

## Color Glass Condensate and initial stages of heavy-ion collisions

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**Abstract.** We introduce the concept of Color Glass Condensate, that describes the wave-function of a nucleon or nucleus at high energy. Then, we explain the relevance of this effective theory in the calculation of particle production in heavy ion collisions, and we show how it can be used in order to make predictions for the initial stages of these collisions – in particular in the task of providing initial conditions for hydrodynamical simulations.

### References

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# Color Glass Condensate and and initial stages of heavy-ion collisions

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**Reminder on hydrodynamics**

**Equations of hydrodynamics :**

$$\partial_\mu T^{\mu\nu} = 0 \text{ (energy-momentum conservation)}$$

$$\partial_\mu J_B^\mu = 0 \text{ (baryon number conservation)}$$

- These equations contain only **first order time derivatives**
- Required initial conditions :

$$T^{\mu\nu}(\tau = \tau_0, \eta, \vec{x}_\perp), J_B^\mu(\tau = \tau_0, \eta, \vec{x}_\perp)$$

**Additional inputs :**

Equation of state:  $p = f(\epsilon)$   
Transport coefficients:  $\eta, \zeta, \dots$



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## Initial correlations and hydrodynamics

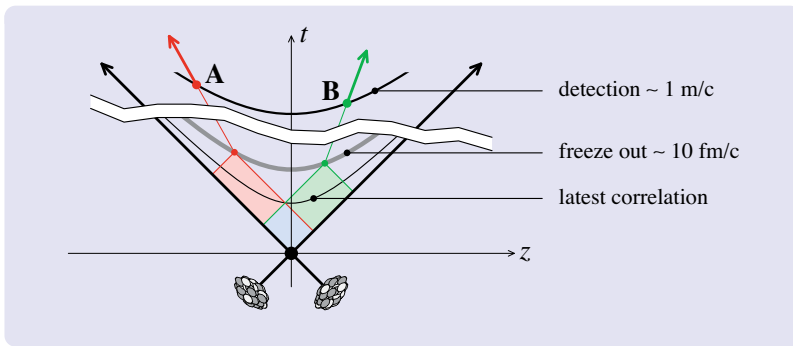
- The equations of hydrodynamics are **non-linear**. Therefore, solving hydro evolution for event averaged initial conditions is not the same as solving hydro event-by-event, and averaging observables at the end :

$$\text{HYDRO} \left[ \left\langle T_{\text{init}}^{\mu\nu} \right\rangle \right] \neq \left\langle \text{HYDRO} \left[ T_{\text{init}}^{\mu\nu} \right] \right\rangle$$

- To study hydrodynamics event by event, one needs an event generator for  $T^{\mu\nu}(\tau_0, \eta, \vec{x}_\perp)$
- To achieve this, it is not sufficient to know the average  $\left\langle T^{\mu\nu}(\tau_0, \eta, \vec{x}_\perp) \right\rangle$ . We also need correlations :

$$\begin{aligned} & \left\langle T^{\mu_1\nu_1}(\tau_0, \eta_1, \vec{x}_{1\perp}) T^{\mu_2\nu_2}(\tau_0, \eta_2, \vec{x}_{2\perp}) \right\rangle \\ & \left\langle T^{\mu_1\nu_1}(\tau_0, \eta_1, \vec{x}_{1\perp}) T^{\mu_2\nu_2}(\tau_0, \eta_2, \vec{x}_{2\perp}) T^{\mu_3\nu_3}(\tau_0, \eta_3, \vec{x}_{3\perp}) \right\rangle \\ & \dots \end{aligned}$$

## Long range rapidity correlations probe early dynamics



### Long range rapidity correlations are created early

From causality, the latest time at which a correlation between two particles can be created is :

$$t_{\text{correlation}} \leq t_{\text{freeze out}} e^{-\frac{1}{2}|y_A - y_B|}$$

Example:  $t_{\text{freeze out}} = 10 \text{ fm/c}$ ,  $|y_A - y_B| = 6$  :  $t_{\text{correlation}} \leq 0.5 \text{ fm/c}$



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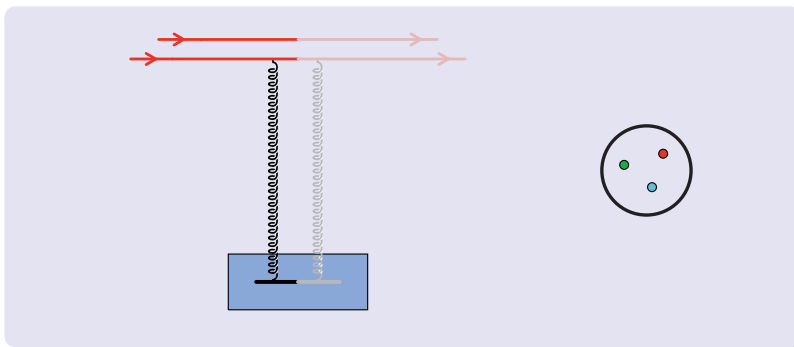
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Gluon saturation



- consider a hadron or nucleus probed via gluon exchange
- at low energy, only valence quarks are present in the hadron wave function

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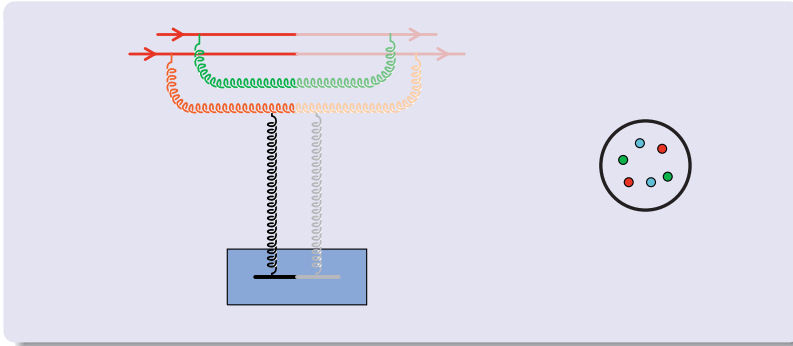
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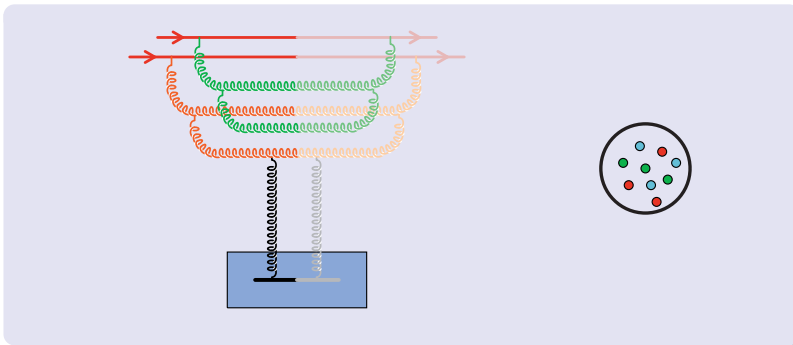
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## Gluon saturation



- when energy increases, new partons are emitted
- the emission probability is  $\alpha_s \int \frac{dx}{x} \sim \alpha_s \ln\left(\frac{1}{x}\right)$ , with  $x$  the longitudinal momentum fraction of the gluon
- at small- $x$  (i.e. high energy), these logs need to be resummed

## Gluon saturation



- as long as the density of constituents remains small, the evolution is **linear**: the number of partons produced at a given step is proportional to the number of partons at the previous step (BFKL)

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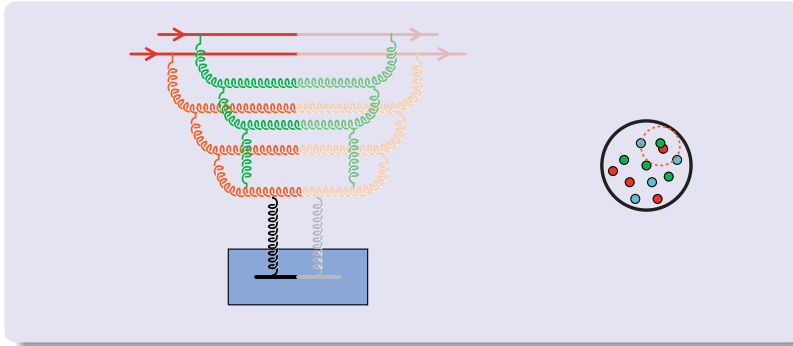
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## Gluon saturation



- eventually, the partons start overlapping in phase-space
- **parton recombination** becomes favorable
- after this point, the evolution is **non-linear**:  
the number of partons created at a given step depends non-linearly on the number of partons present previously

## Criterion for gluon recombination

Gribov, Levin, Ryskin (1983)

Number of gluons per unit area :

$$\rho \sim \frac{xG_A(x, Q^2)}{\pi R_A^2}$$

Recombination cross-section :

$$\sigma_{gg \rightarrow g} \sim \frac{\alpha_s}{Q^2}$$

Recombination happens if  $\rho \sigma_{gg \rightarrow g} \gtrsim 1$ , i.e.  $Q^2 \lesssim Q_s^2$ , with :

$$Q_s^2 \sim \frac{\alpha_s xG_A(x, Q_s^2)}{\pi R_A^2} \sim A^{1/3} \frac{1}{x^{0.3}}$$

Note: At a given energy, the saturation scale is larger for a nucleus (for  $A = 200$ ,  $A^{1/3} \approx 6$ )

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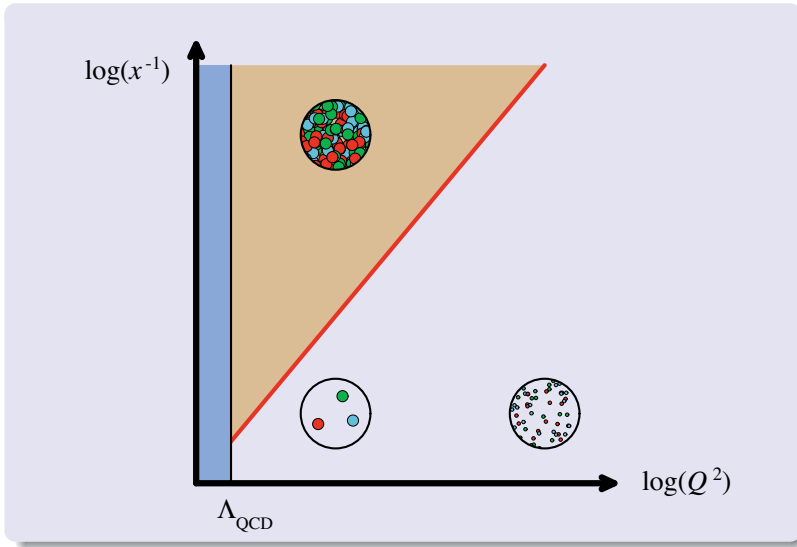
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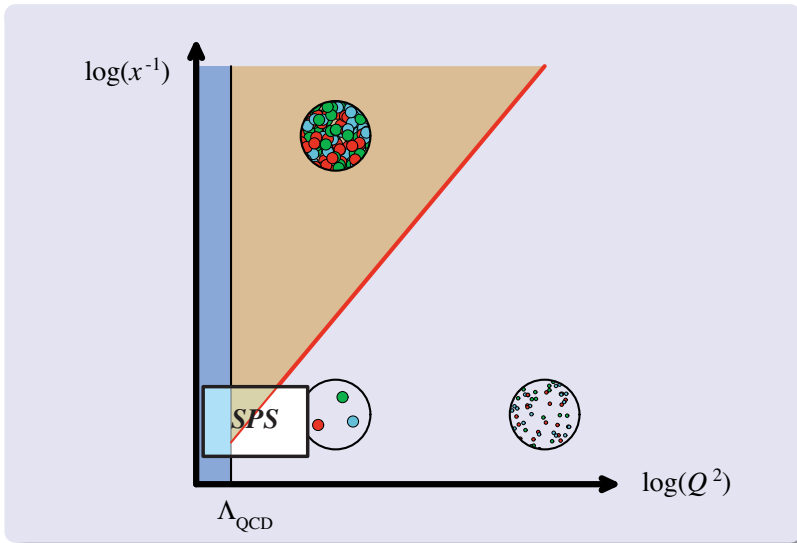
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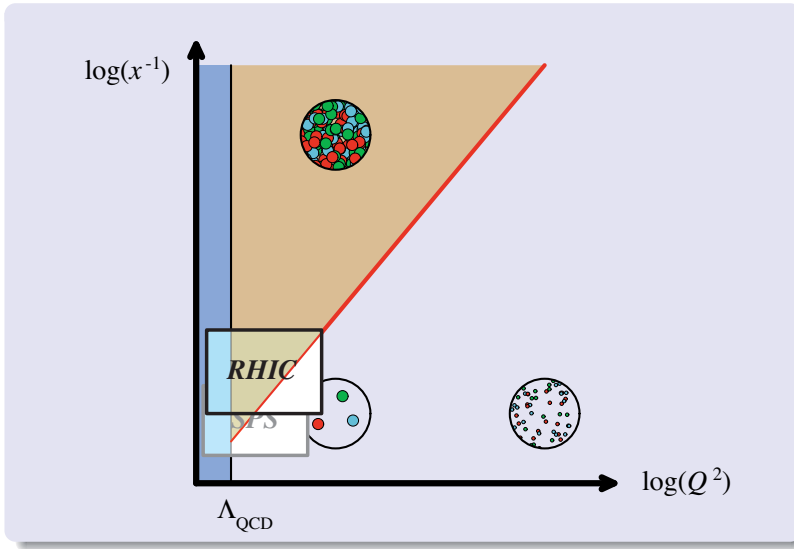
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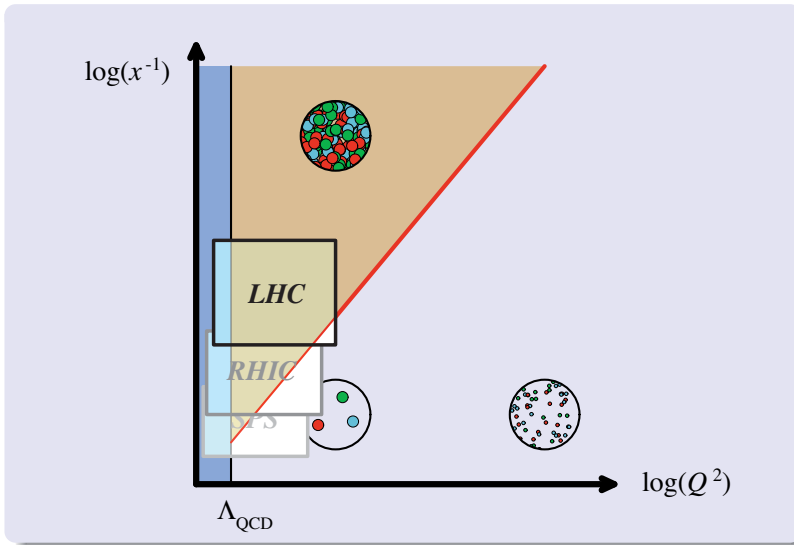
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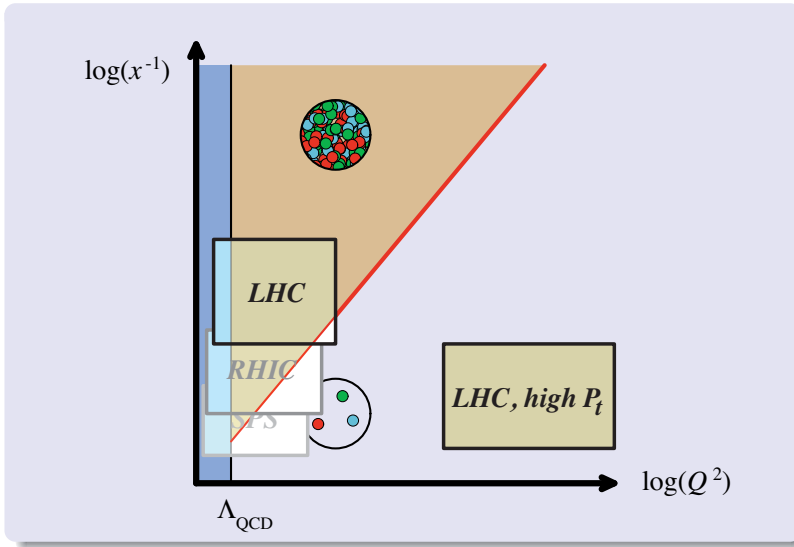
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## CGC: Degrees of freedom

CGC = effective theory of small  $x$  gluons

- The fast partons (large  $x$ ) are frozen by time dilation  
▷ described as **static color sources** on the light-cone :

$$J^\mu = \delta^{\mu+} \delta(x^-) \rho(\vec{x}_\perp) \quad (x^- \equiv (t - z)/\sqrt{2})$$

- Slow partons (small  $x$ ) cannot be considered static over the time-scales of the collision process  
▷ they must be treated as standard gauge fields

Eikonal coupling to the current  $J^\mu$  :  $A_\mu J^\mu$

- The color sources  $\rho$  are **random**, and described by a **distribution functional**  $W_Y[\rho]$ , with  $Y$  the rapidity that separates “soft” and “hard”

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## CGC: renormalization group evolution

### Evolution equation (JIMWLK) :

$$\frac{\partial W_Y}{\partial Y} = \mathcal{H} W_Y$$

$$\mathcal{H} = \frac{1}{2} \int_{\vec{x}_\perp, \vec{y}_\perp} \frac{\delta}{\delta \tilde{\mathcal{A}}^+(\epsilon, \vec{y}_\perp)} \eta(\vec{x}_\perp, \vec{y}_\perp) \frac{\delta}{\delta \tilde{\mathcal{A}}^+(\epsilon, \vec{x}_\perp)}$$

where  $-\partial_\perp^2 \tilde{\mathcal{A}}^+(\epsilon, \vec{x}_\perp) = \rho(\epsilon, \vec{x}_\perp)$

- $\eta(\vec{x}_\perp, \vec{y}_\perp)$  is a non-linear functional of  $\rho$
- This evolution equation resums all the powers of  $\alpha_s \ln(1/x)$  and of  $Q_s/p_\perp$  that arise in loop corrections
- This equation simplifies into the BFKL equation when the source  $\rho$  is small (one can expand  $\eta$  in powers of  $\rho$ )

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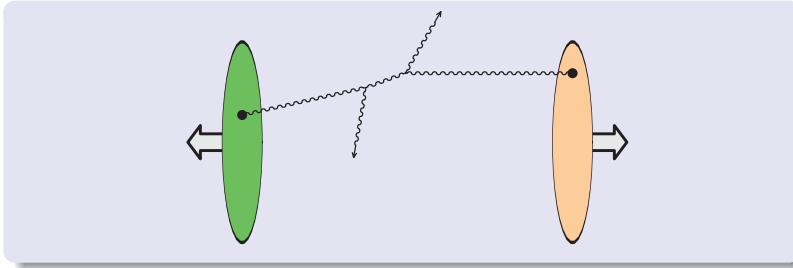
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## Power counting

$$J^\mu \equiv \delta^{\mu+} \rho_1(x^-, \vec{x}_\perp) + \delta^{\mu-} \rho_2(x^+, \vec{x}_\perp)$$

$$S = \underbrace{-\frac{1}{2} \int d^4x \operatorname{tr} F_{\mu\nu} F^{\mu\nu}}_{S_{YM}} + \int d^4x J^\mu A_\mu$$

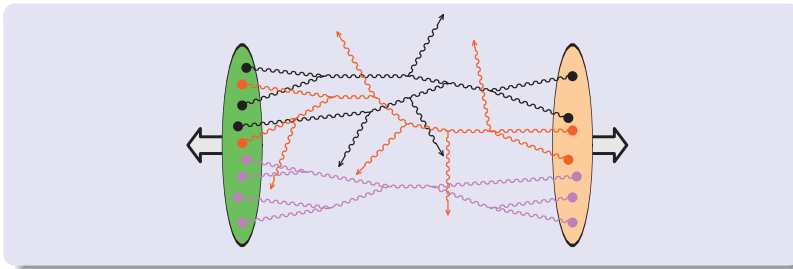


- Dilute regime : one parton in each projectile interact

## Power counting

$$J^\mu \equiv \delta^{\mu+} \rho_1(x^-, \vec{x}_\perp) + \delta^{\mu-} \rho_2(x^+, \vec{x}_\perp)$$

$$S = \underbrace{-\frac{1}{2} \int d^4x \operatorname{tr} F_{\mu\nu} F^{\mu\nu}}_{S_{YM}} + \int d^4x J^\mu A_\mu$$



- Dilute regime : one parton in each projectile interact
- Dense regime : multiparton processes become crucial

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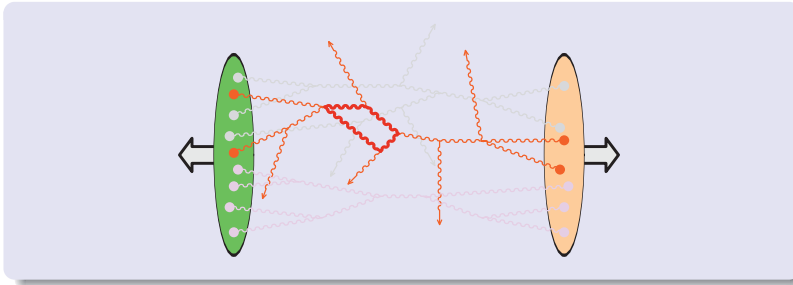
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## Power counting

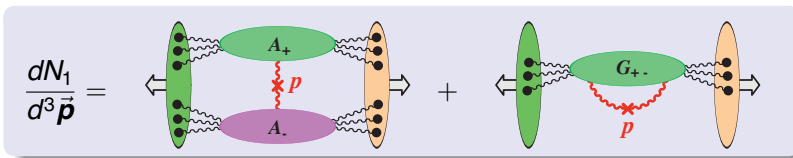


- In the **saturated regime**, the sources are of order  $1/g$  (because  $\langle \rho\rho \rangle \sim$  occupation number  $\sim 1/\alpha_s$ )
- Order of a **connected diagram** :

$$\frac{1}{g^2} g^{\# \text{ produced gluons}} g^{2(\# \text{ loops})}$$

## Diagrammatic expansion of $dN_1/d^3p$

- The single inclusive spectrum has a simple diagrammatic representation :



- There are only connected graphs (AGK cancellation)
- Perturbative expansion in the saturated regime :

$$\frac{dN_1}{d^3\vec{p}} = \frac{1}{g^2} \left[ \underbrace{c_0}_{\text{LO}} + \underbrace{c_1 g^2}_{\text{NLO}} + \underbrace{c_2 g^4}_{\text{NNLO}} + \dots \right]$$

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## Expression in terms of classical fields at LO

### Gluon spectrum at LO :

$$\frac{dN_1}{d^3\vec{p}} \Big|_{LO} \propto \int_{x,y} e^{ip \cdot (x-y)} \square_x \square_y \sum_{\lambda} \epsilon_{\mu}^{(\lambda)} \epsilon_{\nu}^{(\lambda)} \mathcal{A}^{\mu}(x) \mathcal{A}^{\nu}(y)$$

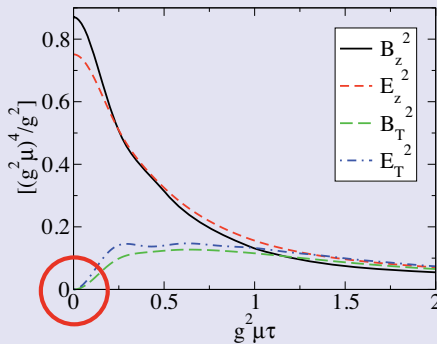
- $\mathcal{A}$  obeys the classical EOM :  $\frac{\delta \mathcal{S}_{YM}}{\delta \mathcal{A}} + \mathbf{J} = 0$
- The boundary conditions are very simple:

$$\lim_{x^0 \rightarrow -\infty} \mathcal{A}(x) = 0$$

## Initial classical fields

Lappi, McLerran (2006)

- Immediately after the collision, the chromo- $\vec{E}$  and  $\vec{B}$  fields are purely longitudinal :



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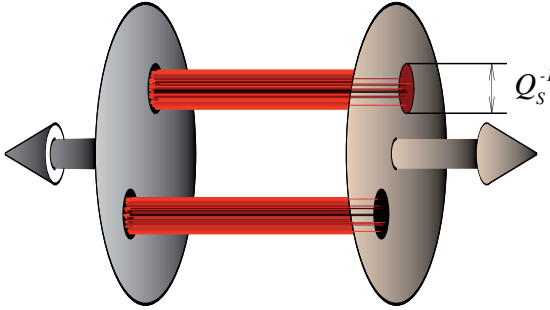


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## Initial classical fields

- The initial chromo- $\vec{E}$  and  $\vec{B}$  fields form longitudinal “flux tubes” extending between the projectiles:

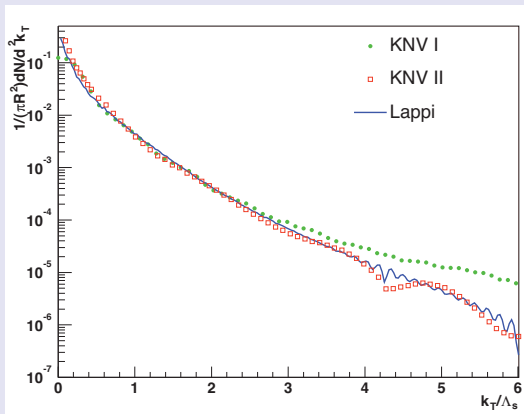


- The color correlation length in the transverse plane is  $Q_s^{-1}$   
 ▷ flux tubes of diameter  $Q_s^{-1}$ , filling up the transverse area

## Single gluon spectrum at LO

Krasnitz, Nara, Venugopalan (1999 – 2001), Lappi (2003)

- No analytic solution for the Yang-Mills equations, but straightforward numerically



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## What is factorization ?

- The naive perturbative expansion of  $dN_1/d^3\vec{p}$ ,

$$\frac{dN}{d^3\vec{p}} = \frac{1}{g^2} \left[ c_0 + c_1 g^2 + c_2 g^4 + \dots \right],$$

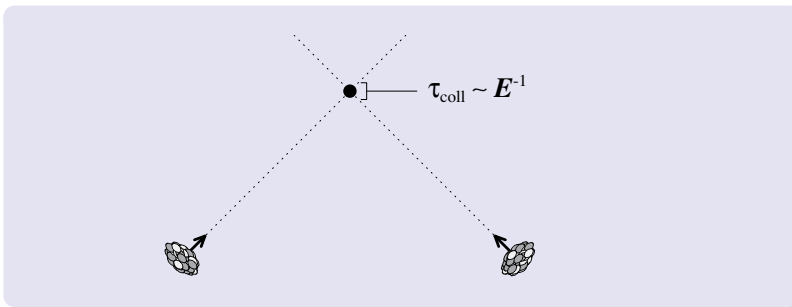
assumes that the coefficients  $c_n$  are of order one

- This assumption is upset by large logarithms of  $1/x_{1,2}$  :

$$\begin{aligned} c_1 &= d_{10} + d_{11} \ln\left(\frac{1}{x_{1,2}}\right) \\ c_2 &= d_{20} + d_{21} \ln\left(\frac{1}{x_{1,2}}\right) + \underbrace{d_{22} \ln^2\left(\frac{1}{x_{1,2}}\right)}_{\text{Leading Log terms}} \end{aligned}$$

- Factorizability:** the logarithms must be **universal** and resumable into functionals that depend only on the projectiles being collided

## Why factorization works: causality



- The duration of the collision is very short:  $\tau_{\text{coll}} \sim E^{-1}$

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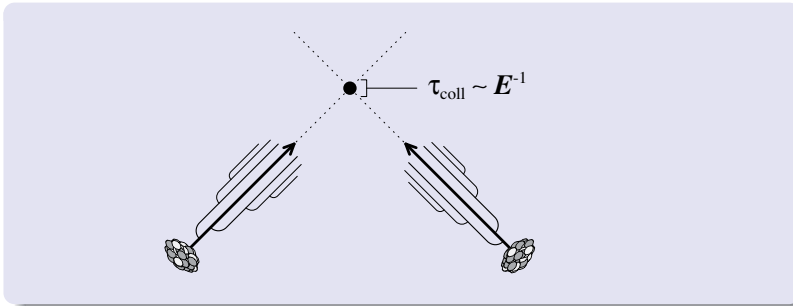
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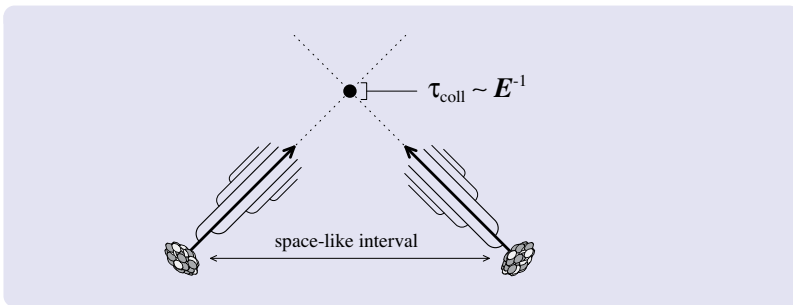


## Why factorization works: causality



- The duration of the collision is very short:  $\tau_{coll} \sim E^{-1}$
- The logarithms we want to resum arise from the radiation of soft gluons, which takes a long time
  - ▷ it must happen (long) before the collision

## Why factorization works: causality



- The duration of the collision is very short:  $\tau_{coll} \sim E^{-1}$
- The logarithms we want to resum arise from the radiation of soft gluons, which takes a long time
  - ▷ it must happen (long) before the collision
- The projectiles are not in causal contact before the impact
  - ▷ the logarithms are intrinsic properties of the projectiles, independent of the measured observable

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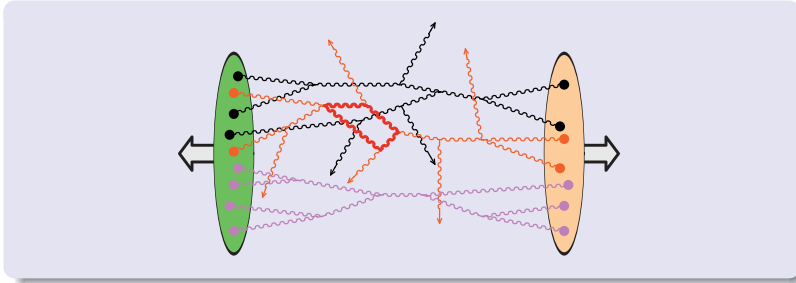
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## Why the proof is complicated: strong fields



- **Procedure:** (i) calculate the 1-loop corrections, (ii) disentangle the logarithms from the finite contributions, (iii) show that the logs can be assigned to the projectiles
- **Problem:** strong fields, analytic calculation not feasible
  - ▷ Take advantage of the retarded nature of the boundary conditions in order to separate the initial state evolution (calculable analytically) from the collision itself (hopeless)

## Factorization in two steps: FG, Lappi, Venugopalan (2008)

I : The NLO gluon spectrum can be written as a perturbation of the initial value of the classical fields on the light-cone :

$$\left. \frac{dN_1}{d^3\vec{p}} \right|_{\text{NLO}} = \left[ \frac{1}{2} \int_{\vec{u}, \vec{v} \in \text{LC}} \mathcal{G}(\vec{u}, \vec{v}) \mathbb{T}_u \mathbb{T}_v + \int_{\vec{u} \in \text{LC}} \beta(\vec{u}) \mathbb{T}_u \right] \left. \frac{dN_1}{d^3\vec{p}} \right|_{\text{LO}}$$

( $\mathbb{T}_u \sim \delta/\delta \mathcal{A}_{\text{initial}}(\mathbf{u})$  ,  $\mathcal{G}, \beta$  are calculable analytically)

II : The operator  $[\dots]$  is related to the JIMWLK Hamiltonian:

$$\frac{1}{2} \int_{\vec{u}, \vec{v} \in \text{LC}} \mathcal{G}(\vec{u}, \vec{v}) \mathbb{T}_u \mathbb{T}_v + \int_{\vec{u} \in \text{LC}} \beta(\vec{u}) \mathbb{T}_u = \log\left(\frac{\Lambda^+}{p^+}\right) \times \mathcal{H}_1 + \log\left(\frac{\Lambda^-}{p^-}\right) \times \mathcal{H}_2 + \text{finite terms}$$

▷ Factorization follows easily



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## Leading Log factorization

- By averaging over all the configurations of the sources in the two projectiles, we get a factorized formula for the resummation of the leading log terms to all orders :

$$\left\langle \frac{dN_1}{d^3\vec{p}} \right\rangle_{\text{LLog}} = \int [D_{\rho_1} D_{\rho_2}] W_{Y_1}[\rho_1] W_{Y_2}[\rho_2] \left. \frac{dN_1}{d^3\vec{p}} \right|_{\text{LO}}$$

with :  $\frac{\partial}{\partial Y} W_Y = \mathcal{H} W$  ,  $Y_1 = \log(\sqrt{s}/p^+)$  ,  $Y_2 = \log(\sqrt{s}/p^-)$

- The distributions  $W[\rho_{1,2}]$  must be evolved up to the rapidity of the produced gluon

## Multigluon spectrum at LO

FG, Lappi, Venugopalan (2008)

- In the saturated regime, the inclusive  $n$ -gluon spectrum at Leading Order is the product of  $n$  1-gluon spectra:

$$\left. \frac{dN_n}{d^3\vec{p}_1 \cdots d^3\vec{p}_n} \right|_{\text{LO}} = \left. \frac{dN_1}{d^3\vec{p}_1} \right|_{\text{LO}} \times \cdots \times \left. \frac{dN_1}{d^3\vec{p}_n} \right|_{\text{LO}}$$

- At LO, in a given configuration of the sources  $\rho_{1,2}$ , the  $n$  gluons are not correlated
- Note: this is true for the bulk ( $p_\perp \lesssim Q_s$ ), but not for the tail of the distribution

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## Multigluon spectrum at NLO

- At NLO, one has again:

$$\begin{aligned} \left. \frac{dN_n}{d^3\vec{p}_1 \cdots d^3\vec{p}_n} \right|_{\text{NLO}} &= \\ &= \left[ \frac{1}{2} \int_{\vec{u}, \vec{v} \in \text{LC}} \mathcal{G}(\vec{u}, \vec{v}) \mathbb{T}_u \mathbb{T}_v + \int_{\vec{u} \in \text{LC}} \beta(\vec{u}) \mathbb{T}_u \right] \left. \frac{dN_n}{d^3\vec{p}_1 \cdots d^3\vec{p}_n} \right|_{\text{LO}} \end{aligned}$$

- Correlations appear at NLO thanks to the operator  $\mathcal{G}(\vec{u}, \vec{v}) \mathbb{T}_u \mathbb{T}_v$ , which can link two different gluons
- Thanks to their universal structure, we can factorize these correlations into the distributions  $W[\rho_{1,2}]$

## Leading Log factorization

### Factorization formula for the $n$ -gluon spectrum

$$\begin{aligned} \left\langle \left. \frac{dN_n}{d^3\vec{p}_1 \cdots d^3\vec{p}_n} \right\rangle_{\text{LLog}} &= \int [D\rho_1 D\rho_2] W[\rho_1] W[\rho_2] \\ &\times \left. \frac{dN_1}{d^3\vec{p}_1} \right|_{\text{LO}} \times \cdots \times \left. \frac{dN_1}{d^3\vec{p}_n} \right|_{\text{LO}} \end{aligned}$$

- This formula tells us that (in the Leading Log approximation) all the correlations arise from the  $W[\rho]$ 's
  - ▷ they pre-exist in the wave-function of the projectiles
- Note: some short range correlations will also arise from splittings in the final state (not taken into account here, because does not come with a  $\ln(s)$ )

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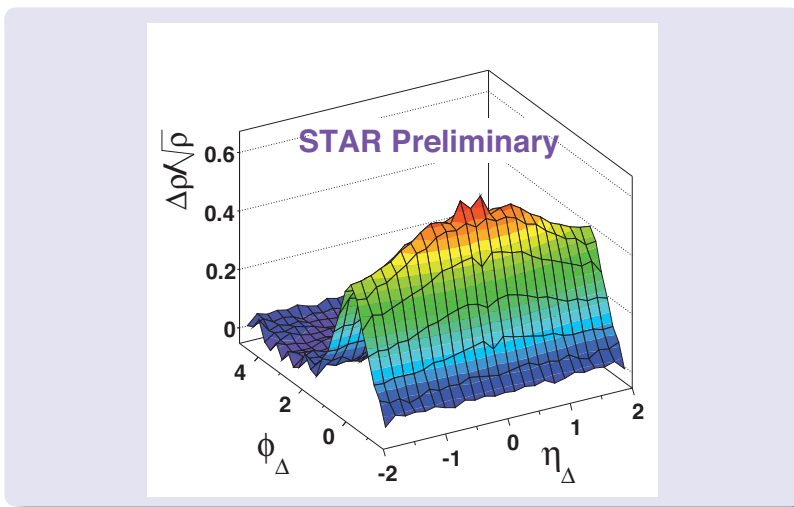
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**2-hadron correlations at RHIC**



- Long range correlation in  $\Delta\eta$  (rapidity)
- Narrow correlation in  $\Delta\phi$  (azimuthal angle)

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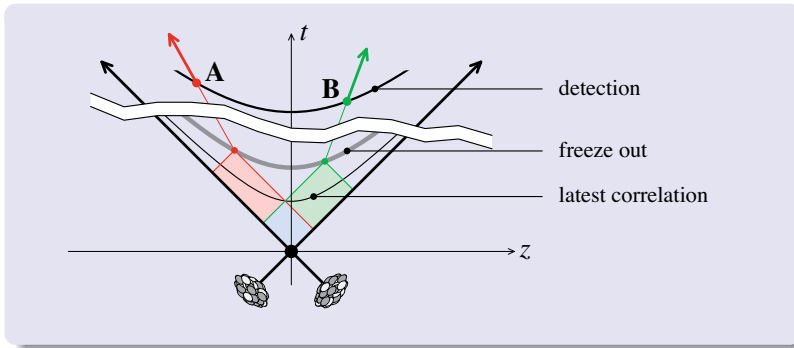
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## Causality constraint



### Long range rapidity correlations are created early

From causality, the latest time at which a correlation between two particles can be created is :

$$t_{\text{correlation}} \leq t_{\text{freeze out}} e^{-\frac{1}{2}|y_A - y_B|}$$

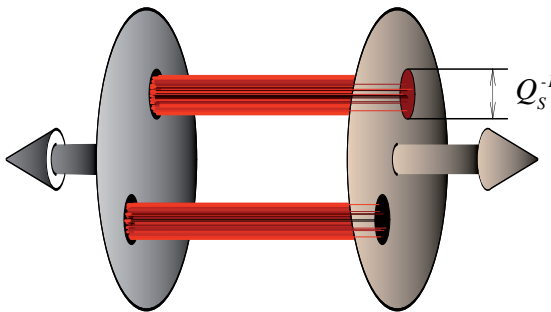
## Glasma flux tubes

Dumitru, FG, McLerran, Venugopalan (2008)

Gavin, McLerran, Moschelli (2008)

Dusling, Fernandez-Fraile, Venugopalan (2009)

- Was there something independent of  $\eta$  at early times?
  - ▷ the chromo- $\vec{E}$  and  $\vec{B}$  fields produced in the collision



- The color correlation length in the transverse plane is  $Q_s^{-1}$ 
  - ▷ flux tubes of diameter  $Q_s^{-1}$ , filling up the transverse area

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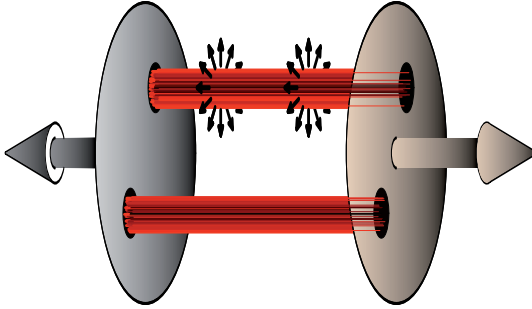
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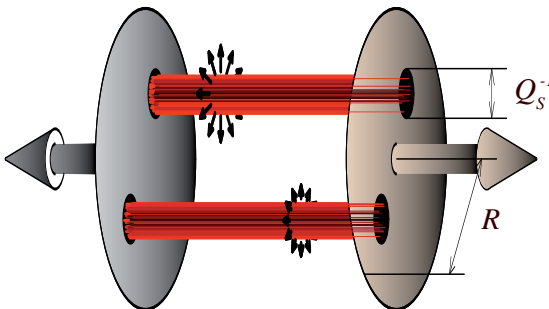
## Glasma flux tubes

- $\eta$ -independent fields lead to long range correlations in the 2-particle spectrum :



## Glasma flux tubes

- $\eta$ -independent fields lead to long range correlations in the 2-particle spectrum :



- Particles emitted by different flux tubes are not correlated
  - ▷  $(RQ_s)^{-2}$  sets the strength of the correlation

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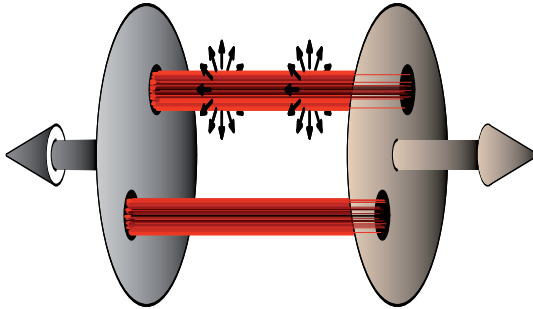
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## Glasma flux tubes

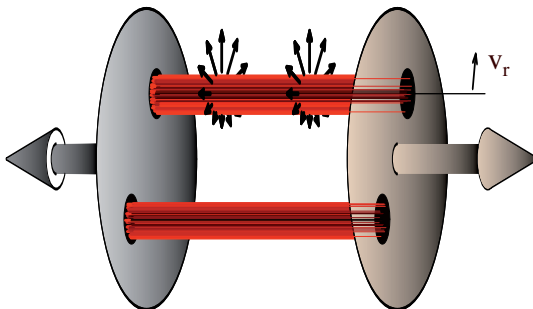
- $\eta$ -independent fields lead to long range correlations in the 2-particle spectrum :



- Particles emitted by different flux tubes are not correlated
  - ▷  $(RQ_s)^{-2}$  sets the strength of the correlation
- At early times, the correlation is flat in  $\Delta\varphi$

## Glasma flux tubes

- $\eta$ -independent fields lead to long range correlations in the 2-particle spectrum :



- Particles emitted by different flux tubes are not correlated
  - ▷  $(RQ_s)^{-2}$  sets the strength of the correlation
- At early times, the correlation is flat in  $\Delta\varphi$   
A collimation in  $\Delta\varphi$  is produced later by radial flow

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## Gluon multiplicity

FG, Lappi, McLerran (2009)

- The combinatorics of color source averages in a single glasma flux tube leads to:

$$\langle N(N-1)\cdots(N-p+1) \rangle - \text{disc. terms} = (p-1)! \langle N \rangle^p$$

- Bose-Einstein distribution
- If one superimposes  $k$  such flux tubes emitting independently:

$$\langle N(N-1)\cdots(N-p+1) \rangle - \text{disc. terms} = (p-1)! \left[ \frac{\langle N \rangle}{k} \right]^p$$

- **Negative binomial distribution** with parameters  $\langle N \rangle, k$
- $k$  is the number of flux tubes:  $k \sim Q_s^2 R^2 \sim \#$  participants
- Experimentally: it seems to work...

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