elephant in the Room

Friese
Dinkelaker
Blume
Speltz



Difficult to avoid, Hard to Model

→ But no unambiguous corroborating evidence

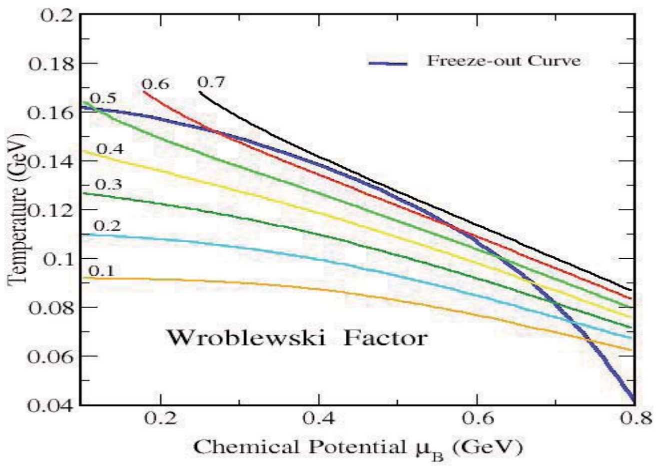
Strangeness in Heavy Ion Collisions
vs
Strangeness in pp - collisions

Use the Wroblewski factor

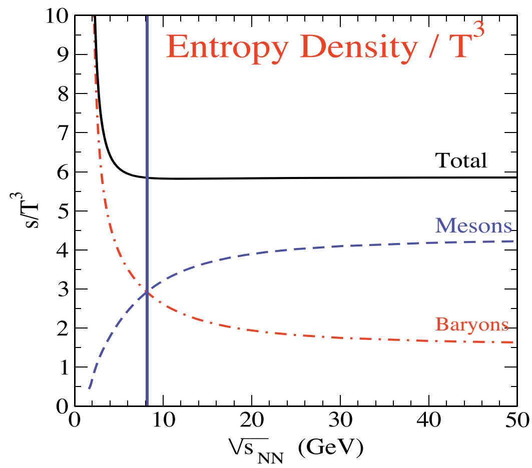
$$\lambda_s = \frac{2\langle s\bar{s}\rangle}{\langle u\bar{u}\rangle + \langle d\bar{d}\rangle}$$

This is determined by the number of **newly** created quark – anti-quark pairs and **before** strong decays, i.e. before ρ's and Δ's decay.

Limiting values :
$\lambda_s = 1$ all quark pairs are equally abundant, SU(3) symmetry.
$\lambda_s = 0$ no strange quark pairs.

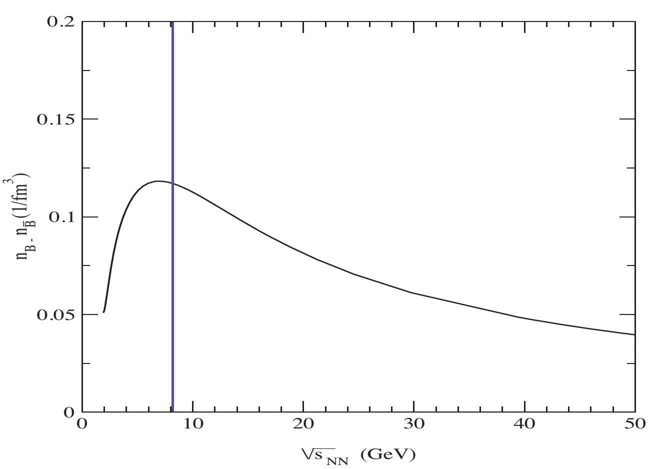


Statistical Model | Strangeness | E_T/N_{ch} vs. E/N | The Horn in the K^+/π^+ Ratio



J. C., H. Oeschler, K. Redlich and S. Wheaton, Physics Letters B615 (2005) 50-54.

Statistical Model | Strangeness | E_T/N_{ch} vs. E/N | The Horn in the K^+/π^+ Ratio



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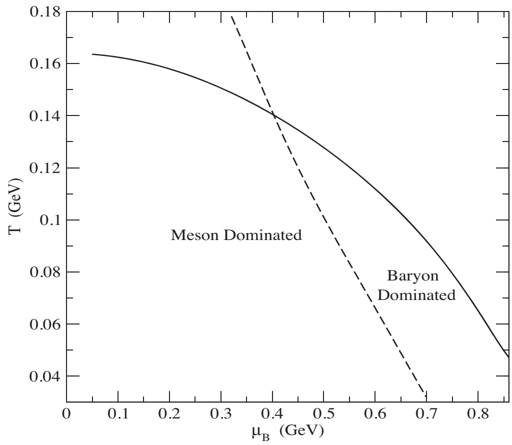

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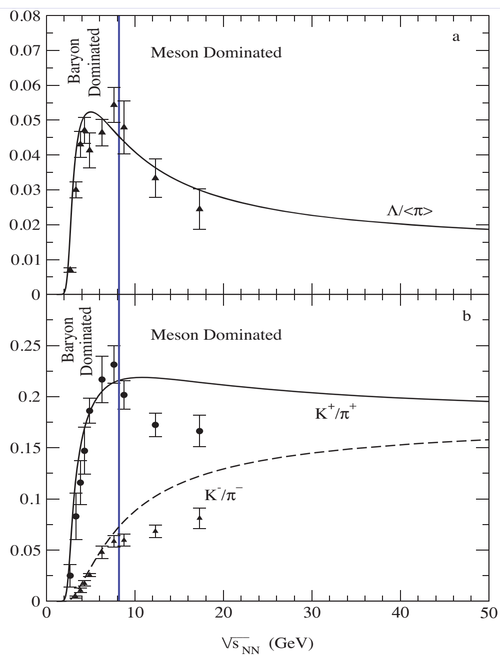
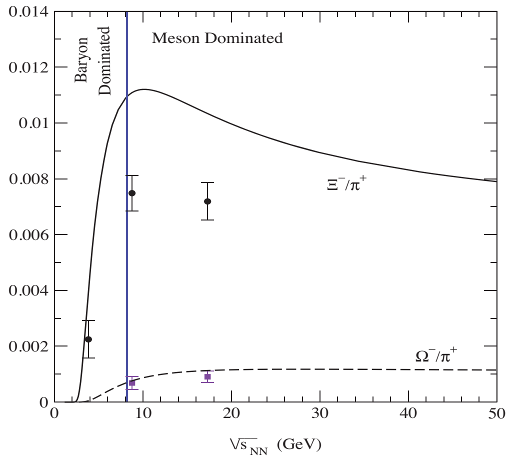


J. C., H. Oeschler, K. Redlich and S. Wheaton, Physics Letters B615 (2005) 50-54.



J. C., H. Oeschler, K. Redlich and S. Wheaton, Physics Letters B615 (2005) 50-54.

Statistical Model Strangeness E_T/N_{ch} vs. E/N The Horn in the K^+/π^+ Ratio

Maxima in Particle Ratios predicted by the Thermal Model.

Ratio	Maximum at $\sqrt{s_{NN}}$ (GeV)	Maximum Value
$\Lambda/\langle\pi\rangle$	5.1	0.052
Ξ^-/π^+	10.2	0.011
K^+/π^+	10.8	0.22
Ω^-/π^+	27	0.0012

J. C., H. Oeschler, K. Redlich and S. Wheaton, Physics Letters B615 (2005) 50-54.

The Thermal-Statistical Model for Particle Production II.

J. Cleymans

25 July 2008 / Dubna, Russia

Hydrodynamic Flow Excluded Volume Corrections Canonical Corrections Transverse Momentum Distributions Rapidity Distributi

Outline

Hydrodynamic Flow.

Cooper-Frye formula

From

$$N = \int j^\mu d\sigma_\mu \text{ and } j^\mu = \int d^3p \frac{p^\mu}{E} f(r,p,t)$$

obtain

$$E\frac{dN}{d^3p} = \int p^\mu d\sigma_\mu \;\; f(r,p,t)$$

The number of particles of type i is determined by:

$$E\frac{dN_i}{d^3p} = \frac{g_i}{(2\pi)^3}\int d\sigma_\mu p^\mu \exp\left(-\frac{p^\mu u_\mu}{T} + \frac{\mu_i}{T}\right)$$

Integrating this over all momenta

$$N_i = \frac{g_i}{(2\pi)^3}\int d\sigma_\mu \int \frac{d^3p}{E} p^\mu \exp\left(-\frac{p^\mu u_\mu}{T} + \frac{\mu_i}{T}\right)$$

or

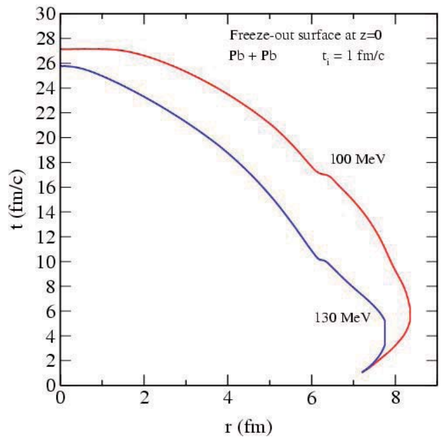
$$N_i = \int d\sigma_\mu u^\mu n_i(T,\mu)$$

If the temperature and chemical potential are unique along the freeze-out curve

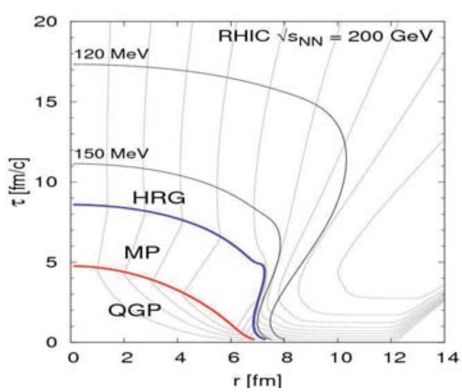
Particle Yield:

$N_i = n_i(T,\mu)\int d\sigma_\mu u^\mu$

i.e. integrated (4π) multiplicities are the same as for a single fireball at rest (apart from the volume).



J. Cleymans, K. Redlich, D.K. Srivastava
Phys. Rev. C55 (1997) 1431



K. Eskola, H. Honkanen, H.Niemi, P.V. Ruuskanen, S.S. Räsänen, hep-ph/0506049

Full Hydrodynamic Flow

Bjorken scaling + Transverse expansion after integration over m_T

$$\left(\frac{dN_i}{dy}\right)_{y=0} = \frac{g}{\pi}\int_\sigma r\,dr\,\tau_F(r)$$
$$\left\{\cosh(y_T) - \left(\frac{\partial\tau_F}{\partial r}\right)\sinh(y_T)\right\} m_i^2 T K_2\left(\frac{m_i}{T}\right)$$

$$\text{Consequence}: \frac{dN_i/dy}{dN_j/dy} = \frac{N_i^0}{N_j^0}$$

Effects of hydrodynamic flow cancel out in ratio.

Excluded Volume Corrections.

$$Z = \exp\left\{V\int\frac{d^3p}{(2\pi)^3}e^{-\frac{E}{T}+\frac{\mu}{T}}\right\}$$
$$= \sum_{N=0}^{\infty}\frac{V^N}{N!}e^{\mu N/T}\left[\int\frac{d^3p}{(2\pi)^3}e^{-\frac{E}{T}}\right]^N$$

with excluded volume corrections

$$Z \rightarrow \sum_{N=0}^{\infty}\frac{(V-V_0N)^N}{N!}e^{\mu N/T}$$
$$\left[\int\frac{d^3p}{(2\pi)^3}e^{-\frac{E}{T}}\right]^N \theta(V-V_0N)$$

Excluded Volume Corrections.

It is more convenient to consider these corrections in the pressure ensemble:

$$Z_p \equiv \int_0^\infty dV\ e^{-PV/T} \sum_{N=0}^{\infty} \frac{V^N}{N!} e^{\mu N/T} \left[\int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} \right]^N$$

$$\begin{aligned} Z_p \quad \rightarrow \quad & \sum_{N=0}^{\infty} \int_0^\infty dV\ e^{-PV/T} \\ & \frac{(V - V_0 N)^N}{N!} e^{\mu N/T} \\ & \left[\int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} \right]^N \theta(V - V_0 N) \end{aligned}$$

introduce $x \equiv V - V_0 N$.

Excluded Volume Corrections.

$$\begin{aligned} Z_p \quad = \quad & \sum_{N=0}^{\infty} \int_0^\infty dx\ e^{-Px/T} \\ & \frac{x^N}{N!} e^{-PV_0 N/T} e^{\mu N/T} \\ & \left[\int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} \right]^N \end{aligned}$$

a new variable $\bar{\mu} \equiv \mu - PV_0$

Excluded Volume Corrections.

$$Z_p \rightarrow \sum_{N=0}^{\infty} \int_0^{\infty} dx\ e^{-Px/T} \frac{x^N}{N!} e^{\bar{\mu}N/T} \left[\int \frac{d^3p}{(2\pi)^3} e^{-\frac{E}{T}} \right]^N$$

which is the original partition function with the replacement

$$\boxed{\bar{\mu} = \mu - P\ V_0}$$

Excluded Volume Corrections.

The particle number density now becomes:

$$\begin{aligned} n &= \frac{T}{V} \frac{\partial}{\partial \mu} \ln\ Z \\ &= \frac{T}{V} \frac{\partial \bar{\mu}}{\partial \mu} \frac{\partial}{\partial \bar{\mu}} \ln\ Z \\ &= \frac{\partial \bar{\mu}}{\partial \mu} n_0 \\ &= [1 - V_0\ n]\, n_0 \end{aligned}$$

$$\boxed{n = \frac{n_0}{1 + V_0 n_0}}$$

Effects Cancel Out in Ratios.

J.C., K. Redlich, H. Satz, E. Suhonen, ZfP C33, 151, (1986)

D.H. Rischke, M.I. Gorenstein, H. Stöcker, W. Greiner, ZfP C51, 485 (1991).

Exact Strangeness Conservation.

For a small system at low temperatures ($T \approx 50$ MeV), e.g. at GSI canonical corrections are necessary.
Instead of

$$N_K \approx \exp - M_K/T$$

one gets

$$N_K \approx \exp -2M_K/T$$

Extra suppression is due to strangeness conservation and lack of a large heat bath. This correction disappears quickly at higher energies and is already small at AGS energies.

Hydrodynamic Flow Excluded Volume Corrections Canonical Corrections Transverse Momentum Distributions Rapidity Distributi

Exact Strangeness Conservation.

$$\boxed{Z = \mathrm{Tr}\, e^{-\frac{H}{T} + \frac{\mu N}{T}}}$$

Insert a Kronecker delta in the trace:

$$\sum_i n_i(S=1) + 2\sum_j n_j(S=2) + 3\sum_k n_k(3) =$$

$$\sum_i \bar{n}_i(S=-1) + 2\sum_j \bar{n}_j(S=-2) + 3\sum_k \bar{n}_k(S=-3)$$

and rewrite it as

$$\delta\left(\sum_i n_i(S=1) + ..., \sum_i \bar{n}_i(S=-1) + ...\right)$$

$$= \frac{1}{2\pi}\int_0^{2\pi} d\phi$$

$$\exp\left(i\phi\sum_i n_i(S=1) + ..\ -i\phi\sum_i \bar{n}_i(S=-1)\right)$$

Exact Strangeness Conservation.

$$Z = \frac{1}{2\pi}\int_0^{2\pi} d\phi \exp \Big\{ Z_1 e^{i\phi} + Z_{-1} e^{-i\phi} + Z_2 e^{2i\phi} + Z_{-2} e^{-2i\phi} + Z_3 e^{3i\phi} + Z_{-3} e^{-3i\phi} \Big\}$$

$$= \frac{1}{2\pi}\int_0^{2\pi} d\phi \exp \Big\{ \sqrt{Z_1 Z_{-1}} \left[\sqrt{\frac{Z_1}{Z_{-1}}} e^{i\phi} + \sqrt{\frac{Z_{-1}}{Z_1}} e^{-i\phi} \right] + \sqrt{Z_2 Z_{-2}} \left[\sqrt{\frac{Z_2}{Z_{-2}}} e^{2i\phi} + \sqrt{\frac{Z_{-2}}{Z_2}} e^{-2i\phi} \right] + \sqrt{Z_3 Z_{-3}} \left[\sqrt{\frac{Z_3}{Z_{-3}}} e^{3i\phi} + \sqrt{\frac{Z_{-3}}{Z_3}} e^{-3i\phi} \right] \Big\}$$

Z_1: sum of all particles with strangeness 1, e.g. K^+

Hydrodynamic Flow Excluded Volume Corrections Canonical Corrections Transverse Momentum Distributions Rapidity Distributi

Exact Strangeness Conservation.

Use

$$\exp\left\{\frac{x}{2}\left(t+\frac{1}{t}\right)\right\} = \sum_{n=-\infty}^{\infty} I_m(x) t^m$$

to obtain

$$Z = \frac{1}{2\pi}\int_0^{2\pi} e^{3im\phi+2in\phi+ip\phi} \sum_{p=-\infty}^{\infty} I_p(x_1) \sum_{n=-\infty}^{\infty} I_n(x_2) \sum_{m=-\infty}^{\infty} I_m(x_3) \, y_1^p y_2^n y_3^m$$

where

$$y_i = \sqrt{\frac{Z_i}{Z_{-i}}} \qquad x_i = 2\sqrt{Z_i Z_{-i}}$$

Exact Strangeness Conservation.

$$Z = \sum_{m=-\infty}^{\infty} \sum_{n=-\infty}^{\infty} I_{3m+2n}(x_1)\, y_1^{-3m-2n} I_n(x_2)\, y_2^n\, I_m(x_3)\, y_3^m$$

Exact Strangeness Conservation.

$$\begin{aligned} Z &= \frac{1}{2\pi}\int_0^{2\pi} d\phi \exp\left\{ Z_1 e^{i\phi} + Z_{-1} e^{-i\phi} \right\} \\ &= \frac{1}{2\pi}\int_0^{2\pi} d\phi \exp\left\{ \sqrt{Z_1 Z_{-1}} \left[\sqrt{\frac{Z_1}{Z_{-1}}} e^{i\phi} + \sqrt{\frac{Z_{-1}}{Z_1}} e^{-i\phi} \right] \right\} \end{aligned}$$

Z_1: sum of all particles with strangeness 1, e.g. K^+
Z_{-1}: sum of all particles with strangeness -1, e.g. Λ

Exact Strangeness Conservation.

Use

$$\exp\left\{\frac{x}{2}\left(t+\frac{1}{t}\right)\right\}=\sum_{n=-\infty}^{\infty} I_m(x)t^m$$

to obtain

$$Z=\frac{1}{2\pi}\int_0^{2\pi} e^{ip\phi}\sum_{p=-\infty}^{\infty} I_p(x_1)y_1^p$$

where

$$y_1 = \sqrt{\frac{Z_1}{Z_{-1}}} \qquad x_1 = 2\sqrt{Z_1 Z_{-1}}$$

$$\boxed{Z = I_0(x_1)}$$

Hydrodynamic Flow Excluded Volume Corrections Canonical Corrections Transverse Momentum Distributions Rapidity Distributi

Exact Strangeness Conservation.

In more detail, e.g. the multiplicity of K^+

$$N_{K^+} = \frac{T}{Z}\frac{\partial I_0(x_1)}{\partial \mu_{K^+}}\Bigg|_{\mu_{K^+}=0}$$

Use

$$\frac{d}{dz}I_0(z) = I_1(z)$$

Exact Strangeness Conservation.

$$\begin{aligned} N_{K^+} &= \frac{T}{Z}\frac{\partial}{\partial \mu_{K^+}} I_0(x_1) \\ &= \frac{T}{I_0(x_1)} I_1(x_1) \frac{\partial x_1}{\partial \mu_{K^+}} \\ &= \frac{T}{I_0(x_1)} I_1(x_1) \frac{\partial 2\sqrt{Z_1 Z_{-1}}}{\partial \mu_{K^+}} \\ &= \frac{I_1(x_1)}{I_0(x_1)} \sqrt{\frac{Z_{-1}}{Z_1}} N^0_{K^+} \end{aligned}$$

where $N^0_{K^+}$ refers to the "unmodified" kaon multiplicity.

Hydrodynamic Flow Excluded Volume Corrections Canonical Corrections Transverse Momentum Distributions Rapidity Distributi

Exact Strangeness Conservation.

In the small volume limit this becomes

$$\lim_{z\to 0} I_0(z) = 1$$

and

$$\lim_{z\to 0} I_1(z) = \frac{z}{2}$$

$$\begin{aligned} \lim_{V\to 0} &= N^0_{K^+} Z_{-1} \\ \lim &= N^0_{K^+} Z_{-1} \\ &= N^0_{K^+} \left[N^0_{K^-} + N^0_{\Lambda} + \cdots \right] \end{aligned}$$

i.e., the particle multiplicity is

- proportional to V^2, and not V^1.
- proportional to $\exp(-2m_K/T)$ or to $\exp(-(m_K + m_\Lambda)/T)$ and not simply $\exp(-m_K/T)$, i.e. there is additional suppression of strange particles.

Exact Strangeness Conservation.

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Hydrodynamic Flow Excluded Volume Corrections Canonical Corrections Transverse Momentum Distributions Rapidity Distributi

Exact Strangeness Conservation

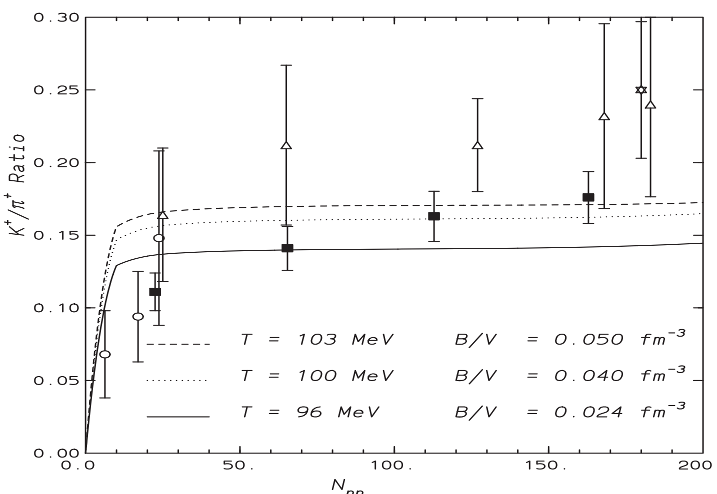


Figure 1

Transverse Momentum in the Thermal Model

$$E_i\frac{dN_i}{d^3p} = \frac{g_i}{(2\pi)^3}VE_i \exp\left(-\frac{E_i}{T}+\frac{\mu_i}{T}\right)$$

$$\frac{dN_i}{dy\ m_T dm_T} = \frac{g_i}{(2\pi)^2}Vm_T\cosh y\ e^{-\frac{m_T}{T}\cosh y+\frac{\mu_i}{T}}$$

$$\frac{dN_i}{m_T dm_T} = \frac{g_i}{2\pi^2}Vm_T\ K_1\left(\frac{m_T}{T}\right)e^{\frac{\mu_i}{T}}$$

For large values of m_T :

$$\boxed{\frac{dN_i}{dm_T} = \frac{g_i}{2\pi^2}Vm_T^{3/2}\ \exp\left(-\frac{m_T}{T}\right)\sqrt{\frac{T\pi}{2}}e^{\frac{\mu_i}{T}}}$$

SCALING in m_T

Rapidity Distribution in the Thermal Model

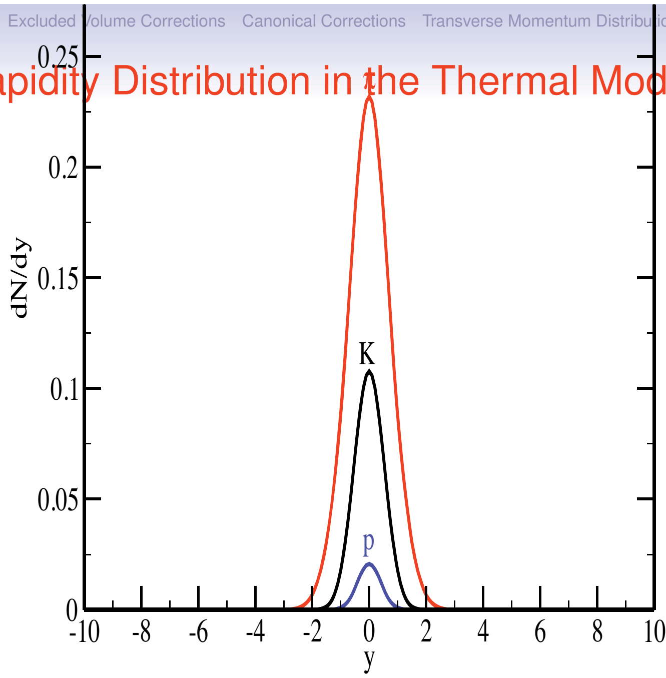
$$\frac{dN_i}{dy\ m_T dm_T} = \frac{g_i}{(2\pi)^2}Vm_T\cosh y\ e^{-\frac{m_T}{T}\cosh y+\frac{\mu_i}{T}}$$

$$\frac{dN_i}{dy} = \frac{g_iV}{2\pi^2}\left[\frac{2T^3}{\cosh^2 y}+\frac{2mT^2}{\cosh y}+m^2T\right]e^{\frac{\mu_i}{T}}$$

$$e^{-\frac{m}{T}\cosh y}$$

Narrow Distribution in Rapidity Approximately Gaussian

Rapidity Distribution in the Thermal Model



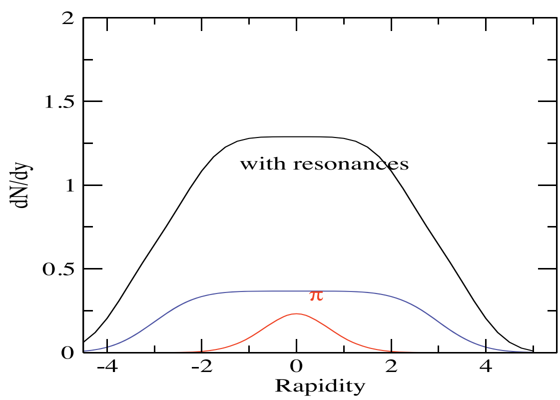
Superposition of Fireballs.

$$\frac{dN_i}{dy} = \int_{-Y}^{Y} dY_{FB}\, \rho(Y_{FB}) \frac{dN_i^0}{dy}(y - Y_{FB})$$

where

$$\frac{dN_i^0}{dy} = \frac{g_i V}{2\pi^2}\left[\frac{2T^3}{\cosh^2(y-Y_{FB})} + \frac{2mT^2}{\cosh(y-Y_{FB})} + m^2 T\right] e^{\frac{\mu_i}{T}}$$

$$e^{-\frac{m}{T}\cosh(y-Y_{FB})}$$



Hydrodynamic Flow Excluded Volume Corrections Canonical Corrections Transverse Momentum Distributions Rapidity Distributi

Superposition of Fireballs.

$$n_i = \int_{-\infty}^{\infty} dy \int_{-Y}^{Y} dY_{FB}\, \rho(Y_{FB}) \frac{dn_i^0}{dy}(y - Y_{FB})$$

$$n_i = \int_{-Y}^{Y} dY_{FB}\, \rho(Y_{FB}) \int_{-\infty}^{\infty} dy \frac{dn_i^0}{dy}(y - Y_{FB})$$

$$n_i = g_i\, V \frac{1}{2\pi^2} T m_i^2 K_2\left(\frac{m_i}{T}\right) e^{\frac{\mu_i}{T}} \int_{-Y}^{Y} dY_{FB}\, \rho(Y_{FB})$$

Equivalent to changing the volume V.

Superposition of Fireballs.

$$\frac{n_i}{n_j} = \frac{\int_{-\infty}^{\infty} dy \int_{-Y}^{Y} dY_{FB}\, \rho(Y_{FB}) \frac{dn_i^0}{dy}(y - Y_{FB})}{\int_{-\infty}^{\infty} dy \int_{-Y}^{Y} dY_{FB}\, \rho(Y_{FB}) \frac{dn_i^0}{dy}(y - Y_{FB})}$$

$$\frac{n_i}{n_j} = \frac{n_i^0}{n_j^0} \frac{\int_{-Y}^{Y} dY_{FB}\, \rho(Y_{FB})}{\int_{-Y}^{Y} dY_{FB}\, \rho(Y_{FB})}$$

$$\frac{n_i}{n_j} = \frac{n_i^0}{n_j^0} = \frac{m_i^2}{m_j^2} \frac{K_2\left(m_i/T\right)}{K_2\left(m_j/T\right)} e^{(\mu_i - \mu_j)/T}$$

Effects Cancel Out in Ratios.